Force control of semi-active valve lag dampers for vibration reduction in helicopters

R. M. Morales, M. C. Turner, P. Court, R. Hilditch and I. Postlethwaite

Abstract—This paper considers the design of a closed-loop force tracking system for a semi-active damper, designed to be used to reduce in-plane vibrations caused by helicopter rotor blades during steady-state forward flight conditions. The paper describes the development of the control law and includes details of (i) how the initial mathematical model of the system is adapted for controller design; (ii) how a nonlinear dynamic inversion (NDI) control law is modified into a form suitable for implementation; and (iii) how the free parameters in the NDI controller can be optimised for various different operational modes. The success of the approach is demonstrated through both force-tracking simulations and also more comprehensive tests in which the controller is incorporated into a large-scale vibration simulation of the AgustaWestland 101 helicopter. The results show that the NDI-based controller can provide a satisfactory level of performance and hence greatly assist in the reduction of unwanted vibrations.

Index Terms—Force tracking, nonlinear control, semi-active dampers, controller tuning, helicopter vibration

I. INTRODUCTION

ARTICULATED and soft-in-plane rotors contain flap and lag hinges at their root hub to prevent the accumulation of large moments and to prevent undue blade stress from occurring. An undesirable side-effect is that these hinges allow in-plane motion which typically induces resonant couplings between the rotor and fuselage undercarriage on rotor run-up and run-down (ground resonance) [32] and also make the coupled rotor-fuselage system more susceptible to the aeromechanical instability of air resonance [23]. To ameliorate these potentially dangerous resonances, so-called “lag dampers” are used in order to provide extra damping and thereby suppress the in-plane excitation. It is standard practice to use hydraulic dampers for this purpose due to the high damping forces required in overground operations, slope landings and high *g*-manoeuvres [3]. Although the use of conventional hydraulic lag dampers can offer fail-safe operation, they include the following drawbacks: i) Lag dampers offer only a narrow level of damping, hence making it difficult to provide the desired

damping levels on each regime of the flight envelop. Such lack of adaptability could potentially lead to unsatisfactory vibration levels found in the fuselage at certain flight conditions, causing discomfort to the crew and lessening the life of various instruments and components. ii) High damping is necessary only over a narrow range of the flight envelope and therefore these dampers are subjected to large damping forces even in those flight conditions when high damping is not required. Such loading contributes to a faster wear and ageing of the damper and other interfaces with the rotor system [3]. iii) Conventional hydraulic piston lag dampers exhibit a large decrease in damping as the amplitude of motion increases, hence increasing excessively the size and weight of the dampers in order to accommodate all operating conditions [18]. For these reasons the helicopter community has begun to explore dampers with some sort of active or semi-active attributes in order to improve over the drawbacks of using passive hydraulic lag dampers.

One ongoing research approach explores the use of so-called magnetorheological fluid elastomeric (MRFE) dampers in an effort to improve the damper’s adaptability to a wider range of operating conditions. Magnetorheological fluids consist of micro-sized, magnetically polarisable particles dispersed in a carrier medium such a silicon or mineral oil [10]. When a magnetic field is applied to the fluid, particle chains form and the fluid becomes semi-solid displaying viscoplastic behaviour. Changes in the applied magnetic field leads to changes in the damper force hence improving the adaptability characteristics [10]. Laboratory work has shown that lag damper versions of MRFE dampers also have amplitude- and frequency-dependent behaviour [13], [12]. However, the damping can be regulated for small amplitudes and across a wide temperature range using conventional feedback control loops [18]. Other research studies in this area explore the potential of using magnetorheological dampers together with feedback control techniques to mitigate ground resonance instabilities [32].

