Large Eddy Simulation with Modelled Wall-Stress for Complex Aerodynamics Applications and Stall Prediction

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The aerodynamics of aircraft high-lift devices at near-stall conditions is particularly difficult to predict numerically. The computational requirements for accurate wall-resolved Large-Eddy Simulations are currently prohibitive, whilst RANS models are generally reliable only for low angles of attack with fully-attached boundary layers. Methods such as DES resolve unsteadiness of the outer boundary layer and can predict separation, but they rely upon a thick RANS layer and highly-stretched cells which damp the resolved turbulent fluctuations near the wall. An alternative approach, adopted here, is to extend the LES down to the wall, employing a relatively large near-wall normal grid spacing and avoiding grid stretching and high aspect ratios near the wall. A boundary condition then applies the correct wall shear stress as provided by a semi-empirical wall model. An adaptive formulation of this Wall-Modelled LES is presented here and validated using realistic test cases. Validation using a channel flow case at a range of Reynolds numbers demonstrates accurate results with a seamless transition between fully resolved ($y^+ \approx 2$) and wall resolved ($y^+ \approx 50$). Predictions of the MD-30P/30N airfoil using a modest grid with $y^+ \approx 100$ gives excellent agreement with experiments and correctly predicts C_{Lmax} . Finally, the method is demonstrated for the NASA HL-CRM providing surface pressure coefficients and velocity profiles. The predictions using a 50M cell mesh (for a full aircraft half model) are in good agreement with considerably finer grid RANS solutions. The presented method has considerable potential as it can produce accurate solutions to challenging engineering problems involving separation with modest grid and computational requirements, whilst being robust to variations in near-wall grid spacing.

Nomenclature

a_∞	=	speed of sound
C	=	airfoil chord length
$\mathbf{F}$	=	inviscid flux tensor
Ma	=	Mach number
p	=	pressure
$\mathbf{Q}$	=	primary flow variable
t	=	physical time
T	=	temperature
u	=	velocity
U	=	mean velocity
$u_{ }$	=	wall-parallel velocity
U_∞	=	free stream velocity
u_τ	=	friction velocity
U_τ	=	mean friction velocity
x	=	axial coordinate
y	=	wall distance
Δ	=	subgrid characteristic length
ε	=	upwinding parameter
θ	=	polar angle
ν	=	viscosity
ν_{sgs}	=	subgrid-scale viscosity
ν_{nw}	=	near-wall turbulent viscosity
ν_w	=	wall turbulent viscosity
ρ	=	density
σ	=	singular value of the velocity gradient tensor
τ_w	=	wall shear stress

I. Introduction

Large-Eddy Simulations (LES) are becoming more common as computational resources improve. They present an intermediate approach between DNS (Direct Numerical Simulations) and RANS (Reynolds Averaged Navier-Stokes equations). LES accuracy relies upon resolving energy-dominant and flow-dependent large eddies rather than modelling

them, as it is done in the RANS approach. Only the small eddies, which are expected to be more homogeneous and isotropic, are modelled by a subgrid scale model (SGS).

However, LES for engineering flows remains expensive due to the high grid requirements for wall-bounded flows, specifically in the near-wall region of the boundary layer. In the inner layer, a strong grid refinement is necessary to capture the dominant eddies, as they scale with wall units [1]. The Reynolds dependence becomes steeper, and the cost of wall resolved LES (WRLES) of high-Reynolds-number wall-bounded flows is similar to that of direct simulation [2]. This makes LES simulations unfeasible for most high-Reynolds-number applications resulting in low-fidelity techniques with lower grid requirements such as RANS often being preferred, especially for design optimization.

Several strategies have been proposed to overcome this issue, one of them being the wall-modelled LES (WMLES) approach. WMLES aims at making LES an affordable methodology by resolving the eddies in the outer layer and by modelling them in the inner layer, thus removing the high grid requirements near the walls. WMLES methods can be classified depending on whether the LES region is formally defined all the way to the wall or not [3]. In hybrid LES/RANS methods [4], the LES region exists only above some distance defined implicitly or explicitly and the RANS approach is employed below this distance. In modelled wall-stress approaches [5–7], the LES equations are defined all the way to the wall, requiring the wall shear stress as a boundary condition.

