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On the Study of Multiple-Source Multiple-Destinations Relay Channels with Network Coding

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Abstract

The essential broadcasting feature of radio propagation channels provides an opportunity for multiple nodes to exchange information and work cooperatively, where each node, e.g. mobile sensor or robot, plays both the role of transmitter and receiver. Especially in such machine-to-machine communication scenarios, the network performance and redundancy of information from different providers highly affect the work efficiency of each individual node and the whole system. To address these problems, a relay assisted centralized network model with physical layer network coding implemented in the relay is proposed in this paper. This structure has the advantage of flexible data exchange and the capability to reduce redundancy in information. Its theoretical performance is analyzed by the Diversity Multiplexing Tradeoff (DMT), which proves the proposed model is versatile in reliable and high spectral-efficiency information exchange. Experiments of multiple nodes in a machine-to-machine scenario - Unmanned Aerial Vehicles (UAVs), further reveal its potential in improving efficiency of communication and cooperation.

Index Terms

Network coding, relay, cooperative communication, diversity-multiplexing tradeoff, machine-to-machine, UAV

I. INTRODUCTION

Recent progresses in wireless communications and mobile network technologies have brought on significant changes in the organization and exchange of information [1]–[3]. The open and easy-access

feature of wireless networks offer simultaneous connections among mobile nodes which are scattered remotely, thus overcoming geographical restrictions. These changes create opportunities, e.g. flexible structure, easy access and connection, and also challenges, e.g. each wireless channel suffers from fading and it is easy for mobile nodes to generate and transmit redundant information. Particularly, in some machine-to-machine networks, each node is not only a provider of information, but also a consumer, and such information is utilized for further actions by each individual and the whole system. We name this scenario as a Multiple-Sources Multiple-Destinations (MSMD) network, which can be found in wireless cloud-computing networks, cellular networks, wireless sensor networks, etc.. Therefore, an efficient and effective wireless model for data exchange is essential.

Traditional communication focuses on a point-to-point radio communication channel (or the so called direct transmission) [4]. However, in the MSMD networks, every node within this cooperative system has the need to exchange data with every other node. Thus a connection failure between any two nodes within the network could significantly decrease the efficiency of the whole system if using this structure. On the other hand, multi-hop ad-hoc networks have been proposed to solve this connection problem when multiple nodes are deployed [5]. Because of fading and interference in wireless channels, to have multiple hops would significantly increase the risk of outage (See Fig. 2, where the increment of hops increases the outage probability). Furthermore, a multi-hop structure not only increases system delay and reduces cooperative efficiency - since some nodes play the role of a bridge, but also increases the complexity of optimization.

In this paper, we propose a centralized relay model for MSMD networks where all the nodes exchange information via the relay. As an information flow-in node, the relay has network coding implemented on itself. Relaying is introduced in wireless networks to achieve a higher data rate and/or spatial diversity. The employment of relaying in a network can also provide higher robustness, extended coverage, and higher spectrum efficiency if well handled [4], [6], [7]. Relay channels were firstly studied in [8] for wireless networks that employ relaying strategies. Recently, it has been comprehensively explored within the fields of user cooperation and relay assisted cooperative networks [6], [9]. Traditional relaying techniques exploit the capability of performance improvement by manipulating the structure of source, relay and destination terminals. With the introduction of network coding in the physical layer [10], [11], the benefits of relay networks can be enjoyed in more scenarios which have the broadcast feature of wireless signals.

Particularly in the cooperative sensing scenario, all nodes need to frequently exchange their acquired data and make decision based on the available information. It thus has high requirements towards the average data rate and robustness of the communication channel. Furthermore, it is usually the case that

the nodes are restricted by battery power and computation capacities. Our proposed model does not only achieve full multiplexing gain, but also carefully addresses these restrictions by shifting the majority of the communication workload to the relay station, thus freeing the nodes from communication burdens. Furthermore, since the model keeps a local copy on the relay, it is thus robust against data loss if some of the nodes lose power or get lost. In this paper, for the purpose of demonstration and verification, we provide an example based on the cooperative sensing scenario, where multiple UAVs are deployed to explore a designated area (See Fig. 1). The scenarios and applications where the proposed model can also be applied include a smart home with wireless connected electrical appliances, teleconferencing, mobile cloud-computing and wireless sensor networks [12], [13].

The key of the proposed scheme is that the relay merges information flow from all the users and reduces redundancy by network coding which further provides the capabilities to improve spectral efficiency and/or diversity gain. The relay will then encode the information flow and transmit it to all the nodes through broadcasting mode. Such a scheme utilizes the advantages of centralized control, network coding and relay communications. The model achieves full data rate and at the same time, the more nodes that are incorporated into the network, the higher diversity gain it can achieve. Similar models have been studied in the literature, e.g. some researchers have considered a different system setup which has multiple relays to assist the multicast or unicast scenario and a maximal diversity gain of $M + 1$ can be achieved (M is the number of relays) [14]. This is particularly useful if there are a lot of relays. A multiple access channel model where multiple end nodes connecting to one destination with the help of multiple relays is studied in [15], which provides necessary supports to the more complicated scenario studied in this paper.

