ITD Double-Edge Diffraction for Complex Source Beam Illumination

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Abstract—A new high-frequency Incremental Theory of Diffraction (ITD) formulation for the double diffraction by metallic wedges when illuminated by Complex Source Points (CSP) is provided. The main motivation is the extension of the class of problems that can be studied using asymptotic (i.e. ray-based and incremental) methods by providing a double diffraction description for CSP, which are considered because they are efficient to analyze electrically large structures. The new formulation provides an accurate asymptotic description of the interaction between two edges in an arbitrary configuration, including slope diffraction contributions. Advantages of the ITD formulation for CSP illumination include avoiding the typical ray-caustic impairments of the GTD/UTD ray techniques and not requiring ray tracing in complex space. Numerical results are presented and compared to a Method-of-Moments analysis to demonstrate the accuracy of the solution.

Index Terms—Incremental Theory of Diffraction, Incremental Theory of Double Diffraction, Geometrical Theory of Diffraction, Diffraction, Complex Source Point, Complex Source Beams, Method of Moments

I. INTRODUCTION

EFFICIENT techniques to represent the illuminating field are often employed in the accurate prediction of the far field radiated or scattered by large structures, such as large reflector antennas. As an example, the Complex-Source-Beam (CSB) expansion is successfully used to represent the radiation from a given realistic source, such as a feed or a feed array, as well as a more general arbitrary directional wave field. An exact representation of scalar fields in terms of CSP launched from a point was first proposed in [1]. An alternative formulation for vector electromagnetic fields has been proposed in [2]–[4], where the field is expanded in terms of a magnetic and an electric CSP-dipole distribution on a sphere enclosing the actual sources. The expansion coefficients may be obtained by matching the CSB expansion to the known far-field pattern of the feed, or to its vector spherical-wave representation obtained from measured data. More recently, Martini and Maci [5], [6] have proposed a non-redundant

CSB representation that has the advantage of minimizing the number of beams and of directly computing the expansion coefficients from the known vector spherical-wave representation. In the application of these techniques, the actual source is replaced with a discrete set of CSBs, whose beams are radially directed from the enclosing sphere [3]. Each complex beam reaches a large object where it may undergo either a reflection or a diffraction by either the surface or the edges, respectively. The overall radiation can be estimated from the superposition of the contributions from only those CSBs that are strongly intercepted by the structure. Thus, a CSP field representation may provide a very efficient tool to estimate the fields radiated by large objects [7], when combined with analytic continuation in complex space of typical ray-techniques such as the Uniform Theory of Diffraction (UTD) [8].

On the other hand, when dealing with large structures, many existing electromagnetic codes resort to Physical Optics (PO) for any type of arbitrary illumination. However, it is well known that the PO approach does not produce accurate field predictions, particularly at side-lobe directions or at shadow zones [9]. To compensate for these inaccuracies, a correction term is added to the PO field estimate by performing a line integration of an incremental fringe field along the edges of the structure. Several techniques have been proposed to derive these elementary fringe contributions, based on PTD [10]–[12], EEW/ILDC [13]–[15] and ITD [16]–[18] for real source illumination. In the last years, an ITD formulation for CSB illumination has been proposed [19] which overcomes both the typical ray-caustic impairments of the GTD/UTD ray techniques and the difficulties of ray tracing in complex space. To augment the accuracy and the efficiency of PO, a fringe formulation of the field diffracted by edges in planar Perfect Electric Conductors (PEC) illuminated by a CSB has been recently presented [20], [21]. There, the total spurious effects due to the edges of the object are corrected by adiabatically distributing and integrating the local incremental fringe field coefficients along the lines of the edges.

However, for certain configurations where more than one metallic edge is present, and for grazing aspects of incidence and/or observation, the effects of multiple interactions between edges can not be overlooked, whether the wedges are either located on the same plane, or they are skewed with respect to each other, or they share a common PEC face [22]–[32]. In fact, at high frequency the dominant

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field associated with a single-diffracted ray is discontinuous due to shadowing by a second wedge. Then, the introduction of a double-diffracted field becomes necessary to compensate for discontinuities occurring mostly at grazing incidence and observation aspects for the diffracted field.