Advances in WMLES have allowed aerodynamic studies in practical engineering applications, which often consist of complex geometries. Kawai and Asada [8] employed wall-modeled large-eddy simulations (LES) to study the subsonic flow over an A-Airfoil near stall conditions at a high Reynolds number ($2.1 \cdot 10^6$). They showed that the wall-modelled LES significantly reduced the computational cost by approximately two-orders of magnitude compared to the traditional wall-resolved LES due to the reduction in the number of grid points and related increase in the time-step size. Bodart et al. [9] performed LES using an equilibrium wall model of a multi-element McDonnell-Douglas 30P/30N airfoil ($Re = 9 \cdot 10^6$) at several angles of attack (AoA). The lift curve was well predicted, but presented a higher stall angle (approx. 23°) compared to the experiments (approx. 21°). The authors attributed this to the blockage corrections applied to the experimental data and to a limited span-wise domain length of the computations. Bose and Moin [10] proposed a dynamic slip boundary condition wall model and simulated the flow over a NACA 4412 airfoil at near-stall conditions. Park and Moin [11] studied the same case with a non-equilibrium wall-model with a new dynamic correction for the wall-model eddy viscosity.

Participants in the BANC-III Workshop [12] were asked to perform computations for an airfoil in high lift configuration at a low angle of attack. Seventeen participants simulated the 30P30N airfoil at 5.5-deg angle of attack with different approaches such as DDES, IDDES, VLES and WMLES. Following the configuration and flow conditions specified in the BANC workshops, Zhang et al. [13] presented a WMLES approach to reproduce the flowfield of a 30P30N three-element airfoil with and angle of attack of 5.5 deg.

Fukushima and Kawai [14] recently used a wall-modeled large-eddy simulation to investigate transonic buffet on the

OAT15A supercritical airfoil ($Re = 3 \cdot 10^6$). Thanks to the wall model the grid spacing and the time-step size could be increased by one order of magnitude compared with the traditional wall-resolved LES.

Goc et al. [15] presented a WMLES approach with an equilibrium wall model which was used to predict the lift and pressure coefficients of the JAXA standard model (JSM), a high-lift aircraft model, at $Re = 1.93 \times 10^6$. The study of Lehmkuhl et al. [16] examined the performance of WMLES over two realistic high-lift aircraft geometries, the JSM and the CRM, using state-of-the-art methodologies for low-dissipation LES and an equilibrium wall model for the prediction of flow separation encountered at high angles of attack. Predictions of global forces and surface pressure distributions by two low-dissipation LES codes are shown to be in reasonable agreement with experimental measurements.

WMLES has not only been employed for aerospace simulations. Favourable predictions of velocity profiles and pressure coefficients were obtained by Alin et al. [17] using an equilibrium wall model to simulate high Re number submarine applications. Garcia et al. [18] performed a numerical study of the aerodynamics of a full-scale train under turbulent crosswind conditions. The application of these methods is also becoming important for automotive applications. Ambo et al. [19] compared wall-modeled and wall-resolved large eddy simulations for the prediction of boundary-layer separation around the side mirror of a full-scale vehicle and in the work of Aljure [20] numerical simulations on the flow over a realistic generic car geometry were carried out.

Most previous published work for aerodynamics applications do not deal with realistic engineering applications and adhere to simple geometries. In addition, most approaches have been tested only for low angles of attacks or have difficulties capturing the stall, either capturing it late or not capturing it at all. This is a key challenge we wish to address - the ability to predict separation and stall correctly without needing to full resolve the wall flow.

The purpose of this work is to overcome this issues presenting a robust wall-modelled LES formulation based on a wall stress boundary condition, and verify its accuracy and efficiency in complex aerodynamics applications and stall prediction. The approach has been tested for a channel flow and an airfoil in high lift configuration. In addition, to show its potential for full aircraft simulations, results for the High-lift Common Research Model (HL-CRM) have been included.

II. Methodology

A. Solvers

The solvers used in this project are built within OpenFOAM[®], a widespread open source CFD software written in C++ and based on the unstructured finite volume method [21], used by the present authors in previous work [22–30]. The validation of the proposed wall modelling was carried out on a channel flow case (see Section III) with the incompressible pressure-based transient solver `pimpleFoam`, while the application of the model to high-lift aerodynamics cases (Sections IV and V) was studied using a newly developed density-based solver for transonic flows.

A description of the two solvers is provided below.

1. Pressure-based solver for incompressible flows

The native incompressible solver `pimpleFoam` is built on a combination of the pressure-implicit split-operator (PISO) [31] and the semi-implicit method for pressure-linked equations (SIMPLE) [32]. The SIMPLE sub-iterations (2 in the present work) allow a more stable convergence for larger time steps. Time integration is here achieved by the second-order implicit backward method, and both convective and diffusive terms are approximated by second-order centered schemes.

