

SUPPLEMENTARY MATERIALS

S1. Small example of the new simple heuristic method

In order to illustrate the proposed heuristic, consider the 5-jobs, 4-machines problem (Table 1) with the following parameters: $x = 2$, $y = 2$, $d = 3$ and $A = 2$.

First $\xi_{j,0}$ is calculated for $j = \{1, 2, 3, 4, 5\}$, resulting in $\xi_{j,0} = \{48.75, 40.75, 42.75, 53.75, 45\}$. The jobs are sorted in non-descending order of $\xi_{j,0}$, obtaining $\alpha = \{2, 3, 5, 1, 4\}$. The set of unscheduled jobs is $U = \{1, 2, 3, 4, 5\}$.

Since $x = 2$, two complete sequences π^l ($l = \{1, 2\}$) are generated by the heuristic. For $l = 1$, a sequence π^1 with $d = 3$ jobs is initially constructed using LR procedure starting from $\pi^1 = \{\alpha_1\} = \{2\}$ and $U = \{1, 3, 4, 5\}$. To select the job to be inserted in position $k = 2$, the index $\xi_{j,k}$ is calculated, $\xi_{j,2} = \{55.67, 55.67, 59, 56\}$. As job 1 has the lowest value, it is selected to be inserted in $\pi^1 = \{2, 1\}$. Also, job 1 is removed from the set of unscheduled jobs, $U = \{3, 4, 5\}$. For $k = 3$, $\xi_{j,3} = \{74.5, 73.5, 66\}$ and job 5 is appended in the end of $\pi^1 = \{2, 1, 5\}$, $U = \{3, 4\}$.

After the LR procedure, the rest of the sequence π^1 is constructed using the NEH, with reinsertions. The jobs in U are sorted in non-descending order of the sum of the processing times, obtaining $\delta = \{3, 4\}$. Job $\delta_1 = \{3\}$ is inserted in π^1 , generating the partial sequences $\{3, 2, 1, 5\}$, $\{2, 3, 1, 5\}$, $\{2, 1, 3, 5\}$ and $\{2, 1, 5, 3\}$. The best partial sequence is $\pi^1 = \{2, 1, 5, 3\}$ and the total flowtime is 118.

After the NEH insertion, jobs are selected to be reinserted in the sequence by the calculus $W_j = \{84, 75, 75, 84\}$ and ordination of W_j in ascending order. The result is $\gamma = \{1, 5, 2, 3\}$. Then, $y = 2$ jobs are selected from γ to form the reinsertion list $L = \{1, 5\}$. The reinsertion of the job $L_1 = \{1\}$ in π^1 results in a new best partial solution $\pi^1 = \{2, 5, 1, 3\}$ and a total flowtime of 117. Job L_1 is introduced in the restriction list R , $R = \{1\}$. The reinsertion procedure is repeated with job $L_2 = \{5\}$, resulting in a new best partial sequence $\pi^1 = \{5, 2, 1, 3\}$ with a total flowtime of 116. The restriction list is updated, $R = \{5, 1\}$.

The next unscheduled job to be inserted is $\delta_2 = 4$. The best NEH generated partial sequence is $\pi^1 = \{5, 4, 2, 1, 3\}$ with a total flowtime of 185. Again, jobs are selected to be reinserted in the sequence by the calculus and ordination of $W_j = \{128, 116, 132, 136, 144\}$, resulting in $\gamma = \{4, 5, 2, 1, 3\}$. As $R = \{5, 1\}$, the reinsertion list is $L = \{4, 2\}$. Reinserting $L_1 = \{4\}$ results in no improvement in the current solution. Reinserting job $L_2 = \{2\}$ in π^1 results in a new best solution $\pi^1 = \{2, 5, 4, 1, 3\}$ and a total flowtime of 182. Thus, the sequence for $l = 1$ is $\pi^1 = \{2, 5, 4, 1, 3\}$ and the total flowtime is 182.