The novelty of this work is the introduction of an incremental double-diffraction formulation for CSP illumination, with the aim to overcome the problem of the discontinuities in the first-order incremental diffracted field contributions under CSP illumination of complex structures. ITD double diffraction coefficients are derived by solving a proper canonical problem for the ITD applied to Complex Source Points [19].

As pointed out in other existing UTD formulations for real source illumination [23], [25], [26], [29], [33], [34], the cascaded application of ordinary UTD coefficients fails when the edge of the second wedge lies within the transition region of the field diffracted by the first wedge. This behavior is due to the rapid spatial variation and to the non ray-optical behavior of the field diffracted by the first edge when it illuminates the second wedge. The same limitation is expected to affect the ITD representation [18], [19], [21], [35], thus preventing a simple cascaded application of the ITD coefficients for single diffraction. Therefore, it is necessary to develop an ITD double-diffraction coefficient that uniformly accounts for the different transitions that may occur. Hence, a formulation based on the application of the reaction principle for the scattering by metallic object is proposed. Equivalent filamentary currents are first introduced; in free space such currents radiate a field that is equivalent to the incremental single-diffracted field by one edge. Then the incremental double-diffracted field is represented as the reaction between the incremental field diffracted by one of the wedges and the filamentary current sources associated with the diffraction by the other wedge. The total double-diffracted field representation requires a two-fold numerical integration in the space domain on each edge of the complex structures. This formulation provides an accurate asymptotic description of the interaction between two edges, which is valid both for skewed separate wedges and for edges joined by a common PEC face. It also explicitly satisfies reciprocity and includes a double incremental slope diffraction augmentation, which provides the correct dominant high-frequency incremental contribution at grazing aspect of incidence and observation.

This article is organized as follows. Section II focuses on the similarities between the UTD and the ITD approach to obtain a double diffraction formulation. Section III presents results for two geometries with interacting edges: a square plate and a disc. All the derivations in this article are carried out for time-harmonic fields at the angular frequency ω ; the time convention $\exp(j\omega t)$ is assumed and suppressed throughout.

II. INCREMENTAL DOUBLE DIFFRACTION FORMULATION

The geometry of the problem consists of a pair of wedges with curved edges l_1 and l_2 , arranged in an ar-

Fig. 1. Geometrical configuration for the Incremental Theory of Double Diffraction.

bitrary configuration as in Fig. 1, and illuminated by a CSP incident beam. The edges may or may not be coplanar, and they may or may not share a common face. The double diffracted field between the two wedges is represented in incremental form on the basis of the ITD formulation for single CSP diffracted fields [19]. In principle, the incremental diffracted field arising at each local point Q_{l_1} , illuminated by a CSP beam, impinges on each local point Q_{l_2} on the second edge l_2 , where it causes a second incremental diffracted field. At high frequency, it is supposed that both the wedges and the local radius of curvature of the two edges l_1 and l_2 are electrically large. Then, the incremental fields at each local point Q_{l_1} and Q_{l_2} may be obtained as those arising from two locally tangent canonical infinite wedges, as sketched out with dashed lines in Fig. 1.

Fig. 2. The canonical configuration for the Incremental Theory of Double Diffraction.

It is now useful to refer to four local spherical edge-fixed coordinate systems: $(\hat{r}'_1, \hat{\beta}'_1, \hat{\phi}'_1)$ and $(\hat{r}_{12}, \hat{\beta}_1, \hat{\phi}_1)$ associated with the incident and diffracted rays at Q_{l_1} on l_1 , respectively; $(\hat{r}_{12}, \hat{\beta}'_2, \hat{\phi}'_2)$ and $(\hat{r}_2, \hat{\beta}_2, \hat{\phi}_2)$ associated with the incident and diffracted rays at Q_{l_2} on l_2 . Using these local reference systems, $S \equiv (\hat{r}'_1, \hat{\beta}'_1, \hat{\phi}'_1)$ denotes the complex coordinates of the illuminating CSP with respect to Q_{l_1}

