Incremental Fringe Formulation for a Complex Source Point Beam Expansion

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Abstract—An Incremental Fringe Formulation (IFF) for the scattering by large metallic objects illuminated by electromagnetic Complex Source Points (CSPs) is presented. This formulation has two main advantages. First, it improves the accuracy of Physical Optics (PO) computations by removing spurious scattered field contributions and, at the same time, substituting them with more accurate Incremental Theory of Diffraction field contributions. Second, it reduces the complexity of PO computations because it is applicable to arbitrary illuminating fields represented in terms of a CSP beam expansion. The advantage of using CSPs is mainly due to their beam-like properties: truncation of negligible beams lowers the computational burden in the determination of the solution. Explicit dyadic expressions of incremental fringe coefficients are derived for wedge-shaped configurations. Comparisons between the proposed method, PO and the Method of Moments (MoM) are provided.

Index Terms—Incremental Fringe Formulation, Incremental Theory of Diffraction, Geometrical Theory of Diffraction, Diffraction, Complex Source Point, Method of Moments

I. INTRODUCTION

A NEW solution to increase the accuracy and improve the numerical efficiency of PO based methods to estimate fields scattered by large structures is presented. The solution consists of an IFF of the field scattered by edges, or shadow lines, in perfect electrically conducting (PEC) objects when they are illuminated by an arbitrary field, which is represented by a CSP expansion.

This work is motivated by the need to efficiently describe directional wave fields in many practical electromagnetic (EM) antenna and scattering problems. In fact, in these problems, a distribution of CSP on a closed regular surface [1]–[3], or collapsed to a single real point, provides an efficient representation of an arbitrary radiating wave field. In addition, a CSP field representation, when combined with the analytic continuation in complex space of ray

techniques such as the Geometrical Theory of Diffraction (GTD) [4] and the Uniform Theory of Diffraction (UTD) [5], may provide a very efficient tool to estimate the fields radiated by large antennas.

Moreover, the scattering by wedges may also be computed with the Incremental Theory of Diffraction (ITD) [6]–[9] and the ITD was recently extended in [10] to compute CSP diffraction by metallic objects. The ITD is considered because, in many cases, it overcomes the typical impairments of the GTD/UTD ray techniques associated with possible ray caustics and with the difficulties of ray tracing in complex space. Furthermore, PO field representation is used by many of the existing electromagnetic codes to deal with the description of fields radiated by large apertures or reflectors, even in the case of illumination by an arbitrary field [11]. Therefore, in order to augment the accuracy and the efficiency of the PO radiated field predictions, this article presents an IFF of the field scattered by edges in perfect electrically conducting objects when illuminated by a CSP representation of an arbitrary field.

To this end, it is supposed that a canonical scatterer is illuminated by a proper linear combination of a finite number N of tilted and scaled CSPs in the complex space $\mathbb{C}^3$. The number of expansion terms required to represent the directional field radiated by general aperture antennas is significantly lower than the one required for a plane-wave expansion for any observation point. The advantage of this representation is that each CSP is in fact treated separately, and then the final representation is recovered using the superposition principle.

This article is organized as follows. Section II provides an overview of the new IFF and its applications. Section III discusses the details of the Incremental End-point Physical Optics (IEPO) coefficients that are introduced in the new IFF formulation. Specifically, the determination of IEPO coefficients proceeds by first identifying the incremental end-point contributions that arise at the truncation of the CSP-PO induced currents over the canonical lit surface [12], [13]. These contributions are obtained by properly applying the generalized ITD localization process [8] to the integral representation of the PO field diffracted by the canonical lit half-plane. Then, the incremental contributions due to the PO end-points are subtracted from the ITD field contributions presented in [10] to obtain the incremental fringe coefficients. Hence, the fringe field to be added to the PO representation is obtained by

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adiabatically distributing and integrating the incremental fringe contributions along the edge discontinuities of the actual surfaces. Section IV contains some numerical examples and additional details of the derivations are presented in the two Appendices.

All the derivations in this article are carried out for time-harmonic fields at the angular frequency ω ; the time convention $\exp(j\omega t)$ is assumed and suppressed throughout.

II. INCREMENTAL FRINGE FORMULATION: AN OVERVIEW

To illustrate this novel IFF, we begin by first describing how it is applied. Then, in Section III, we provide the incremental end-point physical optics coefficients, which are required to apply the IFF.

The PO approximation provides a first order estimate of the field scattered by a metallic object, when illuminated by a single CSP [1], [2], [14], [15] or, more generally, by a discrete distribution of CSPs. PO proceeds by replacing the metallic scatterers with appropriate induced currents $\mathbf{J}^{\text{PO}}(\mathbf{r}_s)$, which are computed as the Geometrical Optics (GO) currents on the lit portions S_{lit} of the scatterer. These currents are allowed to radiate freely in space and the corresponding radiated field $\mathbf{E}^{\text{PO}}(\mathbf{r})$ and $\mathbf{H}^{\text{PO}}(\mathbf{r})$ are determined as [11]

$$\mathbf{E}^{\text{PO}}(\mathbf{r}) = -jk\zeta \iint_{S_{\text{lit}}} \left(\underline{\underline{I}} + \frac{1}{k^2} \nabla \nabla \cdot \right) [\mathbf{J}^{\text{PO}}(\mathbf{r}_s) G_0(\mathbf{r}, \mathbf{r}_s)] ds, \quad (1)$$

$$\mathbf{H}^{\text{PO}}(\mathbf{r}) = \frac{1}{\mu} \iint_{S_{\text{lit}}} \nabla \times [\mathbf{J}^{\text{PO}}(\mathbf{r}_s) G_0(\mathbf{r}, \mathbf{r}_s)] ds, \quad (2)$$

where $G_0(\mathbf{r}, \mathbf{r}_s)$ is the free space Green's function, $\mathbf{r}$ and $\mathbf{r}_s \in S_{\text{lit}}$ denote the position of the observation point and of the current source on the scatterer, respectively.