Another ongoing research approach explores the use of hydraulic dampers equipped with bypass valves in order to manipulate the flow of hydraulic fluid between the damper chambers. These dampers will be referred to in this work as semi-active valve lag dampers (SAVLDs) due to its ability to manipulate the damping characteristics to a certain, but limited extent, to be described later. Despite the limitations in damping manipulation, they can offer significant improvements in terms of adaptability over passive dampers. In addition, they are much simpler to manufacture and install than completely active dampers. This line of research started with the work of Anusonti-Inthra et al [1], [2]. In their work, they report on a configuration of a SAVLD and assumed flap dampers for

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Fig. 1. Overall architecture for vibration helicopter control using SAVLDs.

vibration reduction. The technique was deployed on the light BO-105-type helicopter during high-speed steady flight [30]. The use of SAVLDs to reduce helicopter loads can be done in two ways [3]: i) the opening of the bypass valve is scheduled with fixed values over a number of flight conditions (simple gain scheduling) and hence provide an adequate level of damping on each condition and ii) the valve aperture is modulated as a function of the blade azimuth using a Higher Harmonic Control (HHC) law [15], [16]. Previous studies have demonstrated that the latter approach offer significant improvements when compared to passive hydraulic dampers [31] and also against the gain scheduling approach when reducing vibration on steady-state forward flight conditions [3].

HHC algorithms provide a general platform in order to implement feedback-based vibration reduction techniques [27] which use active and/or semi-active components, such as micro-flaps, active trailing-edge flaps and pitch rod links [5], [6], [19], [8], [24], [9], [25]. HHC algorithms are developed on the representation of the main rotor as that of a linear quasi-static model constructed in the frequency domain which is applicable during steady-state forward flight conditions [15]. The control policies are constructed from the information of vibration sensors (accelerometers) strategically located either across the fuselage of the helicopter or on the main rotor hub and thus avoid vibration propagation to the fuselage. In most active vibration reduction situations, there are two nested control problems: the local control of the device responsible for the actuation; and the global control of the complete system, incorporating many of the active devices and many sensors. This scenario is depicted in Fig. 1 where the actuation element is the SAVLD.

The local control design problem is the main scope of this work and in itself can be challenging due to the nonlinearity and uncertainty associated with the semi-active devices. The scope of this manuscript differs from the work of Botasso et al [3] in the sense that in their work they are concerned with the vibration control design and assumes a linear relation between the harmonics of the vibration loads and the harmonics of the bypass opening. We instead consider the nonlinear dynamics of the damper and wrap a local control loop in order to achieve a satisfactory level of tracking in the damper force with the bypass valve as the control input. We show that embedding the local controller within a conventional HHC vibration control strategy, which assumes a linear relation between the vibration harmonics and the harmonics of the lag damper forces, noticeable reduction in undesired motion can be also be achieved. This approach is expected

to offer better vibration reduction results since the nonlinear characteristics of the damper are directly taken into account in the vibration reduction system. The damper which this work is constructed upon was proposed by Titurus and Lieven in [30]. Simulations are performed with validated models of both, the SAVLD and the coupled rotor-fuselage system of the AgustaWestland five-bade helicopter AW101 during steady-state forward flight conditions. As indicated above, the semi-active damper is a highly nonlinear device and thus a nonlinear dynamic inversion (NDI) control approach [17] is used to provide sufficient levels of reference force tracking in the face of output disturbances and damper variations. This technique is chosen as it is expected to perform better over more traditional control schemes which are based on linear representations of the nonlinear system; provided a reliable nonlinear description is available and uncertainty bounds in the device are relatively small. Due to the limitations of the coupled-rotor fuselage model, we consider the performance of the overall scheme during steady-state forward flight conditions at speeds between 60 and 120 knots and will not address the performance during the regimes where high damping is required.

The paper is structured as follows: section II gives an introduction of the internal composition of the SAVLD and also shows how the general model is reduced to make it suitable for subsequent analysis and control design (section III). Tuning strategies are devised in section IV from three different performance demands which are expressed in the frequency domain. The overall control idea is illustrated through simulation examples; simulations are shown on the local (Section V) and global (Section VI) scale. The paper concludes with some final remarks in section VII.