For $l = 2$, the sequence π^2 with $d = 3$ jobs is constructed using LR procedure starting from $\pi^2 = \alpha_2 = \{3\}$. The solution is completed by following the same steps as described in the previous iteration ($l = 1$). The final solution for $l = 2$ is $\pi^2 = \{3, 5, 4, 2, 1\}$ and the total flowtime is 177. As π^2 is better π^1 , the final solution is $\pi = \{3, 5, 4, 2, 1\}$.

S2. Figures

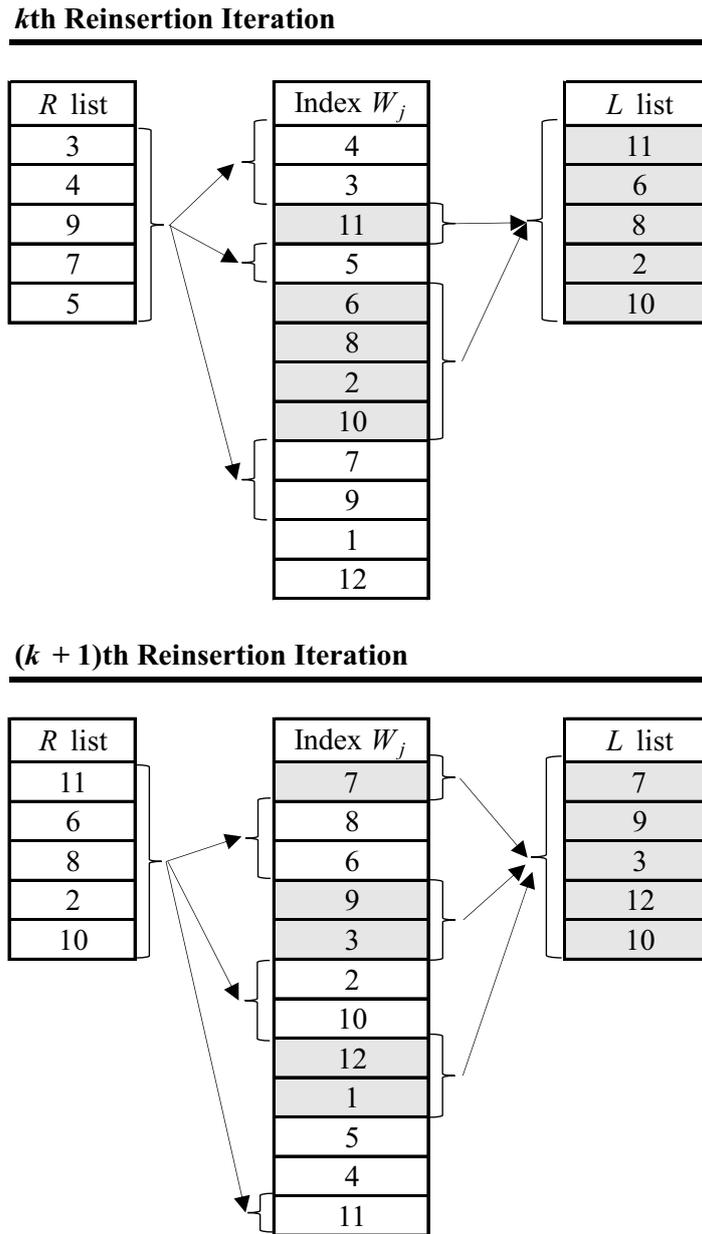
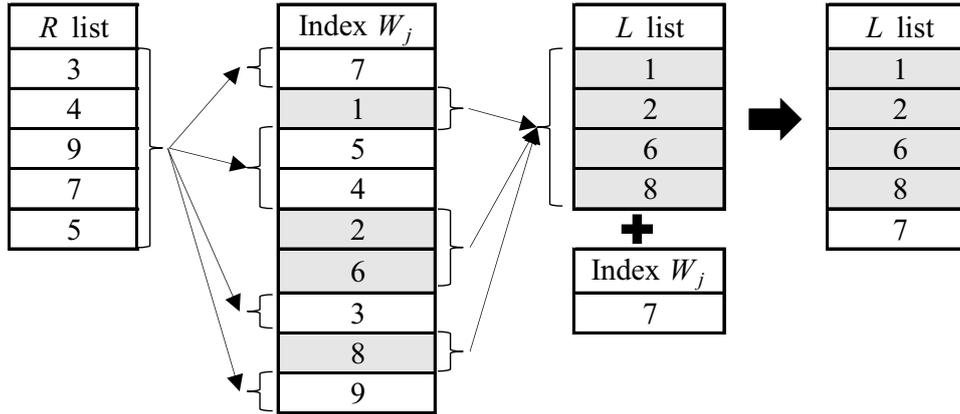