Theoretical performance of the proposed scheme and its application in practice are explored in this paper. The time-division multiplexing channels of the relay communication model can be regarded as multiple access systems [16], thus the diversity-multiplexing tradeoff proposed in [16] is an effective metric to evaluate the theoretical performance of the model. Compared with direct transmission, the DMT result of the proposed model has higher diversity order. And compared with the standard relay scheme [17], the proposed model enjoys a full data rate. We further apply the proposed scheme in a cooperative sensing scenario with multiple searching agents and confirm the performance.

The rest of the paper is organized as follows: Section II introduces the background of this paper; Section III presents the system model and network coding strategy; Section IV analyzes the performance of the communication model; Section V gives an example of applications; Section VI discusses the implementation issues; Section VII concludes the paper and summarizes the key benefits.

Fig. 1. The MSMD relay channel model which has J mobile nodes (U_j , $j = 1, \dots, J$) and one relay station.

II. BACKGROUND

A. Network Coding

Network coding was proposed to increase the capacity of multicast networks in communications, which was further extended to wireless channels, where a large number of potential coding terminals can be hosted and connected simultaneously. The key of network coding is to aggregate information flows. For example, when an intermediate node receives multiple messages, instead of simply relaying them to the destinations individually, a relay can combine these data flows using network coding for higher

efficiency [1], [18]. The principle of network coding is to mix the data flows at intermediate network nodes and separate them at the destination, thus the system capacity can be increased, especially in multicast networks.

In this paper, Analog Network Coding (ANC) is used so that we implement the coding strategy in an analog domain [11]. The essential part of ANC is that each terminal in the wireless network deletes its own interference from the received data before it recovers useful information from the other nodes. The proposed model manipulates the interference similarly to improve the system performance. If applied in wireless networks, the benefit of network coding is two-fold: suppressing interference and/or increasing system capacity [11]. Let us look at the following simplified example:

Suppose A and B need to exchange information via an intermediate node R . At the first time slot, the two sources transmit their own messages to relay R , given by

$$y_R = h_{AR}x_A + h_{BR}x_B + w_R, \quad (1)$$

where h_{AR} and h_{BR} are the channel coefficients from A and B to R respectively. x_A and x_B are the two messages. w_R is noise.

Relay R uses analog network coding to generate x_R from its received data y_R , which contains both the messages: x_A and x_B . x_R is then broadcasted to A and B . Since they already have the knowledge of their own message, useful information from the other can be obtained by reducing the already known information (e.g. x_A for A and x_B for B). Asymptotically the capacity of ANC approaches two times over traditional systems as Signal-to-Noise Ratio (SNR) increases, thus it shows significant potential to be used in large cooperative wireless networks.

Traditionally, concurrent transmissions from different users would generate interference on the received signal of some users [19] and this can decrease the system capacity. However, from the information point of view, an interference signal can still be useful in signal processing. If it can be utilized either by coding or new models, an information gain will be enjoyed. The centralized structure of the model is especially suited to implement network coding since every node exchanges information with the others through the intermediate relay, thus network coding can be applied to increase the information gain and further improve cooperative efficiency.

B. Connectivity

The mobile nodes are usually located remotely at different geographical areas. Therefore they have a high requirement for maintaining the connectivity among themselves as well as with the base station.

Similar requirements are also applied in the demo scenario of this paper, where the mobile sensors are often located at dangerous and unpredicted environments. Connectivity is mainly affected by the received signal strength of radio frequency signals which are controlled by several factors such as distance, relative speed between transmitter and receiver, radio frequency, antenna height and gain, [20].

Friis' equation [21] in free space reveals the inverse square law of radio signal strength: if the distance d between transmitter and receiver is doubled, in order to maintain the same level of received power, the transmitted power should be quadrupled. For mobile terminals such as wireless sensors, the limited power budget makes this quite difficult. As a result, multi-hop relay links are often used to reduce the energy consumption for long distance transmission.

In theory, by means of introducing more hops, the coverage of a network is unlimited. However, besides the increase in cost, routing complexity, path planning difficulties and energy consumption [5], [22], one particular difficulty is to maintain the connectivity in fading channels, which can be measured by outage probability $P(\mathcal{O})$ [23].

Outage probability is defined as the probability that the received SNR falls below the minimum required SNR for reliable communications. In practical wireless channels, outage usually happens if deep fading is encountered for a short segment of time or space, thus leading to large error bursts which cannot be corrected by coding with reasonable complexity. The outage probability can be calculated as follows

$$P(\mathcal{O}) = p(\gamma < \gamma_0) = \int_0^{\gamma_0} p_\gamma(\gamma) d\gamma, \quad (2)$$

where γ_0 is the required minimum SNR and γ is the received SNR. p_γ is the Probability Distribution Function (PDF) of γ , which is often modelled by a Rician distribution in wireless channels with line-of-sight paths [24].

Because deep fading is mostly from destructive interference which is independent of the average received power level [23], if there are M hops between information source and destination, the total outage probability can be calculated as

$$P(\mathcal{O}_M) = 1 - (1 - P(\mathcal{O}))^M, \quad (3)$$

where we assume each hop is independent and identical in realization. From (3), it is easy to see that the increase in number of hops decreases the connection quality, as shown in Fig. 2.