II. SEMI-ACTIVE VALVE LAG DAMPER

The model of the SAVLD is based on that introduced in [30]. The SAVLD is comprised typically of two chambers (CH1 and CH2) and a piston-rod, a set of relief valves and bypass valves. See Fig. 2. The working chambers are filled with hydraulic fluid and the damping effect is expressed by a force which opposes the piston velocity. The damper force is approximately proportional to the pressure difference of the fluid in both chambers.

The bypass valve is the component that provides the “active” characteristics of the damper. Without it, the damper would behave as a passive lag hydraulic damper, see [29]. The damping characteristics of the SAVLD are modified by means of a variable orifice area on the bypass valve which either augment or reduce the flow of the fluid between the chambers. Roughly speaking, it allows the damping to be either increased or reduced, but the system cannot exhibit “negative” damping.

Relief valves are introduced as a safety mechanism and are activated when the pressure of the fluid in any of the chambers reaches a critical value. In principle, once one of the chambers reaches or exceeds a permissible level of pressure, the respective relief valve becomes open allowing the hydraulic fluid to flow towards the other chamber. This has the ultimate effect of restoring the pressure on this chamber back to normal working conditions.

Fig. 2. Typical structure of a SAVLD.

A. The model

The damper is a nonlinear dynamic system whose behaviour is roughly described by the following differential equations:

$$\begin{aligned}\dot{x} &= B_o (V_1(d)^{-1} + V_2(d)^{-1}) (A_P \dot{d} - Q_o(x) - Q_A(x, A(u)) - Q_r(x)) \\ y &= A_P x\end{aligned}\quad (1)$$

The model is taken from [30] with the exception of the effects of the relief valves when only one of them becomes active. Dynamics of the servomechanism which operates the orifice area are not included in this model and will not be considered in this work. A physical description of the parameters is given below:

- $x(t)$ is the pressure difference across the piston.
- $y(t)$ is the damper force induced by the forced movement of the piston.
- $d(t)$ is the displacement of the piston.
- $u(t)$ is the input which operates the controllable orifice area $A(u)$ of the bypass valves. This enables the active capabilities of the damper.
- $V_1(d)$ and $V_2(d)$ denote the volumes of the two working chambers of the damper.
- $Q_o(x)$ is the volumetric flow rate through the piston orifice.
- $Q_A(x, A(u))$ accounts for the volumetric flow rate through the orifice of the bypass valve.
- $Q_r(x)$ represent the influence on the dynamics of the pressure difference when only one of the relief valves become active.
- B_o is a constant associated with the fluid within the damper.
- A_P stands for the cross-sectional area of the piston.

For the sake of simplicity in the control design and local stability analysis, it is assumed that the relief valves are inactive ($Q_r(x) = 0$). This assumption holds if the corresponding forces exerted by the blades do not exceed a certain threshold. In addition, the saturation properties of the controllable area $A(u)$ with respect to the control input $u(t)$ will also not be considered for subsequent analysis and control design (see

Figure 3.). Note however that the foregoing assumptions are not taken into account in all simulations presented in this manuscript in order to validate the success of the control design from a more realistic point of view.

The state-space description (1) is driven by the disturbance $d(t)$, and its derivative $\dot{d}(t)$, as well as the control input $u(t)$. For this reason it is sometimes convenient to write the damper as a 3-state model, viz

$$\mathcal{M} \sim \begin{cases} \dot{x} &= B_o \left(\frac{1}{V_1(d)} + \frac{1}{V_2(d)} \right) (A_P \dot{d} - (\alpha_1 + \alpha_2 u(t)) h(x)) \\ \dot{V}_1 &= -A_P \dot{d} \\ \dot{V}_2 &= A_P \dot{d} \\ y &= A_P x \end{cases} \quad (2)$$