At high frequency [7], [16]–[19], up to the first order, the dominant contributions to the scattered field are given by the stationary phase and end-point PO contributions, these latter associated with the abrupt truncation of the currents at the Shadow Boundary Lines (SBL) on the object (for example, the edge of the scatterer). The end-point PO contributions do not accurately describe the diffracted field. Indeed, the associated locally tangent canonical configuration at each diffraction point is constituted by a half-lit infinite plane and this is not an accurate model of the actual object in the neighborhood of the SBL. Therefore, it is important to estimate the inaccurate end-point PO contributions in order to subtract them from the PO surface integral, thus providing an estimate of the surface GO contribution to which a more accurate diffracted field may be later added.

As a consequence, the IFF for the scattering by the edge of a metallic wedge, when illuminated by an electromagnetic CSP, is computed as follows. First, a PO estimate $\mathbf{E}_S^{\text{PO}}(\mathbf{r})$ of the scattered field $\mathbf{E}_S(\mathbf{r})$ is computed using (1) by integrating the PO induced currents over the lit portion of the object. As discussed in [7], this finite integration introduces inaccurate diffraction contributions

by the edges of the lit portion of the scatterer. Such inaccurate diffraction contributions are estimated, at the first order, by integrating the IEPO coefficients (34) along the actual edge contour of the scatterer, i.e.

$$\mathbf{E}_d^{\text{PO}}(\mathbf{r}) \sim \int_l d\mathbf{E}_d^{\text{PO}}(\mathbf{r}) dl. \quad (3)$$

Second, a first order diffraction contribution $\mathbf{E}_d^{\text{ITD}}$, obtained by the line integration along the same edge line of the ITD incremental contribution defined in [10], is computed

$$\mathbf{E}_d^{\text{ITD}}(\mathbf{r}) \sim \int_l d\mathbf{E}_d^{\text{ITD}}(\mathbf{r}) dl. \quad (4)$$

$\mathbf{E}_d^{\text{ITD}}$ is more accurate than (3) since it comes from a suitable local canonical configuration that more precisely models the actual edge discontinuity. Third, the scattered field is computed as

$$\mathbf{E}(\mathbf{r}) \sim \mathbf{E}_S^{\text{PO}}(\mathbf{r}) + \int_l d\mathbf{E}^{\text{IFF}}(\mathbf{r}) dl \quad (5)$$

where, using (3) and (4),

$$d\mathbf{E}^{\text{IFF}}(\mathbf{r}) = d\mathbf{E}_d^{\text{ITD}}(\mathbf{r}) - d\mathbf{E}_d^{\text{PO}}(\mathbf{r}) \quad (6)$$

is the *incremental fringe contribution*. The latter, when distributed and integrated along the actual edge discontinuity, provides a correction of the PO estimate (1) of the scattered field. In fact, the IFF coefficient requires the subtraction of the less accurate estimate (3) and replaces it with the more accurate ITD estimate (4).

The field representation (5) is particularly suitable for numerical calculations. This is justified by the fact that the integrand in (5) provides a precise cancellation of the singularities that occur at the shadow boundary aspects of the GO incident and reflected fields for both the IEPO and the ITD incremental coefficients. This will be particularly evident in the sections to follow for the special case of a local canonical half-plane.

III. INCREMENTAL END-POINT PO CONTRIBUTIONS FOR CSP ILLUMINATION

Consider an edge discontinuity in a perfectly conducting object, whose size is large in terms of the wavelength of the radiation source, and a canonical infinite uniform cylindrical wedge tangent to it with a $n\pi$ exterior wedge angle, as depicted in Fig. 1. The shape of the edge is arbitrary, but regular. Let us define also a local rectangular coordinate system, with the $\hat{z}$ axis tangent to the canonical edge at the point Q_l and the $\hat{x}$ axis tangent to its $n = 0$ face.

Suppose the canonical infinite wedge be illuminated by an electric and magnetic CSP with dipole moments $\mathbf{p}_e = p_e \hat{z}$ and $\mathbf{p}_m = p_m \hat{z}$, respectively. Furthermore, two spherical coordinate systems (r, β, ϕ) and $(\tilde{r}', \tilde{\beta}', \tilde{\phi}')$ are used to identify the location of the observation point and of the illuminating CSP dipole, respectively. The tilde ($\sim$) notation is used to represent complex distances and angles.

Fig. 1. Geometry of a locally tangent wedge.

Fig. 2. Geometry for the canonical problem of a half-plane in the IFF formulation.

For the sake of simplicity suppose that only one face (the $n = 0$ face in Figure 2) be illuminated by the CSP. As a consequence, the canonical problem for the PO diffraction consists of a diffracting sheet on $x \geq 0, y = 0$. The incident magnetic field on the lit half-plane $x \geq 0$ generated by the CSP dipole can be calculated as

$$\mathbf{H}_e^i(\mathbf{r}_s) = \frac{1}{\mu} \nabla \times A_z(\mathbf{r}_s, \tilde{\mathbf{r}}') \hat{z} \quad (7)$$

$$\mathbf{H}_m^i(\mathbf{r}_s) = -\frac{1}{j\omega\epsilon\mu} \nabla \times \nabla \times F_z(\mathbf{r}_s, \tilde{\mathbf{r}}') \hat{z} \quad (8)$$

where subscripts (e) and (m) denote the fields associated with the electric and magnetic CSP, respectively. The $\hat{z}$ -components of the complex vector potentials $\mathbf{A}$ and $\mathbf{F}$ are

$$A_z(\mathbf{r}_s, \tilde{\mathbf{r}}') = \mu p_e G_0(\mathbf{r}_s, \tilde{\mathbf{r}}') \quad (9)$$

$$F_z(\mathbf{r}_s, \tilde{\mathbf{r}}') = \epsilon p_m G_0(\mathbf{r}_s, \tilde{\mathbf{r}}') \quad (10)$$

where $\mathbf{r}_s = x_s \hat{x} + z_s \hat{z}$ is the observation point belonging to the surface of the lit half-plane and $\tilde{\mathbf{r}}' = \mathbf{r}' - j\mathbf{b}$ is the complex source location related to the shape of the beam of the CSP. More precisely, $\mathbf{r}' = x' \hat{x} + y' \hat{y} + z' \hat{z}$ is a real vector that defines the center of the real position of the CSP, $\mathbf{b} = b_x \hat{x} + b_y \hat{y} + b_z \hat{z}$ is a real vector whose magnitude $b = |\mathbf{b}|$ defines the beam-collimation distance and the unit vector $\hat{b} = \mathbf{b}/b$ is the beam direction. In (9), (10) we denote the complex Green's function $G_0(\mathbf{r}, \tilde{\mathbf{r}}')$ as the analytical continuation of its real counterpart