2. A new density-based solver for transonic flows

The only density-based solver available in OpenFOAM is `rhoCentralFoam` [33], which relies on a first-order Euler time scheme and on the discretization of the convective fluxes through the central-upwind scheme of Kurganov and Tadmor [34], while the viscous flux is discretized using a standard central difference. This existing scheme is too dissipative for eddy resolving applications. In the solver developed for the present work an explicit second-order 4-stage Runge-Kutta scheme was implemented for the temporal discretization, together with an extension of the convective schemes to the well known Godunov type, similarly to previous work [35, 36]. A further modification was implemented to reduce the dissipation in the computation of the inviscid flux as follows. Roe’s flux difference splitting approximate Riemann solver [37] is employed at the interface between two neighbouring control volumes:

$$\mathbf{F} \cdot \mathbf{n} = \frac{1}{2} (\mathbf{F}_L + \mathbf{F}_R) \cdot \mathbf{n} - \varepsilon \frac{1}{2} |\mathbf{A}| (\mathbf{Q}_R - \mathbf{Q}_L). \quad (1)$$

In the above, $\mathbf{n}$ is the face unit normal, $|\mathbf{A}| = \mathbf{M}|\mathbf{A}|\mathbf{M}^{-1}$ is the diagonalizing transform, $\mathbf{A} = \partial(\mathbf{F} \cdot \mathbf{n})/\partial\mathbf{Q}$ is the Jacobian, and $\mathbf{F}$ and $\mathbf{Q}$ are the inviscid flux tensor and primary flow variable vector of the filtered Navier-Stokes equations, respectively. The subscripts L and R indicate left and right. A gradient based extrapolation is employed to compute $\mathbf{Q}_L$ and $\mathbf{Q}_R$. The inviscid flux calculation thus takes the form of a central difference plus a controllable function of the primary variable vectors that acts as an upwinding term. Following Bui [38], ε is adopted as a parameter to directly control the amount of upwinding (see Xia et al. [39] and Angelino et al. [40]). Its maximum value of 1 corresponds to fully upwinded flux, which is used in a “sponge” zone to damp the acoustic reflections near the external boundaries. Its minimum value of 0.15 in the area of interest is optimized to reduce the level of dissipation caused by the upwinding without compromising stability.

B. Proposed Wall Model

The Wall-Modelled LES presented in this work was introduced in its early formulation by Angelino et al. [41]. It is based on the wall-stress modelling approach, in which the LES subgrid-scale model is defined all the way down to the

wall, and the under-resolution of the boundary layer is compensated by imposing a wall-stress boundary condition and a near-wall modelled turbulent viscosity.

1. The subgrid-scale model

The LES subgrid-scale (SGS) model chosen is the σ -model [42], successfully used in complex aerodynamics applications in previous work [41, 43]. The proposed wall method has also been tested with the WALE model [44] and the dynamic Smagorinsky model [45], and can be used with any wall-sensitive SGS model. The standard Smagorinsky model is therefore not recommended. The choice to adopt the σ -model was driven by its local definition, contrary to non-local dynamic procedures, and by its improved formulation for pure shear, as compared to the WALE model, as was thoroughly tested in previous work [22, 46].

The σ -model can yield the correct physical behaviour for boundary and free shear layers, by replacing the classical strain rate in the formulation of the subgrid-scale viscosity ν_{SGS} , with an invariant of the velocity field. The subgrid-scale viscosity is defined as

$$\nu_{SGS} = (C_\sigma \Delta)^2 \mathcal{D}_\sigma \quad (2)$$

where $C_\sigma = 1.35$ is the model constant, Δ is the subgrid characteristic length scale and

$$\mathcal{D}_\sigma = \frac{\sigma_3 (\sigma_1 - \sigma_2) (\sigma_2 - \sigma_3)}{\sigma_1^2} \quad (3)$$

where $\sigma_1 \geq \sigma_2 \geq \sigma_3 \geq 0$ are the three singular values of the velocity gradient tensor.

2. The wall shear stress boundary condition

Following the approach of Wengle and Werner [47], the wall boundary condition is specified by assuming that, at the grid points closest to the wall, the wall-parallel velocity component $u_{||}$ is in phase with the instantaneous wall shear stress τ_w and the instantaneous velocity distribution is assumed to follow the law-of-the-wall. Here Spalding's single formula for the law-of-the-wall [48] was chosen for its continuous nature:

$$y^+ = u^+ + \frac{1}{E} \left(e^{\kappa u^+} - 1 - \kappa u^+ - \frac{(\kappa u^+)^2}{2!} - \frac{(\kappa u^+)^3}{3!} \right) \quad (4)$$

where $y^+ = y u_\tau / \nu$, $u^+ = u_{||} / u_\tau$, $\kappa = 0.41$ and $E = 9.8$. The formula is solved iteratively for u_τ through the Newton-Raphson method and the resulting required wall shear stress