Figure S1.: An example of prioritization for reinsertion with 12 jobs and lists L and R with size 5.

k th Reinsertion Iteration



$(k + 1)$ th Reinsertion Iteration

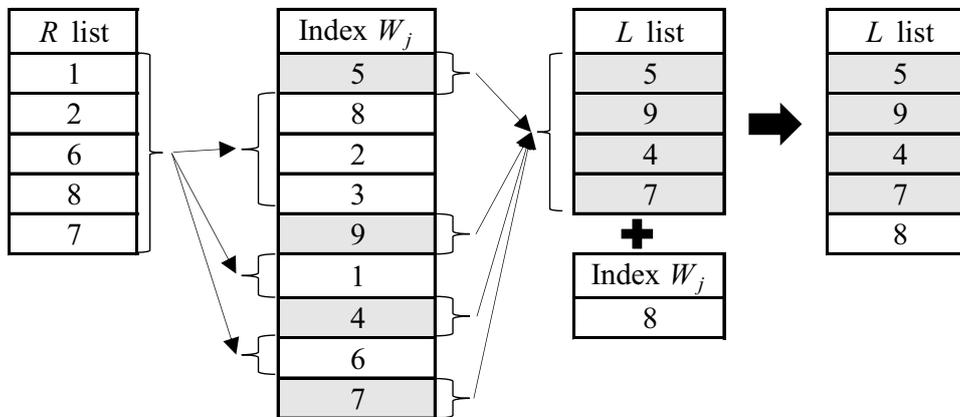


Figure S2.: Example of the prioritization for reinsertion where $|L| < y$ (with 9 jobs and lists L and R with size 5).

S3. Tables

Table S1.: Small example with 5 jobs and 4 machines.

Job	Machine 1	Machine 2	Machine 3	Machine 4
1	7	5	4	2
2	3	4	6	2
3	1	8	1	2
4	7	6	8	2
5	5	4	7	5

Table S2.: ARPD of the different parameters combinations in the Proposed(x,y) heuristic. The values in bold are the best ARPD, in each line.

Instances	$d = \frac{n}{4}$			$d = \frac{n}{2}$			$d = \frac{3n}{2}$		
	$A = \frac{n}{4}$	$A = \frac{n}{2}$	$A = \frac{3n}{2}$	$A = \frac{n}{4}$	$A = \frac{n}{2}$	$A = \frac{3n}{2}$	$A = \frac{n}{4}$	$A = \frac{n}{2}$	$A = \frac{3n}{2}$
50	7.00	6.78	6.64	6.16	6.44	6.44	6.18	6.77	6.46
100	6.25	6.64	6.94	6.23	6.50	6.95	6.16	6.39	6.94
150	5.13	5.42	5.67	5.17	5.43	5.62	4.75	5.15	5.31
200	4.49	4.91	4.97	4.50	4.76	5.03	4.19	4.70	5.07
250	4.30	4.69	4.78	4.23	4.74	4.49	4.03	4.26	4.66
300	3.67	4.20	4.19	3.59	3.77	4.06	3.33	3.70	3.89
350	3.19	3.30	3.39	3.16	3.31	3.80	2.92	3.10	3.50
400	2.75	2.99	3.02	2.89	3.06	3.21	2.55	2.86	3.25
450	2.76	2.95	3.09	2.53	2.74	3.02	2.55	2.91	3.17
500	2.58	2.76	2.94	2.42	2.80	2.93	2.28	2.64	2.93
Avarage	4.21	4.46	4.56	4.09	4.36	4.56	3.89	4.25	4.52

Table S3.: CPU time, in seconds, of the different parameters combinations in the Proposed(x,y) heuristic. The values in bold are the best CPU time, in each line.