$$G_0(\mathbf{r}, \tilde{\mathbf{r}}') = \frac{e^{-jk\tilde{R}}}{4\pi\tilde{R}}, \quad (11)$$

in which $\tilde{R}(\mathbf{r}, \tilde{\mathbf{r}}') = \sqrt{(\mathbf{r} - \tilde{\mathbf{r}}') \cdot (\mathbf{r} - \tilde{\mathbf{r}}')}$ is the complex distance from the source $\tilde{\mathbf{r}}'$ to the observation $\mathbf{r}$ [1].

As mentioned above, a PO description of the problem is obtained by replacing the metallic wedge with a half-sheet of PO induced currents, $\mathbf{J}_e^{\text{PO}}(\mathbf{r}_s) = 2\hat{y} \times \mathbf{H}_e^i(\mathbf{r}_s)$ and $\mathbf{J}_m^{\text{PO}}(\mathbf{r}_s) = 2\hat{y} \times \mathbf{H}_m^i(\mathbf{r}_s)$. By using the analytical continuation of the Debye spectral integral representation of the free-space Green's function $G_0(\mathbf{r}_s, \tilde{\mathbf{r}}')$ in (7) and (8) and taking into account that the differential operators act only on the observation variables, these currents can be calculated as (see Appendix I)

$$\mathbf{J}_e^{\text{PO}}(\mathbf{r}_s) = -\hat{z} \frac{p_e}{4\pi^2} \iint_{-\infty}^{+\infty} e^{-j\mathbf{k}' \cdot (\mathbf{r}_s - \tilde{\mathbf{r}}')} dk'_x dk'_z \quad (12)$$

$$\mathbf{J}_m^{\text{PO}}(\mathbf{r}_s) = -\frac{p_m}{4\pi^2\omega\mu} \iint_{-\infty}^{+\infty} [(k^2 - k_z'^2)\hat{x} + k'_x k'_z \hat{z}] e^{-j\mathbf{k}' \cdot (\mathbf{r}_s - \tilde{\mathbf{r}}')} \frac{dk'_x dk'_z}{k'_y}. \quad (13)$$

It is worth noting that the electric PO currents in (12), due to a $\hat{z}$ -directed electric CSP, have only a $\hat{z}$ -directed component. On the other hand, the electric PO currents in (13), due to a $\hat{z}$ -directed magnetic CSP, have both $\hat{z}$ -directed and $\hat{x}$ -directed components. When the currents (12)-(13) are used in (1)-(2), they provide vector expressions for the PO scattered field by the local canonical half-plane. However, since we are dealing with a canonical problem with a uniform configuration along the $\hat{z}$ -axis, we may restrict our attention only to the $\hat{z}$ -components of $\mathbf{E}^{\text{PO}}(\mathbf{r})$ and $\mathbf{H}^{\text{PO}}(\mathbf{r})$. Indeed, according to a well-established procedure [20], these expressions may be used to find a dyadic formulation for the transverse electric and magnetic field components in the local coordinate system at the diffraction point Q_l . It can be shown that (see Appendix II) the $\hat{z}$ -component spectral representation of the total PO scattered field by the local canonical half-plane may be written as

$$E_{z,e}^{\text{PO}}(\mathbf{r}) = \frac{j\zeta p_e}{16\pi^3} \iiint_{-\infty}^{+\infty} \frac{k_\rho^2}{k^2} \frac{k}{k_y} \frac{1}{k'_x - k_x} e^{j(\mathbf{k}'_\rho \cdot \tilde{\rho}' - \mathbf{k}_\rho \cdot \rho)} dk_x dk'_x e^{-jk_z(z-z')} dk_z \quad (14)$$

for the electric CSP illumination and

$$E_{z,m}^{\text{PO}}(\mathbf{r}) = \frac{j p_m}{16\pi^3 \zeta} \iiint_{-\infty}^{+\infty} \frac{k_\rho^2}{k^2} \frac{k}{k_y} \frac{k_z}{k} e^{j(\mathbf{k}'_\rho \cdot \tilde{\rho}' - \mathbf{k}_\rho \cdot \rho)} dk_x dk'_x e^{-jk_z(z-z')} dk_z, \quad (15)$$

$$H_{z,m}^{\text{PO}}(\mathbf{r}) = \frac{j p_m}{16\pi^3 \zeta} \iiint_{-\infty}^{+\infty} \frac{k_\rho^2}{k^2} \frac{k}{k_y} \frac{1}{k'_x - k_x} e^{j(\mathbf{k}'_\rho \cdot \tilde{\rho}' - \mathbf{k}_\rho \cdot \rho)} dk_x dk'_x e^{-jk_z(z-z')} dk_z \quad (16)$$

for the magnetic CSP illumination. Since both fields are present in this configuration, a cross-polar component will be found in the final result. In the previous equations, $\tilde{\rho}' = \tilde{x}'\hat{x} + \tilde{y}'\hat{y}$, $\rho = x\hat{x} + y\hat{y}$, $\mathbf{k}_\rho = k_x\hat{x} + k_y\hat{y}$ and $\mathbf{k}_\rho' = k'_x\hat{x} + k'_y\hat{y}$. It is worth noting that in the above expressions we considered the case $y > 0$, however the final results apply to $y < 0$ as well. In order to come to canonical expressions suitable for application of the ITD Fourier Transform (ITD-FT) convolution process, as illustrated in [8], let us introduce the spherical coordinates $k'_x = -k'_\rho \cos \gamma'$, $k'_y = -k'_\rho \sin \gamma'$, $k_x = k_\rho \cos \gamma$ and $k_y = k_\rho \sin \gamma$ in the spectral domain ($\gamma' = \alpha' - \tilde{\phi}'$ and $\gamma = \alpha - \phi$) and $\tilde{x}' = \tilde{\rho}' \cos \tilde{\phi}'$, $\tilde{y}' = \tilde{\rho}' \sin \tilde{\phi}'$, $x = \rho \cos \phi$ and $y = \rho \sin \phi$ in the spatial domain, with $\tilde{\rho}' = \tilde{r}' \sin \tilde{\beta}'$ and $\rho = r \sin \beta$. This leads to the expressions for the $\hat{z}$ -components of the total PO diffracted field as