Fig. 2. Outage probability of multi-hop channels which are modelled as independent Rayleigh distribution. The numbers on the figure denote hops.

C. Diversity Multiplexing Tradeoff

The diversity multiplexing tradeoff in multiple access channel was first studied in [16], which reveals the fundamental tradeoff in multiple access channels. Diversity can be enjoyed if more than one copy of the same data is received, so that the system has better robustness at a certain data rate. However, multiple channels can also be used to transmit different copies of information, therefore providing higher spectrum efficiency which is denoted by multiplexing gain. Hence an intrinsic tradeoff between diversity gain and multiplexing gain exists in multiple users system.

Following similar definitions as in [16], a scheme $\mathcal{C}(\text{SNR})$ achieves spatial multiplexing gain r and

diversity gain d if the data rate R is such that

$$\lim_{SNR \rightarrow \infty} \frac{R(SNR)}{\log SNR} \geq r,$$

and the average error probability P_e is such that

$$\lim_{SNR \rightarrow \infty} \frac{\log P_e(SNR)}{\log SNR} \leq -d.$$

If the equality in the above equation holds, a simple expression $P_e(SNR) \doteq SNR^{-d}$ can be used.

The average error probability $P_e(SNR)$ is the error probability of maximum likelihood detection [16]. It can be tightly bounded by the outage probability if the block length is sufficiently long. For a multiple users system with J users, the outage event is defined as

$$\mathcal{O} = \bigcup_S \mathcal{O}_S,$$

where the union is taken over all subsets $S \subseteq \{1, \dots, J\}$, and

$$\mathcal{O}_S \triangleq \{\mathcal{I}(\mathbf{x}; \mathbf{y} | \mathbf{x}_{S^c}, \mathbf{H} = H) < \sum_{k \in S} \mathcal{R}_k : H\}, \quad (4)$$

where S^c is the complementary set of S .

III. MODEL DESCRIPTION AND SYSTEM IMPLEMENTATION

An illustration of the system and model is shown in Fig. 1, where multiple wireless nodes exchange information through the central relay node. In order to reduce ambiguity, the relay node is referred to as relay and others are named as user in the rest of this paper. The relay and users are assumed to have one antenna and work under half-duplex mode. This model expands the two-way relay model [25], [26] to accommodate a flexible number of users. We assume there are J users in the system, denoted as U_j , $j = 1, \dots, J$. The dedicated relay is denoted as R . All users exchange information frequently via the relay through radio propagation channels.

A. Channel Model

The radio frequency channel accommodates the media for wireless signals, thus it is an essential component in the communication system. A typical wireless channel [4] has a Line-Of-Sight (LOS) path and several Non-LOS paths. In order to capture the features of this channel type, a Rician fading model is used in this paper. This is usually the case where LOS and Non-LOS paths coexist in the wireless

sensing networks. The distribution of fading channel coefficient under such model is given by

$$P_X(x) = \frac{x}{\sigma^2} \exp \left[-\frac{(x^2 + u^2)}{2\sigma^2} \right] I_0\left(\frac{xu}{\sigma^2}\right), \quad x \geq 0, \quad (5)$$

where x is the amplitude, u^2 is the power in the LOS component, I_0 is the modified Bessel function of 0th order. The Rice factor K is defined as the ratio of signal power in LOS component over the power in Non-LOS multipath components, e.g., $K = \frac{u^2}{2\sigma^2}$. A special example is when $K = 0$ and the channel becomes Rayleigh fading. In the performance evaluation part of this paper, we consider the scenario where K is the same for all the channels in our performance analysis. Typical values of K for outdoor and indoor environments are usually within $[0 \ 25]$ [27].

The PDF of the power of fading channel $y = |x|^2$ is then given by

$$P_Y(y) = \frac{(1+K)e^{-K}}{\sigma^2} \exp \left[-\frac{(1+K)y}{\sigma^2} \right] I_0 \left(2\sqrt{\frac{K(1+K)y}{\sigma^2}} \right), \quad y \geq 0. \quad (6)$$

B. Signal Model

The scheduling of this model is shown in Fig. 3. Through the uplinks (from remote nodes to relay), all the users simultaneously transmit data to the relay. After the reception of one frame, relay R decodes and prepares for transmission. At the next $J - 1$ time slots, it broadcasts the data to all the users.

In the domain of continuous time, the signal from user j is denoted as $s_j(t)$. We employ the Direct-Sequence Spread Spectrum (DSSS) technique, e.g. Code Division Multiple Access (CDMA) [23] to achieve the required orthogonality for simultaneous communications,

$$s_j(t) = \sqrt{E_j} \sum_k x_{j,k} \sum_{v=0}^{N_s-1} c_{j,k,v} p(t - (kN_s + v)T_c), \quad (7)$$

where E_j is the j -th user chip energy, $x_{j,k}$ is the k -th data symbol, N_s is the spreading factor, $c_{j,k,v}$ is the spreading sequence value for the v -th chip symbol, $p(t)$ is the transmitted pulse shape, T_c is the chip time. Besides CDMA, the system can also be explored in spatial or temporal extensions. Note that $x_{j,k}$ denotes one symbol only for the convenience of expression and it could be expanded to one message or one frame in practice. In the rest of this paper, we use x_j to denote $x_{j,k}$ unless explicitly stated.