and $P \equiv (r_2, \beta_2, \phi_2)$ the coordinates of the observation point with respect to Q_{l_2} . The double diffraction mechanism between edges l_1 and l_2 can be associated with the incremental ray-path contribution $S - Q_{l_1} - Q_{l_2} - P$, where (Q_{l_1}, Q_{l_2}) are all the possible pairs of incremental diffraction points along l_1 and l_2 , respectively. The total double-diffracted field $\mathbf{E}_{12}^{dd}(P)$ at P is then reconstructed by superimposing in integral form all the incremental double-diffracted field contributions $d\mathbf{E}_{12}^{dd}(P, Q_{l_2}, Q_{l_1})$ at P , and namely

$$\mathbf{E}_{12}^{dd}(P) = \int_{l_1} \int_{l_2} d\mathbf{E}_{12}^{dd}(P, Q_{l_2}, Q_{l_1}). \quad (1)$$

It is obvious that a complete double diffraction description of the phenomena includes also the corresponding incremental ray-path contribution $S - Q_{l_2} - Q_{l_1} - P$ between edges l_2 and l_1 . However, this contribution is not considered here, since it is easily obtained by swapping Q_{l_1} with Q_{l_2} .

A. Incremental formulation

According to a well-established locality principle for high-frequency phenomena, the double-diffracted incremental field contribution $d\mathbf{E}_{12}^{dd}(P, Q_{l_2}, Q_{l_1})$ is deduced from the canonical problem of double diffraction between two infinite uniform cylindrical wedges locally tangent at Q_{l_1} and Q_{l_2} , with exterior angles $n_1\pi$ and $n_2\pi$, respectively, as shown in Fig. 2. To obtain a useful expression of the double-diffracted field contribution $d\mathbf{E}_{12}^{dd}(P, Q_{l_2}, Q_{l_1})$, we refer to a particular formulation of the reaction principle for the scattering by metallic objects. Accordingly, we start by representing the incremental field at an observation point P diffracted by a canonical metallic wedge, with straight edge l illuminated by a source at S , as the field radiated in free space by a set of filamentary equivalent electric (e) and magnetic (m) current sources $I_{e,m}(P, Q_{l_i}, S)$ located at each incremental point Q_{l_i} , $i = 1, 2$, and flowing along the line l of the edge. Next, we consider the canonical metallic wedge tangent at Q_{l_2} as the scatterer, which is illuminated by two set of sources. One is the equivalent filamentary current sources distributed on the edge l_1 (labeled as a) that radiate the field diffracted by the first wedge; the other is a local $\hat{z}$ -directed unit dipole located at the observation point P (labeled as b). Limiting the analysis to the field generated by the equivalent filamentary electric currents $\mathbf{I}_e$ flowing along the canonical edges and by the illuminating electric dipoles, the reaction principle states that

$$\begin{aligned} \mathbf{E}_{12}^{dd}(P) &= \int_{-\infty}^{\infty} [\mathbf{E}_i^a(Q_{l_2}) \cdot \mathbf{I}_e^a(P, Q_{l_2}, Q_{l_1})] dz_2 = \\ &= \int_{-\infty}^{\infty} [\mathbf{E}_i^a(Q_{l_2}) \cdot \mathbf{I}_e^b(Q_{l_1}, Q_{l_2}, P)] dz_2, \end{aligned} \quad (2)$$

where $\mathbf{E}_i^a(Q_{l_2})$ is the diffracted field incident at Q_{l_2} due to the equivalent currents at Q_{l_1} , $\mathbf{I}_e^a(P, Q_{l_2}, Q_{l_1})$ is the