Instances	$d = \frac{n}{4}$			$d = \frac{n}{2}$			$d = \frac{3n}{2}$		
	$A = \frac{n}{4}$	$A = \frac{n}{2}$	$A = \frac{3n}{2}$	$A = \frac{n}{4}$	$A = \frac{n}{2}$	$A = \frac{3n}{2}$	$A = \frac{n}{4}$	$A = \frac{n}{2}$	$A = \frac{3n}{2}$
50	0.10	0.11	0.10	0.08	0.08	0.08	0.05	0.06	0.06
100	0.39	0.47	0.39	0.34	0.36	0.34	0.23	0.24	0.28
150	0.90	1.25	0.95	0.84	0.84	0.84	0.58	0.59	0.68
200	1.74	2.22	1.77	1.54	1.68	1.58	1.08	1.10	1.28
250	3.11	3.40	3.13	2.79	2.66	3.03	1.99	2.10	2.00
300	4.68	5.11	5.18	4.48	4.58	4.42	3.19	3.17	3.14
350	7.26	7.01	7.16	6.46	6.25	6.45	4.71	4.85	4.74
400	9.66	9.59	9.67	8.61	8.56	7.41	6.09	6.07	5.41
450	12.57	12.76	12.71	11.54	11.33	9.80	7.20	7.32	7.16
500	15.63	14.16	15.14	15.01	14.85	12.21	9.35	9.52	9.23
Average	5.60	5.61	5.62	5.17	5.12	4.62	3.45	3.50	3.40

Table S4.: ARPD of each heuristic. The values in bold are the best ARPD, in each line.

Instances	FF ($\frac{n}{10}$)	FF (n)	LR ($\frac{n}{10}$)	LR (n)	NEH	LR-NEH(5)	LR-NEH(10)	LR-NEH(15)	WY	FL-LS	Proposed (5,15)	Proposed (5,20)	Proposed (10,15)	Proposed (10,20)
50	23.84	20.88	20.05	17.47	16.26	11.95	11.42	11.17	12.03	9.07	6.18	6.02	5.94	5.43
100	24.94	22.38	20.79	18.25	14.80	11.37	10.78	10.49	10.78	8.30	6.16	5.52	5.48	5.23
150	24.95	22.80	20.70	18.49	12.66	9.78	9.28	9.05	9.13	6.01	4.75	4.54	4.46	4.25
200	24.31	22.64	19.67	17.56	11.48	8.84	8.42	8.18	8.47	5.44	4.19	4.10	3.82	3.60
250	24.01	22.39	19.49	17.41	11.25	7.96	7.60	7.46	6.96	4.54	4.03	3.67	3.59	3.21
300	24.01	22.76	19.55	17.32	9.78	7.31	6.93	6.83	6.60	4.07	3.33	2.90	2.82	2.67
350	22.44	21.29	17.88	16.49	8.43	6.24	5.85	5.71	5.24	3.15	2.92	2.71	2.73	2.45
400	22.45	21.48	18.35	16.26	8.35	5.34	5.14	4.97	5.37	2.54	2.55	2.40	2.12	2.12
450	23.21	21.99	18.92	16.89	7.80	5.61	5.28	5.13	5.01	2.22	2.55	2.25	2.25	1.81
500	23.14	22.16	18.38	16.83	7.58	5.06	4.86	4.75	4.75	2.23	2.28	2.17	2.08	1.92
Avarage	23.73	22.08	19.38	17.30	10.84	7.95	7.55	7.37	7.43	4.76	3.89	3.63	3.53	3.27

Table S5.: CPU time, in seconds, of each heuristic. The values in bold are the best CPU time, in each line.