$$\begin{bmatrix} E_z^{\text{PO}}(\mathbf{r}) \\ \zeta H_z^{\text{PO}}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} G_{11}(\mathbf{r}) & G_{12}(\mathbf{r}) \\ 0 & G_{22}(\mathbf{r}) \end{bmatrix} \begin{bmatrix} \zeta p_e \\ p_m \end{bmatrix}, \quad (17)$$

where $G_{il}(\mathbf{r})$ is defined in (18), in which

$$g_{il}(\gamma, \gamma', \phi, \tilde{\phi}') = \left\{ \frac{1}{2} \left[\frac{\sin(\phi - \tilde{\phi}')}{\cos(\gamma - \gamma') + \cos(\phi - \tilde{\phi}')} + \frac{(-1)^{\frac{(i+l)}{2}} \sin(\phi + \tilde{\phi}')}{\cos(\gamma + \gamma') + \cos(\phi + \tilde{\phi}')} \right] \right\}^{(i-l+1)}. \quad (19)$$

The contour integrations C_γ and $C_{\gamma'}$ are defined along $(-j\infty, \pi + j\infty)$ in the complex γ and γ' planes, with clockwise indentations around the relevant poles in order to exclude the GO contributions. Taking into account the symmetry property of the original integration contours, only the even parts of the integrand are retained in the previous expressions. It is clear that (17) together with (19) provides an exact expression of the $\hat{z}$ -component of the PO diffracted field, when a half-plane (or a face of a PEC wedge) is illuminated by an electromagnetic CSP. A uniform asymptotic evaluation of the above equations leads to a PO uniform diffraction coefficient, useful in all those situations where a ray field regime is well established and a ray description of the PO diffraction by large structures may be computationally very efficient [21]. However this is beyond the scope of this work, in which the same expressions will be used to derive incremental PO diffraction coefficients useful for a high-frequency diffracted field description for those situations where the ray field regime is not yet established. To this end, it is worth noting that expression (18) can be cast as the result of the application of a linear operator $L[\cdot]$ to the product of two spectral functions $U_{il}^{(l)}(\cdot)$ as

$$G_{il}(\mathbf{r}) = L \left[\frac{1}{2\pi} \int_{-\infty}^{+\infty} U_{il}(k_\rho, \rho) U_{il}'(k_\rho, \tilde{\rho}') e^{-jk_z(z - \tilde{z}')} dk_z \right], \quad (20)$$

where $i = 1, 2$ and $l = 1, 2$. By using the ITD-FT localization process outlined in [8], it is found that the

desired PO incremental contributions associated with the generic edge point Q_l are

$$\begin{bmatrix} dE_z^{\text{PO}}(\mathbf{r}) \\ \zeta dH_z^{\text{PO}}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} dG_{11}(\mathbf{r}) & dG_{12}(\mathbf{r}) \\ 0 & dG_{22}(\mathbf{r}) \end{bmatrix} \begin{bmatrix} \zeta p_e \\ p_m \end{bmatrix}, \quad (21)$$

where

$$dG_{il}(\mathbf{r}) = L[u_{il}(-z, \rho) u_{il}'(\tilde{z}', \tilde{\rho}')]. \quad (22)$$

It is straightforward to show that in (22) the Fourier transforms of the two functions $U_{il}^{(l)}(\cdot)$ are

$$u_{il}(-z, \rho) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\sqrt{\frac{k_z}{k}} \right)^{(l-i)} \frac{k_\rho}{k} e^{-k_\rho \rho \cos \gamma} e^{-jk_z z} dk_z, \quad (23)$$

$$u_{il}'(\tilde{z}', \tilde{\rho}') = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left(\sqrt{\frac{k_z^{(l)}}{k}} \right)^{(l-i)} \frac{k'_\rho}{k} e^{-k'_\rho \tilde{\rho}' \cos \gamma'} e^{jk_z \tilde{z}'} dk_z'. \quad (24)$$

Upon substituting (23) and (24) in (22), and after introducing the change of variable $k_z^{(l)} = k \cos \theta^{(l)}$, we obtain

$$dG_{il}(\mathbf{r}) = j \frac{k^2}{32\pi^4} \int_{C_\theta} \int_{C_{\theta'}} \int_{C_\gamma} \int_{C_{\gamma'}} g_{il}(\gamma, \gamma', \phi, \tilde{\phi}') \left(\sqrt{\cos \theta \cos \theta'} \right)^{(l-i)} \sin^2 \theta \sin^2 \theta' e^{-jkr f(\theta, \gamma; \beta)} e^{-jk\tilde{r}' f(\theta', \gamma'; \tilde{\beta}')} d\gamma' d\gamma d\theta' d\theta \quad (25)$$

where $f(\tau, \eta; \omega) = \sin \tau \sin \omega \cos \eta + \cos \tau \cos \omega$, and the contour integrations C_θ and $C_{\theta'}$ are defined along $(-j\infty, \pi + j\infty)$ in the complex θ and θ' planes. The expressions (21) and (25) define the $\hat{z}$ -components of the incremental field diffracted by a uniform half-sheet of PO currents in the form of an angular spectral four-fold integral. Of course, the obtained incremental contribution needs to be asymptotically approximated to provide coefficients usable in engineering applications. For this purpose, the same asymptotic analysis presented in Appendix I of [8] can be applied to expressions in (25) thus obtaining

$$dG_{il}(\mathbf{r}) = j \frac{k}{8\pi^2} D_{il}^{\text{PO}}(\tilde{\nu}; \phi, \tilde{\phi}') \sin \beta \sin \tilde{\beta}' \frac{e^{-jkr}}{r} \frac{e^{-jk\tilde{r}'}}{\tilde{r}'}, \quad (26)$$