equivalent free-space electric currents on l_2 that radiate the diffracted field at P when the second wedge is illuminated by $\mathbf{E}_i^a(Q_{l_2})$, $\mathbf{E}_i^b(Q_{l_2})$ is the incident field at Q_{l_2} due to a local $\hat{z}$ -directed unit electric dipole placed at P , and $\mathbf{I}_e^b(Q_{l_1}, Q_{l_2}, P)$ is the equivalent free-space currents on l_2 that radiate the diffracted field at Q_{l_1} when the second wedge is illuminated by $\mathbf{E}_i^b(Q_{l_2})$. The analysis is limited to the local $\hat{z}$ -component of the electric field (soft component), although a similar procedure can be repeated for the local $\hat{z}$ -component of the magnetic field (hard component).

To apply the reaction principle in (2), it is supposed that the single-diffracted field by the first edge $\mathbf{E}_i^a(Q_{l_2})$ is generated by equivalent filamentary free-space sources distributed along the edge l_1 that radiate in the presence of the second wedge. The component of the above incremental field aligned along $\hat{\beta}'_2$ (in the Q_{l_2} reference system) is

$$E_{i,\beta'_2}^a(Q_{l_2}) = \mathbf{M}_\beta(\gamma_{12}) \cdot \mathbf{E}_i^a(Q_{l_2}), \quad (3)$$

where $\mathbf{M}_\beta(\gamma_{12})$ is the first row of the transformation matrix $\underline{\underline{M}}(\gamma_{12})$ between the two local spherical systems at Q_{l_1} and Q_{l_2} , and γ_{12} is the rotation angle defined between the ray-fixed unit vectors $(\hat{\beta}_1, \hat{\phi}_1)$ and $(\hat{\beta}'_2, \hat{\phi}'_2)$, as illustrated in [26]. As a consequence, the $\hat{z}_{l_2}$ -component of the incident field at Q_{l_2} is then

$$\begin{aligned} E_{i,z_{l_2}}^a(Q_{l_2}) &= E_{i,\beta'_2}^a(Q_{l_2})(\hat{\beta}'_2 \cdot \hat{z}_{l_2}) = \\ &= \sin \beta'_2 \mathbf{M}_\beta(\gamma_{12}) \cdot \int_{-\infty}^{\infty} \underline{\underline{D}}(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1) \cdot \mathbf{E}^i(Q_{l_1}) \frac{e^{-jkr_{12}}}{2\pi r_{12}} dz_1, \end{aligned} \quad (4)$$

where $\mathbf{E}^i(Q_{l_1})$ is the incident field at Q_{l_1} by the CSP at S and

$$\begin{aligned} \underline{\underline{D}}(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1) &= \\ &= D_s(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1) \hat{\beta}_1 \hat{\beta}'_1 + D_h(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1) \hat{\phi}_1 \hat{\phi}'_1 \end{aligned} \quad (5)$$

is the incremental diffraction dyadic. Explicit expressions of D_s and D_h with the corresponding arguments are given in [19]. The above field component is parallel to the local equivalent filamentary current source $I_{e,z_{l_2}}^b(Q_{l_1}, Q_{l_2}, P)$ at Q_{l_2} , that radiates at Q_{l_1} the incremental field

$$\begin{aligned} E_{z_{l_2}}^b(Q_{l_1}) &= E_{\beta'_2}^b(Q_{l_1})(\hat{\beta}'_2 \cdot \hat{z}_{l_2}) = \\ &= jk\zeta \sin \beta'_2 \frac{e^{-jkr_{12}}}{4\pi r_{12}} I_{e,z_{l_2}}^b(Q_{l_1}, Q_{l_2}, P) dz_2 \sin \beta'_2. \end{aligned} \quad (6)$$

Since the latter represents the incremental field at Q_{l_1} single-diffracted by the edge l_2 when illuminated by the unit electric dipole in P , we obtain

$$I_{e,z_{l_2}}^b(Q_{l_1}, Q_{l_2}, P) = \frac{2D_s(-\nu_2; \phi'_2, \phi_2) [\mathbf{E}_i^b(Q_{l_2}) \cdot \hat{\beta}_2]}{jk\zeta \sin \beta'_2}. \quad (7)$$