For the benefit of theoretical analysis, we make the following assumptions. The channels suffer from slow fading which remain unchanged for one time slot. The channel state information is known by destination nodes through training sequences. All terminals are synchronized to the relay, which can be

Time Stamp	Source	Information flow	Sink
1	$\{ U_1, U_2, \dots, U_J \}$	$\longrightarrow$	Relay
2	Relay	$\longrightarrow$	$\{ U_1, U_2, \dots, U_J \}$
...	...	...	...
J	Relay	$\longrightarrow$	$\{ U_1, U_2, \dots, U_J \}$

Fig. 3. The scheduling of signals. Network coding is implemented on the relay.

achieved by broadcasting a synchronization code word from the relay. There are enough spreading codes to accommodate all the terminals.

However, in a practical system, perfect synchronization is hardly achieved, thus asynchronous CDMA system was studied and adopted in this paper. A series of pseudo-noise (PN) codes (e.g. Gold codes) can be used in asynchronous CDMA systems [4]. Since the users are not synchronized perfectly, the sum of the other users' PN sequences appears as interference which can be approximated by a Gaussian process if there are a big number of users. Its variance increases proportionally to the number of users if all of the users are received with the same power, which contributes to the noise term $w(t)$.

This paper uses Gold sequences of length 31, which were generated by a pair of polynomials ($g_1(p) = p^5 + p^2 + 1$ and $g_2(p) = p^5 + p^4 + p^2 + p + 1$). Since the number of mobile sensors or UAVs is usually small (less than 10), Viterbi algorithm is used to decode each users symbols. The computational complexity of Viterbi algorithm increases exponentially with the increase in users. If there are more users, suboptimum detectors such as decorrelating or MMSE detector can be used. Detailed practical issues and the corresponding solutions associated with CDMA can be found in [28], [29], which are neglected in this paper.

At the first time slot, all the users transmit their own data simultaneously to the relay. After the matched

filter, the k th sample in the relay output is given by

$$y_k = \int_{-\infty}^{\infty} p(t - kT_c) \left(\sum_{j=1}^J h_j s_j(t) + w(t) \right) dt, \quad (8)$$

where h_j is the j th user channel coefficient with the distribution given by (5). The user-relay channels are assumed to be independent of each other. $w(t)$ is complex Additive White Gaussian Noise (AWGN) with power spectral density $N_0/2$. The source signals are later separated by correlating the received signal with the code generated by relay for the corresponding source.

In this paper, we assume a complete decoding of all the received symbols at the relay without focusing on a particular decoding algorithm, e.g. zero-forcing or maximum likelihood.

At the next $J - 1$ time slots, relay R broadcasts signals to the users based on the employed network coding strategy. Relay transmitted symbol at time n is given by

$$x_r(t) = \sqrt{E_R} \sum_{j=1}^J g_j(t) x_j(t), \quad (9)$$

where E_R is the transmitting energy in the relay, $g_j(t)$ is the network coding factor on all the symbols and $\sum_{j=1}^J |g_j(t)|^2 = 1$. The network coding factor can be designed to compensate for the received power imbalance caused by a different forward channel from user to relay, similar to the water filling technique [4].

We can express the system model in the discrete time domain by sampling and stacking all the received signals by the user U_j ,

$$\mathbf{y}_j = \sqrt{E_R} \mathbf{H}_j \mathbf{G} \mathbf{x} + \mathbf{w}_j, \quad (10)$$

where $\mathbf{y}_j = [y_j(1), \dots, y_j(J - 1)]^T$, $\mathbf{H}_j = \text{diag}\{h_{r,j}(1), \dots, h_{r,j}(J - 1)\}$, $\mathbf{x} = [x_1, \dots, x_J]^T$, $\mathbf{w}_j = [w_j(1), \dots, w_j(J - 1)]^T$.

$\mathbf{G}$ can be regarded as a matrix for data fusion in the physical layer or the precoding matrix. Thus there are a large number of contributions about how to build a full rank precoding matrix [30]. Without losing generality, we generate $\mathbf{G}$ by using a circulant matrix [31], whose $J - 1$ rows are shifted versions of $\mathbf{g} = [g_1, g_2, \dots, g_J]^T$. And $g_i \neq g_j \neq 0$ for any $i \neq j$. This matrix is known to all users. Other full rank matrix (e.g. Vandermonde matrix) generating methods are available (e.g. [30]); however the details are neglected in this paper.

C. Detection at Nodes

For the success of detection, destinations should have the corresponding channel information which can be obtained by adding a training sequence to the head of each frame [4]. For every user, after the reception of all $J - 1$ frames from the relay, the Maximum Likelihood (ML) [4] technique is applied to obtain useful information from the sources. There are other detection methods which could be considered, e.g. zero-forcing [32] technique. Both these ways are introduced here, their performances and complexity are discussed in Section VI, where we briefly discuss the two and show the reasons why the former is chosen in the proposed model.