$$\tau_w = \rho \cdot u_\tau^2 \quad (5)$$

is applied by defining the wall turbulent viscosity

$$\nu_w = \frac{\tau_w}{\rho \cdot \partial u / \partial y} - \nu \quad (6)$$

3. The near-wall modelled turbulent viscosity

This wall-stress function needs to be balanced by an appropriate internal field in the immediate proximity of the wall. Wengle and Werner [47] replaced the Smagorinsky length scale with the minimum value between the Prandtl mixing length κy and the LES filter width. Shur et al. [49] adopted a similar approach for a Hybrid RANS-LES with the Smagorinsky model in fully-resolved boundary layers, with the addition of the wall-damping function by Piomelli [50] $\left(1 - e^{-(y^+/A^+)^3}\right)$. In both these studies a near-wall treatment was necessary due to the high levels of the Smagorinsky SGS viscosity near the wall resulting in excessive turbulence damping.

In the present work, the grids are designed to be only mildly anisotropic near the wall, with cell aspect ratios below 20, and therefore suitable for LES near the wall. The under-resolution of the boundary layer, on the other hand, means that the subgrid-scale viscosity near the wall needs to be corrected, similarly to the above-mentioned studies. In this case this is necessary because the SGS viscosity obtained with the σ -model is *underestimated*, as it is zero by construction at the wall. Kawai and Larsson [51] tackled the same problem for the dynamic Smagorinsky model by using one-sided formulas for the derivatives and by extrapolating from the interior nodes the model coefficient. Here we choose a less complex approach, by adjusting the near-wall turbulent viscosity, similarly to Templeton et al. [52], in a RANS fashion:

$$\nu_{nw} = (\kappa y)^2 \left(1 - e^{-(y^+/A^+)^3}\right) S \quad (7)$$

with $S = \sqrt{2S_{ij}S_{ij}}$, $\kappa = 0.41$ and $A^+ = 25$.

This near-wall correction is necessary only in the first two cells from the wall, since the LES model can quickly resume the LES behaviour in the adjacent layers. Tests with one cell and three cells were performed, showing one cell not being sufficient to ensure the appropriate wall shear stress with the chosen finite-volume code, and three cells not yielding a significantly different result compared to two. It is possible that this choice could be code dependent. This method is adaptive, as it can be used seamlessly with any resolution of the boundary layer (provided that the grid is relatively isotropic and the modelling only involves the inner boundary layer), since it vanishes with a physical behaviour when the boundary layer is fully resolved. In OpenFOAM this correction has been implemented together with the wall shear stress function within a custom library for the turbulent viscosity boundary condition, allowing the separate selection of the SGS model.

In conclusion, the proposed formulation can be classified at the edge between wall-stress models and hybrid RANS-LES as traditionally defined [3]. Its strength is in the low grid requirements, the absence of zonal blending parameters, and its simple and robust implementation. A schematic of the use of ν_{sgs} , ν_{nw} and ν_w is reported in

Figure 1.

Fig. 1 Schematic of the implementation of the wall modelling.

III. Wall modelling validation - Channel flow

A. Simulation details

The validation of the proposed wall modelling was carried out on a turbulent channel flow driven by a pressure gradient at varying Reynolds numbers, to demonstrate its adaptive behaviour with wall resolution. Three LES simulations at $Re_\tau = U_\tau \delta / \nu = 180, 550$ and 5200 (δ being the half-height of the channel and U_τ the mean friction velocity) were performed with `pimpleFoam` on the same numerical grid. The domain size is $2\pi\delta$ in the streamwise direction, 2δ in the wall-normal direction and $\pi\delta$ in the spanwise direction, with 128, 64 and 128 cells respectively for a total of around 1M. The cells are equally spaced in the streamwise and spanwise direction, while a grid stretching in the wall-normal direction ensures a cell aspect ratio on the wall of $\Delta x^+ / y^+ = 5$ and $\Delta x^+ / y^+ = 2.5$, where y^+ is defined from the wall distance to the cell centre, while the notation Δx^+ and Δz^+ corresponds to the cell size. The time step in each simulation satisfies the condition $CFL < 0.5$. The streamwise bulk velocity is imposed to be equal to the average DNS value [53] by adjusting a streamwise constant pressure gradient. The flow field was initialized by gradually scaling an existing solution result for $Re_\tau = 395$. The statistics presented in the following section were sampled over $10\delta / U_\tau$ time units.