By using (4) and (7), the integral in (2) can be written

as

$$\begin{aligned} & \int_{-\infty}^{\infty} E_{i,zl_2}^a(Q_{l_2}) I_{e,zl_2}^b(Q_{l_1}, Q_{l_2}, P) dz_2 = \\ & = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \mathbf{M}_{\beta}(\gamma_{12}) \cdot \underline{\underline{D}}(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1) \cdot \mathbf{E}^i(Q_{l_1}) \\ & \times \frac{e^{-jkr_{12}}}{2\pi r_{12}} \sin \beta_2 \frac{e^{-jkr_2}}{2\pi r_2} D_s(-\nu_2; \phi'_2, \phi_2) dz_1 dz_2. \end{aligned} \quad (8)$$

A similar procedure exists for the hard polarization. When the notation is compacted in matrix form, the final formulation is

$$\begin{aligned} \mathbf{E}_{12}^{dd}(P) = & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{\underline{D}}_{12}^{dd}(\tilde{\nu}_1, \nu_2; \tilde{\phi}'_1, \phi_1, \phi'_2, \phi_2) \cdot \mathbf{E}^i(Q_{l_1}) \\ & \times \frac{e^{-jkr_{12}}}{2\pi r_{12}} \frac{e^{-jkr_2}}{2\pi r_2} dz_2 dz_1. \end{aligned} \quad (9)$$

Comparing the previous equation with (1), the incremental double-diffracted field contribution at P is

$$\begin{aligned} d\mathbf{E}_{12}^{dd}(P, Q_{l_2}, Q_{l_1}) = & \underline{\underline{D}}_{12}^{dd}(\tilde{\nu}_1, \nu_2; \tilde{\phi}'_1, \phi_1, \phi'_2, \phi_2) \cdot \mathbf{E}^i(Q_{l_1}) \\ & \times \frac{e^{-jkr_{12}}}{2\pi r_{12}} \frac{e^{-jkr_2}}{2\pi r_2} dz_2 dz_1, \end{aligned} \quad (10)$$

which contains the double diffraction dyadic

$$\begin{aligned} \underline{\underline{D}}_{12}^{dd}(\tilde{\nu}_1, \nu_2; \tilde{\phi}'_1, \phi_1, \phi'_2, \phi_2) = \\ = \underline{\underline{D}}(-\nu_2; \phi'_2, \phi_2) \cdot \underline{\underline{M}}(\gamma_{12}) \cdot \underline{\underline{D}}(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1), \end{aligned} \quad (11)$$

and, similarly to (5),

$$\begin{aligned} \underline{\underline{D}}(-\nu_2; \phi'_2, \phi_2) = \\ = D_s(-\nu_2; \phi'_2, \phi_2) \hat{\beta}_2 \hat{\beta}'_2 + D_h(-\nu_2; \phi'_2, \phi_2) \hat{\phi}_2 \hat{\phi}'_2. \end{aligned} \quad (12)$$

The total high-frequency diffracted field by two arbitrary wedges with edges l_1 and l_2 is then obtained by distributing and integrating the incremental contribution in (10) as shown in (1).

B. Even and odd spectral contributions

As in the case of real sources illuminating the wedges [25], [26], it is best to retain in the spectral formulation the product of both the even and the odd parts of each cotangent term associated with the spectral Green's Function of a single wedge [36]. This procedure gives a local double diffraction description that provides, when integrated, the proper asymptotic order in all the different transitions that may occur. As a result, the non-vanishing contributions to the solution for the canonical problem of a pair of wedges are provided by the even parts of the final spectra, i.e. the even parts of $\underline{\underline{D}}_{12}^{dd}$. The latter are obtained from the product of both the two even and odd parts of each cotangent term in the ITD formulation [19]. On the other hand, the mixed terms that result from products between an even and an odd part of the spectrum provide a vanishing contribution when

integrated into the symmetrical spectral domain. The final form of the dyadic double diffraction coefficient is