1) *Maximum Likelihood Detection*: Optimum detector with maximum likelihood criterion is applied to obtain the transmitted symbols. Particularly for user U_j , the decoding strategy is given by

$$\hat{\mathbf{x}}_j = \underset{\mathbf{x} \in \mathcal{A}_r^J}{\operatorname{argmin}} \{ \|\mathbf{y}_j - \sqrt{E_R} \mathbf{H}_j \mathbf{G} \mathbf{x}\|^2 \}, \quad (11)$$

where $\mathbf{x} = [x_1, \dots, x_J]^T$ and x_j is known to user j . $\mathcal{A}_r^J$ is the search space defined by modulated symbol set $\mathcal{A}_r$ and the number of users J .

2) *Zero-Forcing Detection*: Similar, when using Zero-Forcing detection, the transmitted symbols can be detected by user U_j as follows

$$\hat{\mathbf{x}}_j = \mathbf{x} + \{\sqrt{E_R} \mathbf{H}_j \mathbf{G}\}^{-1} \mathbf{w}_j. \quad (12)$$

The performance and complexity of these two detection schemes will be investigated in Section VI, where we can see each of them has its own advantages and disadvantages, therefore they should be carefully chosen in practice.

IV. SYSTEM EVALUATION

A. Diversity Multiplexing Tradeoff (DMT)

In order to obtain the DMT, we assume all relay-user channels support the same data rate $\mathcal{R}_j$, thus for each user, the sum rate of its received data is the combination of the other $J - 1$ users. $\mathcal{R} = \sum_{j=1}^{J-1} \mathcal{R}_j = (J - 1)\mathcal{R}_j$ ($j = 1, \dots, J$).

Theorem 1: If the channels are independently and identically distributed (i.i.d.) Rician fading with PDF given by (5), the diversity multiplexing tradeoff of the proposed system model is given by

$$d_J^*(r) = (J - 1)(1 - r). \quad (13)$$

Proof: In order to prove the theorem, we need to obtain the mutual information corresponding to each outage event $\mathcal{O}_S$. Following the definition in Section II-C, mutual information of the proposed model (10) can be obtained as

$$\mathcal{I}_S = \log \det \left(\mathbf{I} + \frac{\sqrt{E_R}}{N_0} \mathbf{H}_j \mathbf{G} \mathbf{G}^H \mathbf{H}_j^H \right), \quad (14)$$

where $\mathbf{I}$ is an identity matrix with $J \times J$. Since

$$\det(\mathbf{I}_{N \times N} + \mathbf{A}_{N \times M} \mathbf{B}_{M \times N}) = \det(\mathbf{I}_{M \times M} + \mathbf{B}_{M \times N} \mathbf{A}_{N \times M}),$$

the above equation can be transformed as

$$\mathcal{I}_S = \log \det (\mathbf{I} + \gamma \mathbf{Z} \mathbf{G} \mathbf{G}^H), \quad (15)$$

where $\gamma = \frac{\sqrt{E_R}}{N_0}$ is SNR and $\mathbf{Z} = \mathbf{H}_j^H \mathbf{H}_j = \text{diag}\{|h_{r,j}(1)|^2, \dots, |h_{r,j}(J-1)|^2\}$. The probability distribution function of the above equation is obtained as

$$p(\mathcal{I}_S) = \mathbb{E} \{ \delta (\mathcal{I}_S - \log \det (\mathbf{I} + \gamma \mathbf{Z} \mathbf{G} \mathbf{G}^H)) \}, \quad (16)$$

where the expectation $\mathbb{E}\{\cdot\}$ is calculated over the distribution of $\mathbf{Z}$.

The rank of $\mathbf{G}$ is proved first in order to finish the proof of Theorem 1.

Lemma 2: The precoding matrix $\mathbf{G}$ has full row rank.

Proof: We can use mathematical induction to finish the proof. Firstly we expand the matrix $\mathbf{G}$ to a new matrix $\mathbf{G}_S$ which contains one more row. The new row is fulfilled by the last unrepeated shift version of $\mathbf{g}$. Since $\mathbf{G}_S$ is a circulant matrix. The row vectors of $\mathbf{G}_S$ are denoted as $\mathbf{g}_j, j = 1, \dots, J$.

Because $g_i \neq g_j \neq 0$ for $i \neq j$, for $\mathbf{g}_1$, we can always find another vector $\mathbf{g}_j$ which is linearly independent with $\mathbf{g}_1$.

Suppose there are $J-1$ linearly independent row vectors in $\mathbf{G}_S$, denoted as $\mathbf{g}_1, \dots, \mathbf{g}_{J-1}$ and another row vector $\mathbf{g}_J$ which can be linearly denoted by the combination of the previous $J-1$ vectors. Combine all the J vectors together gives us

$$\sum_{j=1}^J \mathbf{g}_j = \{c, \dots, c\},$$

where c is a constant. It is clear that $\mathbf{c} = \{c, \dots, c\}$ is independent with any row vector within $\mathbf{G}_S$. Thus $\mathbf{g}_J$ can only be obtained by combining J independent vectors including $\mathbf{c}$. Therefore $\mathbf{g}_J$ is independent of the other $J-1$ vectors and the rank of $\mathbf{G}_S$ is $J-1$.