B. Results

Figure 2 shows the streamwise mean velocity profiles for the three Reynolds numbers, compared to DNS results [53]. In the case with $Re_\tau = 180$ (calculated $Re_\tau = 182$) the near-wall resolution is sufficient to fully resolve the boundary layer down to the viscous sublayer. The resulting $y^+ = yU_\tau / \nu \approx 2$ indicates that the wall modelling is not active, as designed, and the SGS model is responsible for the correct prediction of the flow. For $Re_\tau = 550$ (calculated

$Re_\tau = 552$) the first cell from the wall falls within the buffer layer at $y^+ \approx 6$, in a range typically avoided in CFD calculations. This case proves more challenging for the model, and the neighbouring two cells present a deviation from the DNS results. Further away from the wall, however, the SGS model quickly recovers the log-law behaviour and obtains good agreement with the true solution, all the way to the centre of the channel. The third and final case, with $Re_\tau = 5200$ (calculated $Re_\tau = 5196$), presents a $y^+ \approx 50$, fully within the log-law region. Here the near-wall correction successfully locates the first velocity values, producing only a slight mismatch between the third and the fourth cell from the wall, before the SGS model takes over.

Fig. 2 Channel flow streamwise mean velocity profiles. The symbols represent the cell-centre values obtained with WMLES ($Re_\tau = 180$, green circles; $Re_\tau = 550$, blue triangles; $Re_\tau = 5200$, red squares). The solid lines are from DNS results [53] (with corresponding colours).

A sensitivity study of the wall modelling to the grid and to the solver was conducted on the case with $Re_\tau = 5200$ and the results are presented in Figure 3. The results obtained with the pressure-based solver on a grid with a $2\times$ refinement in the wall-normal direction ($y^+ \approx 25$, blue $\times$ symbols in the figure) show a very good agreement with those obtained with the reference grid (red squares). This is a first evidence of the grid independence of the wall model, which will be confirmed for more complex geometries in the next section. The density based solver (green $+$ symbols) yields a slightly stronger mismatch with the log law compared to the pressure-based solver, but overall it correctly captures the law-of-the-wall.

Fig. 3 Channel flow streamwise mean velocity profiles for $Re_\tau = 5200$, showing the sensitivity to the solver (pressure-based, red squares; density-based, green +) and to the grid (reference grid, red squares; finer grid, blue $\times$). The solid line is from DNS results [53].

Figure 4 shows the profiles of Reynolds stress components for the three Reynolds numbers against DNS results [53]. The normal streamwise component $\langle u'^2 \rangle^+$ (where u' is the velocity fluctuation in the streamwise direction and the notation $\langle \cdot \rangle$ indicates the time average) is accurately captured for $Re_\tau = 180$, where the wall modelling is not active and the SGS model correctly predicts the fluctuations within the viscous sublayer, the buffer layer and the outer layer. Similarly, the other stress components $\langle u'v' \rangle^+$, $\langle v'^2 \rangle^+$ and $\langle w'^2 \rangle^+$ (where v' and w' are the velocity fluctuation in the wall-normal and spanwise direction respectively) are in very good agreement with the DNS results, with only a slight underprediction of the peak stress. In the second case, $Re_\tau = 550$, the first cell value is well predicted and the peak values are approximately the correct magnitude for all stress components. The peak of $\langle u'^2 \rangle^+$ is further from the wall than the DNS, while the other components of normal stress are in good agreement. In the final case, $Re_\tau = 5200$, the near-wall resolution is not capable of resolving the peak of $\langle u'^2 \rangle^+$, as there are no grid points in this region. However, the wall stress boundary condition yields near-wall values which are in good agreement with the true solution for this component, as can be justified by the assumptions discussed in Section II.B.2. For the other three components, on the other hand, the near-wall correction clearly has a damping effect on the fluctuations in the first few cells, and only in the outer layer is the simulation able to restore the expected behaviour.

Overall, despite the simplicity of the proposed WMLES, the only discrepancy with DNS results is in a few cells adjacent to the solid boundary, as expected due to the under-resolution of the grid; but the mean friction velocity is always correctly predicted and the outer part of the boundary layer agrees with the true solution for all Reynolds numbers studied.

Fig. 4 Channel flow Reynolds stresses. The symbols represent the results obtained with WMLES ($Re_\tau = 180$, green circles; $Re_\tau = 550$, blue triangles; $Re_\tau = 5200$, red squares). The solid lines are from DNS results [53] (with corresponding colours).

IV. Application to a high-lift airfoil

A. Simulation details

One goal of the present work is to test the proposed model's suitability for stall prediction of high-lift airfoils. The case chosen is the multi-element airfoil MD-30P/30N, of which experimental data exists from wind tunnel tests [54]. This airfoil has been extensively used as validation for numerical models or other procedures [9, 12, 13, 55]. The Reynolds number is $9 \cdot 10^6$ based on the stowed chord length and the freestream Mach number is 0.2. The simulations were performed at seven different angles of attack between 8 and 24 degrees.