$$\begin{aligned} \underline{\underline{D}}_{12}^{dd}(\nu_2, \tilde{\nu}_1; \phi_2, \phi'_2, \phi_1, \tilde{\phi}'_1) = \\ \underline{\underline{D}}_{12}^{e,e}(\nu_2, \tilde{\nu}_1; \phi_2, \phi'_2, \phi_1, \tilde{\phi}'_1) + \underline{\underline{D}}_{12}^{o,o}(\nu_2, \tilde{\nu}_1; \phi_2, \phi'_2, \phi_1, \tilde{\phi}'_1), \end{aligned} \quad (13)$$

where

$$\begin{aligned} \underline{\underline{D}}_{12}^{e,e}(\nu_2, \tilde{\nu}_1; \phi_2, \phi'_2, \phi_1, \tilde{\phi}'_1) = \\ \underline{\underline{D}}^e(-\nu_2; \phi'_2, \phi_2) \cdot \underline{\underline{M}}(\gamma_{12}) \cdot \underline{\underline{D}}^e(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1), \end{aligned} \quad (14)$$

is the even part of the ITD double diffraction coefficient and

$$\begin{aligned} \underline{\underline{D}}_{12}^{o,o}(\nu_2, \tilde{\nu}_1; \phi_2, \phi'_2, \phi_1, \tilde{\phi}'_1) = \\ \underline{\underline{D}}^o(-\nu_2; \phi'_2, \phi_2) \cdot \underline{\underline{M}}(\gamma_{12}) \cdot \underline{\underline{D}}^o(\tilde{\nu}_1; \phi_1, \tilde{\phi}'_1) \end{aligned} \quad (15)$$

is the odd part. It is worth pointing out that expression (10) together with (13)-(15) is not a merely analytic continuation in complex space of that for real illuminating sources [26]. Here, the presence of the two terms $\tilde{\nu}_1$ and $-\nu_2$ assures the symmetrical behavior of the incremental double diffraction contribution with respect to the source and observation coordinates, and when (10) is integrated along the actual structure, provides the correct dominant high-frequency contribution at grazing aspect of incidence and observation.

Because of space limitations, the attention was focused on the derivation of the incremental double contributions. Considerations analogous to those in [26] are valid for singularity and discontinuity contributions in the representation of the total double-diffracted field and will be the subject of future investigation.

III. NUMERICAL RESULTS

A. Perfectly conducting square plate

Fig. 3. Geometry for the scattering by a square plate illuminated by a CSP.

Consider the scattering from a perfectly conducting square plate illuminated by a single CSP, as shown in Fig. 3. A $\hat{y}$ -directed electric CSP dipole is placed at $(0, 0, h = 8\lambda)$ above a square plate with sides $a = 5\lambda$, and located in the xy plane. The vector associated with the beam is $\mathbf{b} = \lambda(\sin(8\pi/9), 0, \cos(8\pi/9))$, and the beam points towards one of the edges of the plate. Both

$\hat{\theta}$ - and $\hat{\phi}$ -components of the scattered electric field have been calculated for observations at a distance $r = 11\lambda$ and presented in Fig. 4 for the E-plane and Fig. 5 for the H-plane, respectively. Numerical results obtained from the Physical Optics (PO) method, the incremental fringe method (PO+IFF) computed as in [21] and the present method (PO+IFF+DD) are plotted for $\alpha \in [0, 2\pi]$ and compared with a method of moment (MoM) solution.

Fig. 4. $|E_{\theta}^s|$ component of the scattered electric field for the square plate illuminated by a single CSP electric dipole. Observations are made in the E-plane.

Fig. 5. $|E_{\phi}^s|$ component of the scattered electric field for the square plate illuminated by a single CSP electric dipole. Observations are made in the H-plane.