where

$$D_{il}^{\text{PO}}(\tilde{\nu}; \phi, \tilde{\phi}') = \left\{ \frac{1}{2} \left[\underbrace{\frac{\sin(\phi - \tilde{\phi}')}{\cos \tilde{\nu} + \cos(\phi - \tilde{\phi}')}}_{d^{\text{PO}}(\tilde{\nu}, \phi - \tilde{\phi}')} + \frac{(-1)^{\frac{(i+l)}{2}} \sin(\phi + \tilde{\phi}')}{\cos \tilde{\nu} + \cos(\phi + \tilde{\phi}')} \right] \right\}^{(i-l+1)} \left(\sqrt{\cos \beta \cos \tilde{\beta}'} \right)^{(l-i)} \quad (27)$$

$$G_{il}(\mathbf{r}) = j \frac{k}{16\pi^3} \int_{\tilde{C}_\gamma} \int_{\tilde{C}_{\gamma'}} g_{il}(\gamma, \gamma', \phi, \tilde{\phi}') \int_{-\infty}^{+\infty} \left(\frac{k_z}{k}\right)^{(l-i)} \frac{k_\rho^2}{k^2} e^{-jk_\rho \tilde{\rho}'} \cos \gamma' e^{-jk_\rho \rho \cos \gamma} e^{-jk_z(z-\tilde{z}')} dk_z d\gamma' d\gamma \quad (18)$$

in which the arguments $\tilde{\beta}'$, $\tilde{\phi}'$ and $\tilde{\nu}$ are defined by

$$\tilde{\nu} = -j \log \left[\tan \left(\frac{\beta}{2} \right) / \tan \left(\frac{\tilde{\beta}'}{2} \right) \right] \quad (28)$$

$$\cos \tilde{\phi}' = \frac{\tilde{x}'}{\tilde{r}' \sin \tilde{\beta}'}, \quad \sin \tilde{\phi}' = \frac{\tilde{y}'}{\tilde{r}' \sin \tilde{\beta}'} \quad (29)$$

$$\tilde{r}' = \sqrt{\tilde{x}'^2 + \tilde{y}'^2 + \tilde{z}'^2} \quad (30)$$

$$\cos \tilde{\beta}' = -\frac{\tilde{z}'}{\tilde{r}'}, \quad \sin \tilde{\beta}' = \frac{\sqrt{\tilde{x}'^2 + \tilde{y}'^2}}{\tilde{r}'} \quad (31)$$

with the branch of the square root in (30) chosen to be $\text{Re}[\sqrt{\cdot}] > 0$.

The $\hat{z}$ -components dE_z^{PO} and dH_z^{PO} are eventually used to provide the dyadic closed-form expression for the incremental PO field $d\mathbf{E}^{\text{PO}}$ in the local ray-field coordinate system [5]. For this purpose, it is worth noting that

$$\begin{bmatrix} dE_z^{\text{PO}}(\mathbf{r}) \\ dH_z^{\text{PO}}(\mathbf{r}) \end{bmatrix} = \sin \beta \begin{bmatrix} dE_\beta^{\text{PO}}(\mathbf{r}) \\ dH_\beta^{\text{PO}}(\mathbf{r}) \end{bmatrix} \quad (32)$$

and that the $\hat{\beta}'$ -components of the incident field at Q_l due to the $\hat{z}$ -oriented electric and magnetic CSP at $\tilde{\mathbf{r}}'$ may be written as

$$\begin{bmatrix} E_{\beta'}^i(\mathbf{r}) \\ E_{\phi'}^i(\mathbf{r}) \end{bmatrix} = jk \sin \tilde{\beta}' \frac{e^{-jk\tilde{r}'}}{4\pi\tilde{r}'} \begin{bmatrix} \zeta p_e \\ p_m \end{bmatrix} \quad (33)$$

Therefore expressions (21) and (26) may be rewritten in the dyadic form

$$\begin{bmatrix} dE_\beta^{\text{PO}}(\mathbf{r}) \\ dE_\phi^{\text{PO}}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} D_{11}^{\text{PO}} & D_{12}^{\text{PO}} \\ 0 & D_{22}^{\text{PO}} \end{bmatrix} \begin{bmatrix} E_{\beta'}^i(\mathbf{r}) \\ E_{\phi'}^i(\mathbf{r}) \end{bmatrix} \frac{e^{-jkr}}{2\pi r}. \quad (34)$$

Note that the PO incremental contribution (34) is not reciprocal, due to the presence of the cross-polar component D_{12}^{PO} . This is expected since the PO field (14)-(16) exhibits a non-reciprocal behavior. The same expression shows a useful symmetry property with respect to β and $\tilde{\beta}'$. Moreover, when the observation and/or the source approach the edge of the local canonical configurations, the term $\cos \tilde{\nu}$ becomes large and the incremental contribution in (26) tends to vanish.

The total PO diffracted field by the entire canonical configuration is obtained by the spatial integration of the incremental contribution (34) along the straight edge $\hat{z}$. This representation exhibits appropriate discontinuities that occur at the shadow boundaries of the GO field, and are provided by the singularities of the incremental PO coefficients $d^{\text{PO}}(\tilde{\nu}, \phi - \tilde{\phi}')$ and $d^{\text{PO}}(\tilde{\nu}, \phi + \tilde{\phi}')$. It is a straightforward matter to prove that the incident shadow boundary lines (ISBs) occur when $\tilde{\nu} = \pm[(\phi - \tilde{\phi}') - \pi]$ for which two singularity directions (ϕ_i, β_i) ($i = 1, 2$) exist,

where

$$\begin{aligned} \beta_i &= 2 \arctan \left[\left| \tan \frac{\tilde{\beta}'}{2} \right| e^{-(-1)^i \text{Im}\{\tilde{\phi}'\}} \right] \\ \phi_i &= \pi + \text{Re}\{\tilde{\phi}'\} + (-1)^i \arg \tan \frac{\tilde{\beta}'}{2}. \end{aligned} \quad (35)$$