If $\mathbf{G}_S$ has full row rank, every submatrix which contains rows of $\mathbf{G}_S$ also has full row rank, thus $\mathbf{G}$ has full row rank. $\blacksquare$

Since $\mathbf{G}$ has full row rank, the matrix $\mathbf{G}\mathbf{G}^H$ also has the full rank, therefore it can be decomposed by eigendecomposition as

$$\mathbf{G}\mathbf{G}^H = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H,$$

where $\mathbf{Q}$ is a square matrix whose j th column is the eigenvector and $\mathbf{\Lambda}$ is the diagonal matrix whose diagonal elements are the corresponding eigenvalues $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \dots, \lambda_{J-1}\}$.

The mutual information can be approximated as follows if SNR is high [15]

$$\log \det (\mathbf{I} + \gamma \mathbf{Z}\mathbf{G}\mathbf{G}^H) = \log \prod_{k=1}^{J-1} \gamma \lambda_k |h_{r,j}(k)|^2. \quad (17)$$

Combining (16) and (17), we have

$$p(\mathcal{I}_S) = \mathbb{E} \left\{ \delta \left(\mathcal{I}_S - \log \prod_{k=1}^{J-1} \gamma \lambda_k |h_{r,j}(k)|^2 \right) \right\}. \quad (18)$$

The above expectation can be calculated by employing the PDF of $|h_{r,j}(k)|^2$, $k = 1, \dots, J-1$, which is obtained as

$$\begin{aligned} p(\mathcal{I}_S) &= \frac{e^{\mathcal{I}_S} \gamma^{-(J-1)}}{\det[\mathbf{G}\mathbf{G}^H]} \int_1^{e^{\mathcal{I}_S}} \int_1^{\phi_1} \cdots \int_1^{\phi_{J-3}} p_X \left(\frac{e^{\mathcal{I}_S}}{\gamma \lambda_1 \phi_1} \right) \\ &\quad p_X \left(\frac{\phi_1}{\gamma \lambda_2 \phi_2} \right) \cdots p_X \left(\frac{\phi_{J-3}}{\gamma \lambda_{J-2} \phi_{J-2}} \right) \\ &\quad p_X \left(\frac{\phi_{J-2}}{\gamma \lambda_{J-1}} \right) d \log(\phi_1) \dots d \log(\phi_{J-1}), \end{aligned} \quad (19)$$

where $p_X(x)$ is the PDF of Rician fading channel, given by (6). At high SNR region, the equation above can be reduced as

$$\begin{aligned} p(\mathcal{I}_S) &= \frac{e^{\mathcal{I}_S}}{\gamma^{J-1} \det[\mathbf{G}\mathbf{G}^H]} \int_1^{e^{\mathcal{I}_S}} \int_1^{\phi_1} \cdots \int_1^{\phi_{J-3}} \\ &\quad \pi^{J-1} d \log(\phi_1) \dots d \log(\phi_{J-1}), \end{aligned} \quad (20)$$

where the modified Bessel function $I_0(0) = 1$, $\phi_j \leq e^{\mathcal{I}_S}$ and $\pi = (1 + K)e^{-K}\sigma^{-2}$. Now with the availability of distribution $p(\mathcal{I}_S)$, we can write the outage probability as (4)

$$P(\mathcal{O}_S) = \int_0^{\mathcal{R}} p(\mathcal{I}_S) d\mathcal{I}_S.$$

Note that $\mathcal{R} = \sum_{k=1}^{|S|} \mathcal{R}_k$.

The outage probability of the proposed system is obtained below. Because π^{J-1} remains constant when the SNR γ goes to infinity, following a similar proof of [15, Lemma 5, Theorem 6]: at a data rate $\mathcal{R} = r \log \gamma$,

$$P(\mathcal{O}_S) \doteq \gamma^{-(J-1-\sum_{k=1}^{|S|} r_k)}. \quad (21)$$

It is easy to see this outage probability increases with the increase of $|S|$ and the maximum value of $|S|$ is $J - 1$. With the same transmitting power and the same data rate r_j for every user, we can have

$$P(\mathcal{O}_S) \doteq \gamma^{-(J-1-(J-1)r)}, \quad (22)$$

so that $d_J^*(r) = (J - 1)(1 - r)$. ■

The DMT equation $d_J^*(r) = (J - 1)(1 - r)$ reveals the potential of the proposed model and can be used to support the design of wireless networks. Based on Theorem 1, the relay can adaptively allocate resources to meet the required performance. For example, as in [33], if some users (e.g. $J - 1$) have the same message to transmit, the relay can allocate the corresponding proportion of resources to this message, therefore $J - 1$ independent copies of the same data can be received by the destinations, and a full diversity order of $J - 1$ can be achieved. On the other hand, if all the users have the same amount of individually generated data to transmit, the relay can assign each one the same share of resource so that total J different data can be exchanged within J time slots. Thus the model enjoys full data rate (multiplexing gain = 1), corresponding to $d_J^*(1) = 0$.