where

$$h(x) := h_1(x) = \sqrt{|x|} \text{sign}(x) \quad (3)$$

The parameters α_1 and α_2 are defined as

$$\alpha_1 = C_{Do} A_o \sqrt{\frac{2}{\rho}} \quad (4)$$

$$\alpha_2 = C_{DA} k \sqrt{\frac{2}{\rho}} \quad (5)$$

C_{Do} and C_{DA} denote the flow discharge coefficients for the flows through the piston orifice and the bypass orifice, respectively. ρ is the fluid density and A_o is the cross-sectional area of the piston orifice. k is the slope of the linear approximation between the effective area of the bypass valve $A(u)$ and the spool position $u(t)$, see Figure 3. This model is valid for mixed flow models when the pressure difference x can be expressed as a memoryless directional quadratic function of the directional flows $Q_A(x, A(u))$ and $Q_o(x)$, see [30] and [20].

From a control perspective, model (2) simply has two inputs: u (control) and $\dot{d}$ (disturbance). The above model with $h(x) = h_1(x)$ is defined as

$$\mathcal{M}_1 = \mathcal{M} \quad \text{s.t.} \quad h(x) = h_1(x) \quad (6)$$

B. Insight from linear analysis

Treating the model in its original form leads to various problems in linearisation. In particular, at the origin (a key equilibrium of interest), $h(x) = h_1(x)$ is not differentiable and hence it is not possible to obtain a linear model here. To overcome this difficulty, the function $h(x)$ has been approximated by another function which is differentiable at the origin. This function is

$$h_2(x) = m_1 \text{atan}(m_2 x) \quad (7)$$

where m_1 and m_2 are positive constants which have been optimised using numerical algorithms to minimise the difference between $h_1(x)$ and $h_2(x)$ over a certain range of x . Thus an approximation of model (2) is

$$\mathcal{M}_2 = \mathcal{M} \quad \text{s.t.} \quad h(x) = h_2(x) \quad (8)$$

For both models (6) and (8) the equilibrium points of interest are given by

$$\begin{aligned} \begin{bmatrix} x \\ V_1 \\ V_2 \end{bmatrix} &= \begin{bmatrix} 0 \\ V_{10} \\ V_{20} \end{bmatrix} \\ \begin{bmatrix} \dot{d} \\ u \end{bmatrix} &= \begin{bmatrix} 0 \\ u_o \end{bmatrix} \end{aligned} \quad (9)$$

where $V_{10}, V_{20} \geq 0$ and $u_o \in \mathbb{R}_+$. Note that the control effort must be positive in the physical device.

In addition, note also that $\alpha_2 \gg \alpha_1$ for large openings of the bypass valve $A(u)$ and similar discharge coefficients. In this operating region, we can therefore disregard the contributions of $Q_o(x)$ and make the following approximations in order to simplify subsequent analysis and control design:

$$\begin{aligned} \alpha_1 &\approx 0 \\ \alpha_2 &:= \alpha \end{aligned} \quad (10)$$

Linearisation of $\mathcal{M}_2$ around the equilibrium points (9) and taking into account the effects of the bypass valve only yields a linear state-space model

$$\mathcal{M}_3 \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] \quad (11)$$

where the state-space matrices are given by equation (12) overleaf. Note that the linear model is not *controllable*, see [28]. This is perhaps not surprising, given that the lag damper is *semi*-active, meaning that the influence of u on the system is limited. It also shows that caution must be exercised when drawing conclusions from the linear model.

Neglecting the control channel, the transfer function of the system, from $\dot{d}(t)$ to $y(t)$ is given by

$$Y(s) = \frac{A_P^2 B_o (V_{10}^{-1} + V_{20}^{-1})}{s + B_o (V_{10}^{-1} + V_{20}^{-1}) m_1 m_2 \alpha u_o} (s D(s)) \quad (13)$$

If u_o is positive, then the system is guaranteed to be asymptotically stable around the equilibrium point (neglecting uncontrollable modes). $Y(s)$ and $D(s)$ denote the Laplace transform of the damper force and the piston displacement, respectively.