The tangent function is inverted with branch cuts $(-j\infty, -j)$ and $(j, +j\infty)$ on the top Riemann sheet of its argument. Analogous considerations hold for the reflection shadow boundaries (RSBs) for which singularities occur when $\tilde{\nu} = \pm[(\phi + \tilde{\phi}') - \pi]$ along the two singularity directions (ϕ_i, β_i) , ($i = 3, 4$), where

$$\begin{aligned} \beta_i &= 2 \arctan \left[\left| \tan \frac{\tilde{\beta}'}{2} \right| e^{-(-1)^i \text{Im}\{\tilde{\phi}'\}} \right] \\ \phi_i &= \pi - \text{Re}\{\tilde{\phi}'\} - (-1)^i \arg \tan \frac{\tilde{\beta}'}{2}. \end{aligned} \quad (36)$$

One observes that the above singularities are exactly the same as in the ITD coefficients in [10] associated with the upper face of the wedge. This is evident when computing the incremental fringe coefficient for the special case of a local canonical half-plane. It is a straightforward matter to prove that this coefficient assumes the useful form

$$\begin{bmatrix} dE_\beta^{\text{IFF}}(\mathbf{r}) \\ dE_\phi^{\text{IFF}}(\mathbf{r}) \end{bmatrix} = \begin{bmatrix} D_{11}^{\text{IFF}} & D_{12}^{\text{IFF}} \\ 0 & D_{22}^{\text{IFF}} \end{bmatrix} \begin{bmatrix} E_{\beta'}^i(\mathbf{r}) \\ E_{\phi'}^i(\mathbf{r}) \end{bmatrix} \frac{e^{-jkr}}{2\pi r}, \quad (37)$$

where

$$D_{il}^{\text{IFF}}(\tilde{\nu}; \phi, \tilde{\phi}') = \left\{ \frac{1}{2} \left[\frac{\cos \frac{(\phi - \tilde{\phi}')}{2}}{\cos \frac{\tilde{\nu}}{2} + \sin \frac{(\phi - \tilde{\phi}')}{2}} + \frac{(-1)^{\frac{(i+l)}{2}} \cos \frac{(\phi - \tilde{\phi}')}{2}}{\cos \frac{\tilde{\nu}}{2} + \sin \frac{(\phi - \tilde{\phi}')}{2}} \right] \right\}^{(i-l+1)} \left(-\sqrt{\cos \beta \cos \tilde{\beta}'} \right)^{(l-i)} \quad (38)$$

An inspection of the above incremental contributions reveals that this latter is free of singularities at the reflection and incident shadow boundaries, thus making the calculations of the total PO correction term in (5) easy to determine.

Finally, it is worth noting that, when both faces of a wedge with exterior angle $n\pi$ are illuminated, two IEPO coefficients are computed and subtracted from the ITD one: one for the 0-face and one for the n -face. The coefficient for the n -face is calculated by substituting $\tilde{\Phi}^- = \phi - \tilde{\phi}'$ with $-\tilde{\Phi}^-$ and $\tilde{\Phi}^+ = \phi + \tilde{\phi}'$ with $2n\pi - \tilde{\Phi}^+$ in (34).

IV. NUMERICAL RESULTS

Let us consider the scattering from a perfectly conducting square plate illuminated by a single CSP. The geometrical configuration is shown in Figure 3. A $\hat{y}$ -directed electric CSP dipole is placed at a distance $h = 8\lambda$ from a

Fig. 3. Geometry for the scattering by a square plate illuminated by a CSP.

square plate with sides $a = 5\lambda$, and located in the xy plane. The vector associated with the axis of the beam is defined by $\mathbf{b} = \lambda(\sin(8/9\pi), 0, \cos(8/9\pi))$, so the beam points towards one of the edges the plate. Numerical results obtained from the MoM (dash-dotted line), PO (dashed line) and this method (PO+IFF) (solid line) are presented in Fig. 4 and Fig. 5 for observations at a distance $r = 11\lambda$ on the yz plane and xz plane, respectively. In Fig. 4 the $\hat{\theta}$ -component of the scattered electric field through the IFF formulation is plotted for $\alpha \in [0^\circ, 360^\circ]$. This curve is found in good agreement with MoM calculations, except for the region close to grazing observation. As expected, at these observation aspects the first-order ITD incremental diffraction coefficients exhibit a discontinuity that can be overcome with the introduction of double diffraction contributions [22]. These latter have not been taken into account in this formulation, but will be the subject of further investigations. It is also worth noting that the accuracy of PO calculations is significantly improved when adding to it the incremental fringe calculations.

Fig. 4. Scattered $|E_\theta|$ component of the far field for the square plate illuminated by a single CSP electric dipole. Observations are made in the yz -plane.

The same considerations also apply for the $\hat{\phi}$ -component of the scattered electric field shown in Fig. 5 for $\alpha \in [0, 360^\circ]$. However in this case, at grazing observation aspects, the fringe calculation do not exhibit first-order discontinuity in the ITD field. Around 270° , some discrepancy is noted between MoM and PO+IFF results: at

Fig. 5. Scattered $|E_\phi|$ component of the far field for the square plate illuminated by a single CSP electric dipole. Observations are made in the xz -plane.

first-order, the strongly-lit edge at $\alpha = 90^\circ$ provides a spurious and discontinuous contribution at the observation point, where the correct, but weak contribution from the edge at 270° is not sufficient to correct this behavior. Furthermore, the contribution from the vertices in this illumination condition becomes more important, but it is not taken into consideration in this formulation.