The PO+IFF curve is in good agreement with MoM calculations, except for the region close to grazing observation. As expected, at these observation aspects, the first-order ITD incremental diffraction coefficients exhibit a discontinuity that is removed with the introduction of the new ITD double diffraction contributions, as shown in Fig. 4. Additionally, the accuracy of PO calculations is significantly improved when the incremental fringe contributions are added. The same considerations also apply for the $\hat{\phi}$ -component of the scattered electric field shown in Fig. 5. However, in this case, the ITD double diffraction contribution is not as important, because, at grazing observation aspects, the fringe calculation do not exhibit first-order discontinuities in the ITD field. Furthermore, correct contributions from vertices in this illumination condition become important, but they are not properly taken into consideration in this formulation since they are reconstructed only by the truncation of the IFF integration

along the edges of the plate. For both polarizations, once the double diffracted field contribution is added to the PO+IFF formulation, a much closer result to the MoM computations is achieved, especially in the region near to the xy -plane.

The effects of the double-diffracted field are more evident in the E-plane cut: double diffraction contributes to recover the solution for the electric field close to the xy -plane. In the H-plane cut, however, double diffraction plays a negligible role: we believe that the discrepancies found with the MoM are due to the fact that the corner diffraction contributions play a significant role.

B. Perfectly conducting disc

To investigate the effectiveness of the formulation when the scatterer does not include vertices, the scattering from a perfectly conducting disc illuminated by a CSP is now considered, as shown in Fig. 6. A horizontal $\hat{y}$ -directed CSP dipole is placed at $(0, 0, h = 8\lambda)$ above a PEC disc with radius $a = 2.5\lambda$, centered at the origin of the coordinate system and perpendicular to the $\hat{z}$ axis. The vector associated with the beam is $\mathbf{b} = \lambda(\sin(8/9\pi), 0, \cos(8/9\pi))$, so the beam points toward the edge of the disc. The observation is made at a distance $r = 10\lambda$ in the yz plane for $0 \leq \theta \leq \pi$.

Fig. 6. Geometry for the scattering by a planar disc illuminated by a CSP.

In Fig. 7, the $\hat{\theta}$ -component of the total electric field obtained from the proposed PO+IFF+DD formulation is compared with the results from a first order (single diffracted) PO+IFF calculation, and with a MoM prediction. Good agreement between MoM and first order PO+IFF is found everywhere except for the region close to grazing observation, where double diffraction coefficients need to be introduced to compensate for the discontinuity of the ITD incremental diffraction field, as expected. Fig. 8 shows the good matching between PO+IFF+DD and MoM, with the compensating effect over the ITD discontinuity. To provide a better understanding of the double diffraction phenomenon, the scattered electric field $|E_{\theta}^s|$ is plotted in Fig. 9. It can be seen that the double diffraction component compensates the effect of the IFF field and provides a good matching with the MoM solution.

Fig. 7. $|E_\theta|$ component of the total electric field for the case of a disc.

Fig. 8. Detail of the $|E_\theta|$ component of the total electric field for the case of a disc.

Fig. 9. $|E_\theta^s|$ component of the scattered electric field for the case of a disc.

IV. CONCLUSIONS

A double-diffraction formulation for the incremental diffraction by two interacting wedges under CSP illumination has been introduced. The formulation has been obtained by using a reciprocal reaction analysis. In particular, the incremental double-diffracted field is represented as the reaction between the incremental field diffracted by one of the two wedges and equivalent filamentary current sources associated with the diffraction by the other wedge. The derivation of total double-diffracted field requires a two-fold numerical integration in the space domain on each edge of the complex structures. The application to the high-frequency scattering from double-edges structures has been investigated. It has been shown that the for-

mulation satisfies reciprocity and, when combined with a single-diffracted field representation, provides a valid high-frequency description of the scattering by an obstacle illuminated by a CSP including grazing aspects of incidence and observation. Numerical results obtained for the scattering by a square plate and a circular disc are found in very good agreement with those calculated from a MoM solution. These simple examples have been chosen to illustrate the accuracy of the proposed method that uniformly accounts for the different transitions that may occur in reconstructing the total scattered or diffracted field.

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