B. Performance comparison and Simulation Results

In order to show the fundamental advantages of the proposed model, we compare the DMT lines of the following three schemes: *Proposed Model*, *Direct Transmission* and *Standard Relay Model* [17], as shown in Fig. 4. From the results, it is easy to see that since each orthogonal channel is only occupied by one user if using *Direct Transmission*, it can achieve the full data rate. However, no diversity gain can be obtained. The *Standard Relay Model* can achieve the maximum diversity gain $J - 1$, but its maximum multiplexing gain is only 0.5 because of the delay in relay. Comparing with the previous two schemes, the proposed model utilizes the benefits of wireless access behavior, network coding and centralized structure, thus it achieves higher diversity/multiplexing gain given the same value of the other parameter.

Fig. 5 shows the simulation results of the proposed model when used in an uncoded system. We consider the two cases where there are 2 and 4 users respectively. Both the two detection methods introduced before are examined: zero-forcing and ML. The channels are set as the same in Section III-A,

Fig. 4. The DMT performance of different protocols.

where the Rician factor is $K = 1$. The modulation scheme is Quadrature Phase-Shift Keying (QPSK).

Based on the performance analysis, under the assumption of Rician channel model, the increase of user number J leads to better diversity gain at high SNR region, which is confirmed in Fig. 5. Under the same detection algorithm, more users result higher diversity gain, thus lower BER of the detected symbol.

Fig. 5. Bit Error Rate (BER) of the simulations. Zero-forcing and ML methods are used for detection.

Furthermore, ML can achieve faster performance improvement with the increase of J than Zero-forcing. It is worth noticing that the DMT is obtained under high SNR assumption [16]. Therefore at low SNR region, the result is not accurate, which is also demonstrated in both the cases. Even though both of them can work at high SNR, they offer different BER. As shown in the figure, at $E_b/N_o = 7\text{dB}$ and $J = 4$, the BER of ML is 3×10^{-3} while that for zero-forcing is 10^{-2} . The complexity of these two schemes is discussed in Section VI.

V. APPLICATION: FRONTIER-BASED COOPERATIVE SENSING

In this section, we integrate Frontier-based Exploration (FE) [34] with the proposed communication model to show its application and efficiency improvement in the cooperative sensing scenarios. The FE technique is able to efficiently detect an unknown area by guiding the mobile robots to the boundary between an unexplored area and an open area, where new information is most likely to gain. In the simulations, we deploy multiple UAVs as mobile sensors to explore an unknown area. Each UAV is

guided by the FE algorithms and connected by a central relay which will process the data from all UAVs and provide support for communications. A detailed FE experiment with UAVs can be found in our recent paper [35].

The experimental conditions are set as follows. Four UAVs are sent out to explore an unknown wildness area with a relay for communications. They all have a map of the whole area, which is divided into 20×20 cells. During the working procedure, each UAV collects data and updates its own map while at same time transmitting and receiving information from the relay, which maintains a global map. There are 3 kinds of cells. The first kind includes those with the probability of 0.5, which means they have not been explored yet. The second ones have lower probabilities than 0.5, which means that they have already been explored and the value of their information is denoted by the number. The third ones are with the probability of 1, which means they are currently occupied by either UAVs or obstacles.

For the benefit of understanding the basic algorithm, the pseudocode of FE is briefly introduced in Algorithm 1.

Algorithm 1 Frontier Exploration of u_j

Input: M_j (Map of u_j), P_j (position of u_j), D_r (data from relay)

Output: The next position of u_j

```

1: Integrate  $D_r$  into  $M_j$ 
2: Mark the positions of other UAVs on  $M_j$  as 1
3: Set the search radius  $R = 1$ 
4: while  $R \leq R_{max}$  do
5:   if Find a cell  $C_i$  within  $(M_j, R)$  with probability 0.5 then
6:     if  $C_i$  is not marked as 1 then
7:       Output coordinates of  $C_i$ 
8:       return
9:     end if
10:     $R = R + 1$ 
11:   end if
12: end while
13: Output (0,0) ▷ The whole map has been explored.
14: return.

```

In this algorithm, UAVs are assumed to have knowledge of their own geographical information, which can be retrieved from the onboard Global Positioning System (GPS) receiver. This information is frequently transmitted to the relay by attaching it to data messages. The relay will then fuse all the data given this geographical information.

The channels between UAVs and relay are modelled as Rician fading as stated before. The symbols

Fig. 6. The explored map and the position of UAVs and relays when 70% of the area is opened. (The number '1's and letter 'R' denote the positions of UAVs and relay respectively.)

are modulated by QPSK. SNR is set to 20dB. All the transmitted messages are encoded by convolutional code and decoded at receiver by Viterbi algorithm. During the simulations, all UAVs are constantly connected to the relay. Other less important implementation details are neglected here for the benefit of concise expression.

Fig. 6 captures the picture of the map when 70% of it has been explored. The white areas have been visited by UAVs and dark areas are still unexplored. The current positions of UAVs are marked on the map, and are clearly at the frontier between explored and unknown areas. The red numbers on the map denote the UAVs and R denotes the position of relay, which always maintains its position in the centre of all UAVs.