Instances	FF ($\frac{n}{10}$)	FF (n)	LR ($\frac{n}{10}$)	LR (n)	NEH	LR-NEH(5)	LR-NEH(10)	LR-NEH(15)	WY	FL-LS	Proposed (5,15)	Proposed (5,20)	Proposed (10,15)	Proposed (10,20)
50	0.00	0.03	0.01	0.10	0.00	0.01	0.02	0.03	0.02	0.04	0.05	0.07	0.11	0.13
100	0.02	0.20	0.09	0.88	0.01	0.05	0.10	0.16	0.21	0.32	0.23	0.27	0.44	0.55
150	0.06	0.62	0.37	3.63	0.02	0.14	0.28	0.43	0.69	1.10	0.58	0.64	1.05	1.27
200	0.15	1.51	1.00	10.10	0.03	0.29	0.57	0.87	1.70	2.72	1.08	1.19	1.97	2.40
250	0.29	2.85	2.24	23.25	0.04	0.52	1.01	1.52	3.32	5.44	1.99	2.00	3.33	4.01
300	0.49	4.77	4.39	44.69	0.05	0.81	1.66	2.47	5.84	10.05	3.19	3.05	5.09	6.12
350	0.77	7.63	7.78	77.06	0.07	1.22	2.46	3.71	8.60	16.37	4.71	4.44	7.41	8.87
400	1.16	11.53	13.10	128.21	0.13	1.79	3.60	5.43	13.09	23.81	6.09	6.09	10.25	12.20
450	1.69	17.42	20.15	207.24	0.13	2.48	4.99	7.42	19.39	36.17	7.20	8.16	13.75	17.43
500	2.32	23.75	30.00	305.61	0.20	3.35	6.62	9.67	27.87	50.01	9.35	10.56	17.88	22.77
Avarage	0.69	7.03	7.91	80.08	0.07	1.07	2.13	3.17	8.07	14.60	3.45	3.65	6.13	7.58

Table S6.: Tukey test results of the simple heuristics, with significance level of 95%. The values in bold mean that there is a significant statistical difference between the algorithms in the first and second column.

Heuristic (I)	Heuristic (J)	Mean difference (I-J)	Std. Error	Significance
FL-LS	WY	-2.67600	0.22411	0.000
	LR-NEH(15)	-2.61772	0.22411	0.000
	Proposed(5, 15)	0.86368	0.22411	0.002
	Proposed(5, 20)	1.12948	0.22411	0.000
	Proposed(10, 15)	1.22796	0.22411	0.000
	Proposed(10, 20)	1.48804	0.22411	0.000
WY	FL-LS	2.67600	0.22411	0.000
	LR-NEH(15)	0.05828	0.22411	1.000
	Proposed(5, 15)	3.53968	0.22411	0.000
	Proposed(5, 20)	3.80548	0.22411	0.000
	Proposed(10, 15)	3.90396	0.22411	0.000
	Proposed(10, 20)	4.16404	0.22411	0.000
LR-NEH(15)	FL-LS	2.61772	0.22411	0.000
	WY	-0.05828	0.22411	1.000
	Proposed(5, 15)	3.48140	0.22411	0.000
	Proposed(5, 20)	3.74720	0.22411	0.000
	Proposed(10, 15)	3.84568	0.22411	0.000
	Proposed(10, 20)	4.10576	0.22411	0.000
Proposed(5, 15)	FL-LS	-0.86368	0.22411	0.002
	WY	-3.53968	0.22411	0.000
	LR-NEH(15)	-3.48140	0.22411	0.000
	Proposed(5, 20)	0.26580	0.22411	0.900
	Proposed(10, 15)	0.36428	0.22411	0.666
	Proposed(10, 20)	0.62436	0.22411	0.079
Proposed(5, 20)	FL-LS	-1.12948	0.22411	0.000
	WY	-3.80548	0.22411	0.000
	LR-NEH(15)	-3.74720	0.22411	0.000
	Proposed(5, 15)	-0.26580	0.22411	0.900
	Proposed(10, 15)	0.09848	0.22411	0.999
	Proposed(10, 20)	0.35856	0.22411	0.682
Proposed(10, 15)	FL-LS	-1.22796	0.22411	0.000
	WY	-3.90396	0.22411	0.000
	LR-NEH(15)	-3.84568	0.22411	0.000
	Proposed(5, 15)	-0.36428	0.22411	0.666
	Proposed(5, 20)	-0.09848	0.22411	0.999
	Proposed(10, 20)	0.26008	0.22411	0.909
Proposed(10, 20)	FL-LS	-1.48804	0.22411	0.000
	WY	-4.16404	0.22411	0.000
	LR-NEH(15)	-4.10576	0.22411	0.000
	Proposed(5, 15)	-0.62436	0.22411	0.079
	Proposed(5, 20)	-0.35856	0.22411	0.682
	Proposed(10, 15)	-0.26008	0.22411	0.909