Let us consider the behaviour of such linear representation from a frequency domain perspective. First note that

$$\left| \frac{Y(j\omega)}{j\omega D(j\omega)} \right| \approx \frac{A_P^2}{m_1 m_2 \alpha u_o} \quad (14)$$

for $\omega \ll B_o (V_{10}^{-1} + V_{20}^{-1}) m_1 m_2 \alpha u_o$. It is evident then that for such a frequency region, the influence of the piston velocity on the damper force is reduced by increasing the control input u (and hence the controllable orifice area $A(u)$).

Consider now the other scenario where $\omega \gg B_o (V_{10}^{-1} + V_{20}^{-1}) m_1 m_2 \alpha u_o$. In this case the control input u_o has no effect on y since

$$\left| \frac{Y(j\omega)}{j\omega D(j\omega)} \right| \approx \frac{A_P^2 B_o (V_{10}^{-1} + V_{20}^{-1})}{j\omega} \quad (15)$$

The above result suggests that for sufficiently high frequencies the behaviour of the SAVLD converges to the behaviour obtained when the control input u_o is set to zero.

III. NDI-BASED CONTROLLER

The linear representation of the damper in (13) suggests that conventional linear control design techniques may not be suitable for the damper due to the controllability characteristics between y and u . For this reason the design strategy developed in this section is based on the concept of Nonlinear Dynamic Inversion (NDI), see [17]. This practice has been increasingly popular among engineers (e.g. [11], [14], [4] and [26] for example) and in this case, it provides promising results.

A. Design Procedure

An NDI controller can be designed in a reasonably straightforward manner, but due to some peculiarities of the SAVLD model, some small modifications to the standard NDI design must be made. Firstly, the model of the SAVLD (2) is further simplified in order to achieve a controller which is easily implementable. The main assumptions we work under are: i) The volumetric flow $Q_o(x)$ is disregarded as we expect the bypass valve to be operating mostly with large opening values (see eq. (10)) ii) The volumes V_1 and V_2 are *constant*. This is not completely accurate, but is a good enough approximation for slowly varying disturbance inputs. Thus in this case, the original model, again with the relief valves inactive, becomes

$$\mathcal{M}_4 \sim \begin{cases} \dot{x} &= \beta (A_P \dot{d} - \text{sign}(x) \sqrt{|x|} \alpha u) \\ y &= A_P x \end{cases} \quad (16)$$

where $\beta := B_o (1/V_1 + 1/V_2)$ is a constant. Next choosing the nonlinear part of the control law as

$$u = \frac{1}{\sqrt{|x|}} \text{sign}(x) v \quad (17)$$

gives the linear system

$$\begin{aligned} \dot{x} &= \beta (A_P \dot{d} - \alpha v) \\ y &= A_P x \end{aligned} \quad (18)$$

Note that the transfer function from v to y is simply an integrator. We can then choose v as the standard linear proportional control law

$$v = -k_N (r - y), \quad k_N > 0 \quad (19)$$

where r denotes the reference force signal that is desired to track. Thus our overall (ideal) control law would be

$$u = -k_N \left(\frac{r}{x} - A_P \right) \sqrt{|x|} \quad (20)$$

However as $1/x = \text{sign}(x)/|x|$, it is clear that this control law becomes singular at $x = 0$, so instead we propose to approximate $1/x$ as $1/x \approx \text{sign}(x)/(\epsilon_N + |x|)$ for some sufficiently small ϵ_N . Thus our ‘practical’ control law becomes

$$u = -k_N \sqrt{|x|} \left(\frac{r \text{sign}(x)}{\epsilon_N + |x|} - A_P \right) \quad (21)$$

The control strategy may also be improved by using the linear control law $v = -k_N (k_F r - y)$ with $k_N > 0$ and where k_F can be used to increase the level of feedforward control. Some remarks are in order:

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] = \left[\begin{array}{ccc|cc} -(V_{10}^{-1} + V_{20}^{-1})B_o\alpha u_o m_1 m_2 & 0 & 0 & (V_{10}^{-1} + V_{20}^{-1})B_o A_p & 0 \\ 0 & 0 & 0 & -A_p & 0 \\ 0 & 0 & 0 & A_p & 0 \\ \hline A_p & 0 & 0 & 0 & 0 \end{array} \right] \quad (12)$$

- Notice that, due to the simplified model, $\mathcal{M}_4$, having simply one-state, the state-feedback law, which involves x (the pressure difference) can be replaced by one involving only force measurement, $y = A_p x$, i.e. $x = y/A_p$. The resulting control law then becomes exceptionally simple: no dynamics (states) are required and only output feedback is necessary.
- Although, in general, NDI control laws can lack robustness due to lack of knowledge about the functions that play part in the state equation, in the SAVLD case, the only knowledge which is required is that of A_p , which is not thought to be a problem and hence the control law is expected to be reasonably robust.
- Depending on the relative sizes of ϵ_N and x , rapid large amplitude control signals (known as “chattering”) may occur. This essentially comes from the fact that we would ideally like to use the singular function $1/x$. Chattering can be avoided by choosing k_N smaller or, similarly, ϵ_N larger.

B. Nominal stability.

The NDI control law above assumes that the control signal u can take any value, but in fact it is restricted to be positive, i.e. $u \in \mathbb{R}_+$. This reflects the fact that the actuator can only enforce a variable orifice area which is always a positive numerical value.

Bearing this in mind we can assess the stability of the approximate model, $\mathcal{M}_4$, using Lyapunov’s second method. To analyse stability of the origin we assume $\dot{d} = 0$ and hence d is constant. We have then that the state equation of $\mathcal{M}_4$ becomes

$$\dot{x} = -\beta \text{sign}(x) \sqrt{|x|} \alpha u \quad (22)$$

where β is a positive constant.

Choosing a Lyapunov function $v(x) = x^2$, it follows that

$$\dot{v}(x) = -2\beta |x|^{3/2} \alpha u \quad (23)$$

As $\alpha > 0$ and $\beta > 0$, it follows, for any $u > 0$, that $\dot{v}(x) < 0$ and thus the system will be globally asymptotically stable.

C. Robust stability.

From the above analysis, the NDI control law will tend to be robust to parameter perturbations providing they are such that $\alpha > 0$ and $\beta > 0$, giving some inherent robustness properties to the system.

However, this analysis is conducted on the basis of the simplified model $\mathcal{M}_4$ and in reality a broader class of dynamic uncertainties may be present. In particular, the model in equation (22) neglects the actuator dynamics of the servo-mechanism [7] to operate the controllable orifice areas, which

can be considered as a dynamic uncertainty at the plant input. Such dynamics may add phase lag to the system which in essence means that, for large enough feedback gain, the system will become unstable. However, a simple linear gain/phase margin analysis will not give accurate information for such a system as the spool dynamics occur *within* the nonlinear part of the system. It is therefore difficult to establish robust stability of the system and, in fact, the lack of robustness guarantees are a well-known problem with NDI control laws. Intuitively, however, one may expect that, providing the feedback gain, k_N , is sufficiently low and ϵ_N sufficiently large, the control signals will be small and within the frequency range deliverable by the spool dynamics.