The grid, shown in Figure 5, is a fully structured c-shaped mesh with a total of 24 million cells. The domain has a radius of 15 chords and is assumed to be periodic in the spanwise direction with a width of 8% of the chord. A preliminary study was conducted at three angles of attack (pre-stall, stall and post-stall) for spans of 4, 8 and 12%, revealing a converged solution for the intermediate one. The near-wall resolution gives a y^+ at the leading edge of the main element of 120 for low AoA up to around 200 for high AoA. The chordwise and spanwise resolutions are $\Delta x^+ = (5-20)y^+$ and $\Delta z^+ = (4-10)y^+$, respectively.

The time step satisfies the condition $CFL < 1$. Each simulation was run for 10 flow through times tU_∞/c to initialize the flow, and another 2.5 for averaging. The computational cost of a simulation was of the order of 100,000 CPUh.

Fig. 5 Grid around the MD-30P/30N airfoil.

B. Results

Figure 6 shows contours of instantaneous velocity magnitude (u/U_∞) around the MD-30P/30N airfoil for four selected angles of attack: 8° , 16° , 21° and 24° . The flow velocity near the leading edge increases with the angle of attack, producing a stronger separation. The wake past the flap, resulting from the merging of the shear layers from the slat and the main element, gets thicker with the AoA. As the angle of attack increases, there is a stronger, thicker, unsteady wake from the slat upper surface that merges with the unsteadiness from the main element top surface. This causes the flap to be fully stalled, and the flow is no longer turned downwards, so losing lift and creating a large wake.

Fig. 6 Contours of velocity magnitude (u/U_∞) around the MD-30P/30N airfoil for four angles of attack (8°, 16°, 21° and 24°, top to bottom).

A grid-independence analysis was conducted for the case with angle of attack 19°. The mean velocity profiles at different locations of the main element ($x/c = 0.11$ and $x/c = 0.85$) and the flap ($x/c = 0.90$ and $x/c = 1.03$) in comparison with experimental data [56] are shown in Figure 7 for the reference grid (red filled circles) and for a grid with a coarser wall-normal spacing (empty red circles). The difference in grid refinement can be seen from the y^+ distribution in Figure 8a. Further work is needed to reveal the limits of grid resolution in all three directions, but the following analysis aims in particular at highlighting the independence on grid aspect ratio, which plays an important role in WMLES. With both grids the effects of the slat wake and of the main element wake on the velocity profiles appears less pronounced than in the experiments. This could be due to the inaccurate prediction of transition producing a thicker boundary layer, or to the diffusive effect of the coarsening of the grid further away from the solid boundaries. Nonetheless, only a slight dependence on the grid is noticeable at the end of the flap, whereas in the vicinity of the walls the predicted velocity is in very good agreement with the experiments.

Fig. 7 Velocity profiles at different locations of the main element and the flap of the MD-30P/30N airfoil compared to experimental data [56].

The friction coefficient C_f was deemed one of the outcomes likely to be most sensitive to the grid resolution and is therefore reported for the two grids in Figure 8b along the top surface of main element, flap and slat (experimental data being available only for the first two). Compared to experimental data [57], C_f is in very good agreement with both grids, especially on the flap and on the main element after $x/c = 0.3$, where the outer boundary layer is very well resolved by the LES and the assumptions of the wall model are respected. On the slat and near the leading edge of the main element a sudden drop appears, indicative of a separation bubble. This could be caused by the under-resolution of the developing boundary layer, given the slight dependency on the grid refinement. Indeed, based on the velocity profiles

on the main element, the near-wall grid spacings for the reference grid expressed as a function of the local boundary layer thickness δ are $\Delta y/\delta \approx 0.02$, $\Delta x/\delta \approx 0.04$ and $\Delta z/\delta \approx 0.03$ near the trailing edge, but only $\Delta y/\delta \approx 0.15$, $\Delta x/\delta \approx 0.8$ and $\Delta z/\delta \approx 0.5$ near the leading edge. The proposed model is also not expected to capture transition accurately, as it assumes fully developed turbulence on all surfaces. However, the obtained result is not incompatible with the information provided by the experimental data. Overall, given the difference in y^+ between the two grids and the very good agreement with the experiments, this is a confirmation of the substantial insensitivity of the proposed wall modelling to grid refinement.

Fig. 8 Plot of y^+ (left) and friction coefficient (right) along the top surface of the MD-30P30N main element and flap. The black symbols are from experimental data [57], while the red filled circles and the red empty circles are from simulations on the reference mesh and on a slightly coarser mesh respectively (only one every two symbols is shown).

The surface pressure distributions obtained in the simulations for angles of attack of 8° , 16° and 21° are presented in Figure 9 in comparison to the experimental ones [54]. The model yields good predictions on pressure and suction sides and captures the suction peak and the subsequent decrease of the pressure coefficient along the suction side. Good agreement is found even on the flap at 21° , where the flow is strongly separated.