Table S7.: ARPD of the IG algorithms. The values in bold are the best ARPD, in each line.

Instances	vIG DE	vIG DE	vIG DE	vIG DE	vIG DE	vIG DE
	$t = 10$	Improved $t = 10$	$t = 30$	Improved $t = 30$	$t = 60$	Improved $t = 60$
50	1.56	1.43	0.66	0.50	0.16	0.22
100	1.93	1.96	0.99	0.84	0.18	0.18
150	1.51	1.42	0.84	0.67	0.15	0.09
200	1.61	1.25	0.78	0.70	0.25	0.09
250	1.49	1.04	0.73	0.54	0.27	0.12
300	1.13	0.67	0.67	0.33	0.29	0.11
350	1.24	0.71	0.71	0.43	0.38	0.17
400	1.09	0.46	0.60	0.30	0.37	0.09
450	1.29	0.46	0.88	0.46	0.48	0.17
500	1.05	0.48	0.92	0.40	0.57	0.23
Average	1.39	0.99	0.78	0.52	0.31	0.15

Table S8.: Tukey test results of the IG algorithms, with significance level of 95%. The values in bold mean that there is a significant statistical difference between the algorithms in the first and second column.

Heuristic (I)	Heuristic (J)	Mean difference (I-J)	Std. Error	Significance
vIG DE t=10	vIG DE Improved t=10	0.40304	0.04300	0.000
	vIG DE t=30	0.61264	0.04300	0.000
	vIG DE Improved t=30	0.87356	0.04300	0.000
	vIG DE t=60	1.08336	0.04300	0.000
	vIG DE Improved t=60	1.24328	0.04300	0.000
vIG DE Improved t=10	vIG DE t=10	-0.40304	0.04300	0.000
	vIG DE t=30	0.20960	0.04300	0.000
	vIG DE Improved t=30	0.47052	0.04300	0.000
	vIG DE t=60	0.68032	0.04300	0.000
	vIG DE Improved t=60	0.84024	0.04300	0.000
vIG DE t=30	vIG DE t=10	-0.61264	0.04300	0.000
	vIG DE Improved t=10	-0.20960	0.04300	0.000
	vIG DE Improved t=30	0.26092	0.04300	0.000
	vIG DE t=60	0.47072	0.04300	0.000
	vIG DE Improved t=60	0.63064	0.04300	0.000
vIG DE Improved t=30	vIG DE t =10	-0.87356	0.04300	0.000
	vIG DE Improved t=10	-0.47052	0.04300	0.000
	vIG DE t=30	-0.26092	0.04300	0.000
	vIG DE t=60	0.20980	0.04300	0.000
	vIG DE Improved t=60	0.36972	0.04300	0.000
vIG DE t=60	vIG DE t=10	-1.08336	0.04300	0.000
	vIG DE Improved t=10	-0.68032	0.04300	0.000
	vIG DE t=30	-0.47072	0.04300	0.000
	vIG DE Improved t=30	-0.20980	0.04300	0.000
	vIG DE Improved t=60	0.15992	0.04300	0.003
vIG DE Improved t=60	vIG DE t =10	-1.24328	0.04300	0.000
	vIG DE Improved t=10	-0.84024	0.04300	0.000
	vIG DE t=30	-0.63064	0.04300	0.000
	vIG DE Improved t=30	-0.36972	0.04300	0.000
	vIG DE t=60	-0.15992	0.04300	0.003