The second example that we consider is the more realistic scattering from a perfectly conducting parabolic reflector when illuminated by a horn antenna. The geometrical configuration is shown in Fig. 6. The parabolic reflector has a radius of 10λ and focal distance 30λ , equivalent to a height of the paraboloid $h = 5/6\lambda$. The horn antenna

Fig. 6. Geometry for the scattering by a planar parabolic reflector illuminated by a horn antenna in the focus of the reflector.

operating at $f = 1.645$ GHz is located at the focus of the paraboloid with the main beam axis pointing to the vertex of reflector, as shown in Fig. 6. The horn antenna radiated field, linearly polarized along the $\hat{x}$ -axis, may be completely represented using a proper CSP expansion [23], [24] consisting of 1122 single CSPs located at the center of the local horn reference system. The expansion is performed in such a way that the CSPs have a uniform beam vector distribution, so as to cover uniformly a sphere of zero real radius. However, to estimate the scattered field from the reflector, it is found that 373 CSPs directly contribute to the scattered field and the incremental quantities, using the truncation algorithm. The observation is made at a distance $r = 60\lambda$ from the center of the aperture, for $\theta \in [0^\circ, 180^\circ]$ on a plane cut at $\phi = 45^\circ$. In Figure 7 and Figure 8 the curves relative to the total electric field $|E_\theta|$

obtained from MoM simulations are compared with those obtained from PO+IFF and PO alone, respectively. One observes that the results obtained introducing the CSP fringe corrections to the PO formulation are in a very good agreement with MoM calculations. One also observes that this method provides improved accuracy as compared with only PO results, especially at grazing incidence and in the back-lobe region.

Fig. 7. Total $|E_\theta|$ far field for the parabolic reflector illuminated by a horn antenna for the geometry in Fig. 6. Observations are made in a plane cut at $\phi = 45^\circ$.

Fig. 8. Total $|E_\theta|$ far field for the parabolic reflector illuminated by a horn antenna for the geometry in Fig. 6. Observations are made in a plane cut at $\phi = 45^\circ$.

In the third example, the same horn antenna illuminating the parabolic reflector is located such that its phase center is at $\mathbf{r}' = (-30\lambda \sin \frac{\pi}{9}, 0, 30\lambda \cos \frac{\pi}{9})$, and tilted so that the main beam axis still points to the center of the parabolic dish, as shown in Fig. 9. The same CSP expansion of the previous example for the horn antenna radiated field has been used. Again, the observation is made at a distance $r = 60\lambda$ from the center of the aperture, for $\theta \in [0^\circ, 180^\circ]$ on a plane cut at $\phi = 45^\circ$. Figure 10 and Figure 11 show the total electric field $|E_\theta|$ obtained from MoM simulations and compared with the PO+IFF and PO formulations, respectively. As expected, an overall significantly improved accuracy is observed when introducing the fringe corrections in the PO formulations. Only in the backward region, near $\theta \simeq 170^\circ$ the curve calculated with the proposed method deviates from that by MoM. This discrepancy may be attributed to creeping wave effects,

caused by the off-axis illumination and not accounted for in this formulation.

Fig. 9. Geometry for the scattering by a parabolic reflector illuminated by an off-set horn antenna.

Fig. 10. Total electric field $|E_\theta|$ for geometry in Fig. 9. Observations are made in a plane cut at $\phi = 45^\circ$.

Fig. 11. Total electric field $|E_\theta|$ for geometry in Fig. 9. Observations are made in a plane cut at $\phi = 45^\circ$.

V. CONCLUSIONS

In this paper, an IFF for the field scattered by edges, or shadow lines, in perfect electrically conducting objects when illuminated by CSPs was introduced. This was motivated by the need for a fast and efficient way to obtain more accurate results than the PO simulations for the scattering by metallic objects. The formulation is obtained by a proper analytical continuation in complex space of the relevant geometrical quantities. It is found that the obtained incremental fringe contributions are free of first order GO singularities, since the developed

IEPO incremental coefficients have singularities that cancel out those of the ITD coefficients. Furthermore, the IFF representation of the scattered correction terms allows a straightforward interpretation of the physical phenomena involved in this extension, thus permitting IFF to be a good candidate to be widely employed in codes for the description of the scattering by complex antenna systems. The formulation was tested with both GO and ITD methods and the MoM for large reflector antennas and the results show the effectiveness of the method.

APPENDIX I

Starting from expressions (7) and (8), the incident magnetic field $\mathbf{H}_e^i(\mathbf{r}_s)$ and $\mathbf{H}_m^i(\mathbf{r}_s)$ at the point $\mathbf{r}_s$ on the illuminated face of the half-plane due to an electric and magnetic CSP with dipole moments $\mathbf{p}_e = p_e \hat{z}$ and $\mathbf{p}_m = p_m \hat{z}$, respectively, may be written as

$$\mathbf{H}_e^i = \frac{1}{\mu} \left[\frac{\partial A_z}{\partial y} \hat{x} - \frac{\partial A_z}{\partial x} \hat{y} \right], \quad (39)$$

$$\mathbf{H}_m^i = \frac{j\omega}{k^2} \left\{ \frac{\partial}{\partial z} \left[-\frac{\partial F_z}{\partial x} \hat{x} - \frac{\partial F_z}{\partial y} \hat{y} \right] + \left[\frac{\partial^2 F_z}{\partial x^2} + \frac{\partial^2 F_z}{\partial y^2} \right] \hat{z} \right\}. \quad (40)$$

where, for the sake of simplicity, the spatial dependence from $\mathbf{r}_s$ and $\mathbf{r}'$ has been omitted. We can now calculate the PO currents on the illuminated face of the semi-plane as

$$\mathbf{J}_e^{\text{PO}}(\mathbf{r}_s) = -2p_e \frac{\partial G_0}{\partial y} \hat{z} \Big|_{\mathbf{r}=\mathbf{r}_s}, \quad (41)$$

$$\mathbf{J}_m^{\text{PO}}(\mathbf{r}_s) = \frac{j\omega 2\epsilon p_m}{k^2} \left\{ \left[\frac{\partial^2 G_0}{\partial x^2} + \frac{\partial^2 G_0}{\partial y^2} \right] \hat{x} + \frac{\partial^2 G_0}{\partial x \partial z} \hat{z} \right\} \Big|_{\mathbf{r}=\mathbf{r}_s}. \quad (42)$$

After introducing in the above expressions the analytical continuation of the Debye spectral integral representation

$$G_0(\mathbf{r}, \mathbf{r}') = \frac{1}{8\pi^2 j} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-j[k'_x(x-\tilde{x}') + k'_y|y-\tilde{y}'| + k'_z(z-\tilde{z}')] } \frac{dk'_x dk'_z}{k'_y}, \quad (43)$$

and performing the derivatives with respect to the observation variables, under the assumption that the source lies in the upper y' semi-space, it is a straightforward matter to obtain (12) and (13) where $\mathbf{k}' = k'_x \hat{x} + k'_y \hat{y} + k'_z \hat{z}$.