Fig. 9 Surface pressure coefficients of the MD-30P/30N airfoil at three angles of attack 8°, 16° and 21°.

Figure 10 shows the lift coefficients of the three elements of the airfoil. The black dashed lines represent the experimental values [54] and the red continuous lines the results obtained with the present model. A good agreement with the experimental results is shown for the slat in Figure 10a, with a bit more deviation as the AoA increases. For the flap, Figure 10b, the lift coefficient is very well predicted even in near-stall conditions, where the flow is detached and the adverse pressure gradient is stronger. With regard to the main element of the airfoil, the model captures the stall at the right angle of attack with only a 1.5% difference in maximum lift, as can be seen in Figure 10c. The overall error in lift prediction for the whole airfoil was found to be always below 4.35%, even at post-stall conditions. This is traditionally where most models struggle to capture the right amount of separation. Even recent DDES attempts [58]

did not show the appropriate drop in lift after stall. A possible explanation for the success of the proposed model is that it removes the damping effect of a thick RANS layer near the wall that is inherent in DES approaches, and the inner boundary layer is therefore favourably affected by the unsteadiness of the outer layer. This is illustrated in Figure 11, which shows for the AoA 24° case instantaneous streamwise velocity fluctuations at the near wall cell centres. Clearly significant velocity fluctuations are present adjacent to the wall, unlike for example in DES.

Fig. 10 Lift coefficients of the MD-30P/30N airfoil elements.

Fig. 11 Details of the airfoil surfaces coloured by near-wall cell-centre streamwise velocity fluctuations u'/U_∞ for angle of attack 24° .

These results for the MD-30P/30N high-lift multi-element airfoil provide evidence of the suitability of the method for the accurate prediction of stall. Unlike other methods in literature, this method has been tested on a wide range of angles of attack and achieves good results predicting stall behaviour.

V. Application to the High-lift Common Research Model (HL-CRM)

A. Simulation details

The suitability of the proposed WMLES for a complex, realistic aircraft model is illustrated by simulating the NASA High-Lift Common Research Model (HL-CRM) that has been widely used as a CFD workshop test case[59].

The high-lift CRM configuration was designed to serve as a test case for high-lift simulations. The geometry is representative for a wide-body commercial aircraft. The model consists of an aircraft in landing configuration, split along a symmetry plane, with fuselage, wing, a deployed leading-edge slat and two (inboard and outboard) trailing-edge flaps separated by a thin gap. To reduce complexity, and matching the workshop, the configuration omitted the tail, pylon, engine, and flap support brackets and fairings. Table 1 provides details about the geometry and the operating conditions. The desired Reynolds number of 3.26 million, based on the mean aerodynamic chord (MAC), is achieved with the full scale geometry by scaling up the viscosity appropriately.

Table 1 HL-CRM geometric parameters and operating conditions.

Ma	AoA	MAC	Re _{MAC}
0.2	16°	7 m	3.26M

Fig. 12 Detail of a slice of the HL-CRM grid, at $y=1\text{MAC}$.

Fig. 13 Detail of the surface grid on the wing of the HL-CRM.

No experimental data is currently available, therefore the following results are compared to state-of-the-art RANS solutions from the High-Lift Prediction Workshop (HiLiftPW-3) [60] obtained with grid-converged simulations on grids up to 400M cells. The grid used in the present work was specifically designed for the proposed WMLES, with hybrid structured-unstructured topology and near-wall hexahedral layers, as depicted in Figure 12. The domain consists of half a cube of side 200MAC , with a symmetry plane cutting the fuselage. The farfield boundaries are therefore around 100MAC from the solid boundaries, with progressive clustering towards the aircraft. The surface mesh on the wing was obtained with a quad-dominant method as shown in Figure 13. The resolution at the wall was not required to reach $y^+ \approx 1$, and was set to around 100. This allowed us to keep the cells aspect ratios $\Delta x^+/y^+$ and $\Delta z^+/y^+$ in the range 4 to

40 (corresponding to surface grid spacings of around 0.0007-0.007 MAC), while limiting the total number of cells to 50M. This is a relatively modest grid requirement and is considerably smaller than industrial best practice for low Reynolds number RANS simulations. The computational cost of the simulation was of the order of 500,000 CPUh. A resolution as fine as that of the MD-30P/30N case was not deemed essential as the AoA is far from stall. However, the relatively coarse resolution, especially in the spanwise direction, can limit the accuracy of the results in a study of stall prediction. The required number of cells of such a study for a full aircraft configuration, estimated by scaling up the span of the grid defined in the MD-30P/30N case, would be of the order of one billion, which would be achievable with current industrial scale HPC resources.