Fig. 7 shows the total time used to finish exploring the whole map, where different numbers of UAVs are

Fig. 7. The total time used by different number of UAVs.

employed. If the number is more than one, the proposed structure allows the relay to reduce redundancy of its received data before broadcasting, thus improving the efficiency of communications and sensing. From the figure, we can see the time used by four UAVs is about a quarter of that by one UAV, which confirms the benefit of data fusion and reduced redundancy as each UAV collects unique information. However, we can still observe that there is an approximate 10 *time units* difference than the ideal case of $101.25 = \frac{1}{4} * 405$ *time units*. The possible reason is that because of the time delay in communications, several cells were explored more than once.

In the next experiment, we compare the performance of the proposed model and multi-hop model. Four UAVs are deployed in the simulation. Other conditions (channel, SNR, coding schemes and decoding algorithm) are the same for both the models except in the data exchange mode. In the proposed model, all the data are transmitted to the relay where they are fused and then broadcasted to the users. In the multihop model, the data are passed from node to node, where the redundancy of data is reduced once a node receives new data from its neighbours.

Fig. 8. Multi UAV cooperative sensing by using centralized relay model and multi-hop model.

Fig. 8 shows the simulation results. To finish exploring 70% of the area, UAVs in the proposed model need about 65 *time units*; while the time spent by those in multi-hop model is about 110 *time units*. The time difference shows the importance of data fusion and redundancy cancellation. Another observation is that the difference in time usage increases when the work process goes on. The reason is that at the beginning, since the whole map has not been explored yet, there is less redundancy in the data collected by each node. When the procedure goes on, some cells on the map will be visited repeatedly, thus leading to more time wasting.

VI. DISCUSSION

A. Distributive Structure or Centralized Structure

Comparing with the distributive structure, this centralized model shifts the majority of the communication and computation power from individual nodes to the relay. Even though the benefits are obvious, there exists a risk in robustness. It is possible that failure of the relay could cause the collapse of the

whole network, as often seen in traditional wired network structures [36]. We address this problem by including basic distributive capability in each individual node as a backup. If the relay fails, the users should have the autonomous functions to control its behaviour and recover communications using a multi-hop structure. A full implementation of the scenario is part of the future work.

B. Orthogonal User-Relay Channels

The uplink channels - users to relay - are orthogonal with each other using different codes in this paper. Other multi-user strategies can be used if the users can transmit data to relay through temporal, frequency or spatial orthogonal channels.

C. Complexity of Detection Algorithms

The performance of ML and zero-forcing scheme is discussed and examined in Section IV-B. We will have a further look at their complexity here. Under M-ary quadrature amplitude modulation (e.g. QPSK), the complexity of ML detection is $\sim \mathcal{O}(2^{(M*J)})$ where J is the number of connected users. For the zero-forcing scheme, its complexity is $\sim \mathcal{O}(J^2)$. We can generally conclude that if M and J are small (e.g. $M=2$ for QPSK, $J=2$), ML is preferred for its superior performance over zero-forcing, which is the case in the simulations of this paper. If M and J are large, zero-forcing can be chosen for its simplicity. Other factors, e.g. computation capability and power budget can also affect the choice in practice.

D. Applications

The proposed scheme can be used in mobile networks where there are several subscribers sharing information with each other. Instead of transmitting each one's data separately, the base station can fuse and encode all the data by network coding before transmission. This can improve the capability of combating fading in wireless channels and reduce the redundancy in data.

When used in a mobile cloud-computing scenario, the relay is more than just an intermediate station, it can be equipped with the required computing platform and software so that each end node can communicate with it and share its provided services.

Wireless Sensor Networks (WSN) can be enhanced if using the proposed scheme. Besides the benefits mentioned previously, there is another gain for WSN. Sensors in WSN usually have limited energy, thus the life of WSN depends on energy consumption of the end nodes. If using the proposed model, since the majority of the communication is shifted to the relay and the redundancy of information is largely

reduced by the relay, the life of sensors and the whole network can be extended. This will be particularly important in large-scale-connected city deployments.

With the increased spreading of wireless networks in more areas, more scenarios which suit and favour this model (with necessary modifications both in structure and coding) will emerge. The expansion and extension of this model will be our work at the next step.

VII. CONCLUSION

This paper proposes a centralized wireless network model with a relay to connect all the access nodes. We further implemented the model with network coding in the relay. The DMT performance shows that the model is superior to the classical relay model and direct transmission. Based on the DMT analysis, the proposed model can be used to transmit the same message for the benefit of maximum diversity gain, or can be configured to transmit different data so that full multiplexing gain can be enjoyed. In this paper, the latter is chosen for higher spectrum efficiency. Simulation results confirm the advantages. We applied this model in a cooperative sensing scenario with multiple UAVs. Comparing with multi-hop structures, the proposed scheme can achieve higher efficiency. Even though this paper only examined and tested the sensing scenario, a considerable number of wireless applications share a similar behaviour and can benefit from this model.

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