APPENDIX II

Starting from expression (1) let us consider the elementary contribution

$$d\mathbf{E}^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) = \mathbf{J}^{\text{PO}}(\mathbf{r}_s) G_0(\mathbf{r}, \mathbf{r}_s) + \frac{1}{k^2} \nabla \nabla \cdot \mathbf{J}^{\text{PO}}(\mathbf{r}_s) G_0(\mathbf{r}, \mathbf{r}_s), \quad (44)$$

where $\mathbf{J}(\mathbf{r}_s) = J_x(\mathbf{r}_s) \hat{x} + J_z(\mathbf{r}_s) \hat{z}$ since the currents are defined over a lit half-plane S ($y_s = 0$, $x_s \geq 0$, $-\infty < z_s < +\infty$). Let us now introduce in the above the well known spectral representation of the Green's function

$$G_0(\mathbf{r}, \mathbf{r}_s) = \frac{1}{8\pi^2 j} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} e^{-jg(\mathbf{r}, \mathbf{r}_s)} \frac{dk_x dk_z}{k_y}, \quad (45)$$

where $g(\mathbf{r}, \mathbf{r}_s) = [k_x(x-x_s) + k_y|y| + k_z(z-z_s)]$ and $k_y^2 = k^2 - k_x^2 - k_z^2$. Performing the derivatives with respect to the observation variables leads to

$$d\mathbf{E}^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) = \frac{1}{8\pi^2 j} \iint_{-\infty}^{+\infty} \underline{\underline{K}} \cdot \mathbf{J}^{\text{PO}}(\mathbf{r}_s) e^{-jg(\mathbf{r}, \mathbf{r}_s)} \frac{dk_x dk_z}{k_y}, \quad (46)$$

where

$$\underline{\underline{K}} = \begin{bmatrix} \left(1 - \frac{k_x^2}{k^2}\right) \hat{x}\hat{x} & -\frac{k_x k_z}{k^2} \hat{x}\hat{z} \\ \mp \frac{k_x k_y}{k^2} \hat{y}\hat{x} & \mp \frac{k_z k_y}{k^2} \hat{y}\hat{z} \\ -\frac{k_x k_z}{k^2} \hat{z}\hat{x} & \left(1 - \frac{k_z^2}{k^2}\right) \hat{z}\hat{z} \end{bmatrix}, \quad (47)$$

is a dyadic, in which the $\mp$ signs apply for $y > 0$ and $y < 0$ respectively. As a consequence, the PO radiation integral in (1) may be written as

$$\mathbf{E}^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) = \frac{k\zeta}{8\pi^2} \iint_S \iint_{-\infty}^{+\infty} \underline{\underline{K}} \cdot \mathbf{J}^{\text{PO}}(\mathbf{r}_s) e^{-jg(\mathbf{r}, \mathbf{r}_s)} ds \frac{dk_x dk_z}{k_y}. \quad (48)$$

Let us now restrict the analysis to the case $y > 0$ and consider only the $\hat{z}$ -component of the scattered electric field. It is a straightforward matter to prove that the final results apply also to $y < 0$ as well. By introducing (12) in (48) and interchanging the spatial and spectral integration one obtains

$$\mathbf{E}_e^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) \cdot \hat{z} = -\frac{\zeta p_e}{32\pi^4} \iint_{-\infty}^{+\infty} \iint_{-\infty}^{+\infty} \frac{k_\rho^2}{k^2} \frac{k}{k_y} e^{j\mathbf{k}' \cdot \mathbf{r}'} e^{-j\mathbf{k} \cdot \mathbf{r}} \iint_S e^{-j\mathbf{r}_s \cdot (\mathbf{k}' - \mathbf{k})} ds dk_x dk_z dk'_x dk'_z. \quad (49)$$

The inner integral in the above expression, evaluated over the lit half-plane, provides

$$\iint_S e^{-j\mathbf{r}_s \cdot (\mathbf{k}' - \mathbf{k})} ds = \frac{2\pi \delta(k'_z - k_z)}{j(k'_x - k_x)}, \quad (50)$$

that, introduced in (49), directly leads to (14). The scattered electric field due to the magnetic CSP illumination may be obtained similarly by first introducing (13) in (48), yielding (51). Next, selecting the $\hat{z}$ -component of the above expression and using the identity (50), after a simple algebraic manipulation one clearly obtains (15). Finally, the scattered magnetic field $\mathbf{H}_m^{\text{PO}}(\mathbf{r}, \mathbf{r}_s)$ due to a magnetic CSP illumination may be derived from (51) as

$$\mathbf{H}_m^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) = -\frac{1}{j\omega\mu} \nabla \times \mathbf{E}_m^{\text{PO}}(\mathbf{r}, \mathbf{r}_s). \quad (52)$$

Taking into account that the derivatives act only on the observation variables, the $\hat{z}$ -component of the scattered

$$\mathbf{E}_m^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) = -\frac{p_m}{32\pi^4} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \underline{K} \cdot [k_\rho'^2 \hat{x} + k_x' k_z' \hat{z}] \frac{1}{k_y' k_y} e^{j\mathbf{k}' \cdot \hat{\mathbf{r}}'} e^{-j\mathbf{k} \cdot \mathbf{r}} \iint_S e^{-j\mathbf{r}_s \cdot (\mathbf{k}' - \mathbf{k})} ds_s dk_x dk_z dk_x' dk_z'. \quad (51)$$

magnetic field may be written as

$$\mathbf{H}_m^{\text{PO}}(\mathbf{r}, \mathbf{r}_s) \cdot \hat{z} = -\frac{p_m}{32\pi^4 \zeta} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{k_\rho^2}{k k_y'} e^{j\mathbf{k}' \cdot \hat{\mathbf{r}}'} e^{-j\mathbf{k} \cdot \mathbf{r}} \iint_S e^{-j\mathbf{r}_s \cdot (\mathbf{k}' - \mathbf{k})} ds_s dk_x dk_z dk_x' dk_z' \quad (53)$$

that, after using again (50), provides (16).

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