B. Results

Figure 14 provides a qualitative representation of the instantaneous flow field around the HL-CRM. The vortical flow structures are depicted by an isosurface of the Q -criterion [61], which defines a vortex as a region where $Q = \frac{1}{2} \left(\|\mathbf{\Omega}\|^2 - \|\mathbf{S}\|^2 \right) > 0$, i.e. where the norm of the vorticity tensor $\mathbf{\Omega}$ dominates that of the strain-rate $\mathbf{S}$. The isosurface is coloured by values of stream velocity. This figure reveals the wing-tip and flap-tip vortices, as well as fine turbulent structures captured by the LES model on the wing. No tripping is required for transition, as the simulation relies on the inherent flow instability to trigger it. It is also interesting to note that separation occurs on the top of the fuselage and these vortical structures eventually merge into those around the wing root. The advantage of an eddy-resolving method against a RANS approach is clear: the unsteady behaviour of the turbulent flow can help trigger flow separation and can potentially provide time-resolved information for aeroacoustics studies.

Fig. 14 Q-criterion of the HL-CRM, coloured by streamwise velocity.

In Figure 15 velocity profiles on the main element and on the flap fall within or very close to the range predicted by RANS simulations from the HiLiftPW-3 workshop [59]. Similarly, Figure 16 shows the surface pressure coefficient at two spanwise locations on the wing, presenting very good agreement with fine-grid RANS solutions [60]. The overall lift coefficient was found to be 2.39, within the range of predictions from the workshop. These results show the possibility of the proposed WMLES to predict essential features such as pressure distribution and mean velocity field, with a computationally efficient eddy-resolving simulation, despite the under-resolution of the boundary layer.

Fig. 15 HL-CRM velocity profiles at 55% of the wing semispan over the main element (left) and the flap (right). For the definition of the location of the profiles, see Rumsey et al. [59]. The symbols are from the present work with WMLES, the grey area is representative of RANS results from the HiLiftPW-3 workshop [59].

Fig. 16 HL-CRM surface pressure coefficients at spanwise locations $y=1\text{MAC}$ (left) and 2.88MAC (right). The symbols are from the present work with WMLES (only one in eight symbols is shown), the solid lines are fine-grid RANS results [60].

VI. Conclusion

A wall modelling approach for Large Eddy Simulation based on a wall stress boundary condition and a thin near-wall correction for the sub-grid scale viscosity is proposed. As there is no longer a need to resolve the viscous sublayer,

it is feasible to generate a grid adjacent to the wall with acceptable aspect ratios that allows the capture of structures close to the wall. The results shown here indicate that this resolution of near wall eddies gives improvements in the prediction of separation as compared to DES - which is limited by the significant RANS layer adjacent to the wall. A further advantage of the approach is that grid sizes can be smaller than typical low Reynolds number RANS grids even at industrial Reynolds numbers. Compared to other more complex wall-stress-based WMLES methods, which require additional near-wall calculations on separate grids or a slip boundary condition, the ease of implementation and robustness of the proposed model make it a good candidate for a variety of industrial applications.

The validation of the model conducted in Section III for a turbulent channel flow case at a range of Reynolds numbers on a fixed grid showed how good accuracy for the mean flow could be obtained even when the first grid point was at $y^+ \approx 50$. When examining velocity fluctuations, errors were observed in the highest Reynolds number case as it is unable to fully capture the peaks near the wall simply due to a lack of resolution there. Nevertheless, beyond $y^+ \approx 200$ good agreement for velocity fluctuations is found with DNS. This demonstrates how the adaptive approach with a single model works well on a range from well resolved through to a very under-resolved near wall, correctly predicting skin friction and the core turbulence fluctuations.

The application of the model to the flow around the MD-30P/30N high lift airfoil in Section IV and the High Lift Common Research Model in Section V proves its suitability for complex aerodynamics cases.

The MD-30P/30N case showed good prediction of the friction coefficient, with evidence of a substantial insensitivity to the near-wall grid refinement. Excellent agreement with experiments is presented for surface pressure at three angles of attack, including the challenging 21 degree case. Comparison with experimental lift coefficient for the slat, flap and main element also gave excellent agreement, in particular being able to predict maximum lift coefficient and post stall behaviour.

Whilst no experimental data (yet) exists for the High Lift Common Research Model, even a modest 50M cell grid showed eddy-resolving capabilities and matched pressure coefficient distributions and lift to a reference RANS solution. Of note is that the grid was only $\approx 1/3$ the size of the RANS low Reynolds number grid.

The proposed adaptive Wall Modelled LES approach shows considerably promising results for a range of industrial applications as it is a robust, reliable and easy to implement algorithm with accurate prediction of flow separation.

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