IV. TUNING AND WORKING REGIMES

In steady flight, the disturbance (piston velocity) is predominantly a harmonic signal with a fundamental equal to that of the rotor frequency revolution. Such a frequency will be treated in Hz and denoted by R . Also, the spectrum of the reference force provided by the outer-loop ACSR controller is typically situated on higher multiples of R , i.e. $N_1, N_2, \dots, N_N$ multiples of R Hz. To obtain accurate force tracking, and thus, it is hoped, good vibration reduction is desirable for the controller to both reject the lower harmonic disturbance content and track the higher frequency reference content. Thus, the tuning of the controller will be carried out to enable: tracking of the harmonic content of the reference; rejection of the frequency spectrum of the disturbance; and minimal use of the control signal. It is also desirable to obtain as smooth a control signal as possible in order to avoid physical deterioration in the SAVLD. A performance index suitable for the afore-mentioned specifications is expressed below

$$J = \|W(Y - R)\|_2^2 + \|W_u U\|_2^2 \quad (24)$$

Y , R and U denote the Fourier transforms of the damper force signal $y(t)$, the desired damper force $r(t)$ and the spool position $u(t)$, respectively. W and W_u are filters which are used in the performance function to obtain an optimal controller with certain desirable characteristics. Noting that the SAVLD is a highly nonlinear device, it transpires that tracking and output disturbance rejection requirements are conflicting, and hence a trade off for these two requirements may be desirable. The performance weight W is then chosen to tune the controller according to reference tracking and disturbance rejection demands. The following operational modes of the controller are defined:

- **Lower Harmonic or 1R Rejection (LHR).** For this scenario the controller focuses control efforts to deliver a desirable level of rejection to the 1R harmonic in the damper force.

Fig. 3. Saturation characteristics of the control input.

- **Higher Harmonic Tracking (HHT).** In this case the controller is configured to accomplish a good level of tracking of the higher frequency band $[N_1 R, N_N R]$ and ignore up to some extent the harmonic content in the other region of the spectrum.
- **Lower Harmonic Rejection and Higher Harmonic Tracking (LHR&HHT).** The controller is configured to deliver a satisfactory trade-off between LHR and HHT.

In order to account for the smoothness of the control actions, the designer could opt to have a filter W_u which has the characteristics of a differentiator

$$W_u(s) = \gamma s \quad (25)$$

where γ is chosen to indicate the influence of smoothness actions in the demanded performance.

The tuning procedure can be implemented by finding controller parameters that brings the performance index J to its lowest value using numerical computational tools. The local loop is highly nonlinear and an analytical solution of the optimisation problem would be very difficult. We suggest to execute off-line simulations and run the optimisation algorithms over a finite interval time. Such a procedure can provide helpful initial values of the controller parameters for real implementation of these.

V. SIMULATION RESULTS: THE NDI CONTROLLER

The simulation uses the same parameters values found in [30]. With the purpose of making the simulation more

Fig. 4. Frequency characterisation of the SAVLD obtained by numerical tools and with a harmonic piston displacement of 1mm.

realistic, the SAVLD dynamics used are those given by $\mathcal{M}_1$ ((2) together with $h(x) = h_1(x)$) and the effects of the relief valves are also included. Also, we assume that the orifice area is a saturation function with respect to the control input, see Figure 3. The largest achievable area is 200mm^2 , which corresponds to a spool displacement of $u = 0.5\text{mm}$. Operation in the linear region of $A(u)$ provides that

$$\alpha_2 = 0.01284 \quad (26)$$

A. Open-loop results

Firstly, a frequency domain characterisation of the SAVLD model, $\mathcal{M}_1$ is carried out for a small perturbation and for admissible values of the control input, see Fig. 4. For this, we have injected a sinusoidal piston displacement with 1mm amplitude. The gain plot was obtained by computing

$$\frac{\max |y|}{\max |\dot{d}|} \quad (27)$$

for a set of frequencies.

The qualitative behaviour seems to agree up to some extent with that expressed in section II-B - the effects of the control input become noticeable at low frequencies whereas for high ones, it seems to have no effect on the delivered damper force. In fact, for high frequencies, the behaviour with constant control inputs seems to converge to that behaviour, particularly when $u = 0$. It is also interesting to observe that for the very low frequency region, increments of the control input above 0.05mm actually increase the gain of the system. This somewhat contradicts the predictions of the linear approximation (13) at low frequencies, although we note that the latter was indeed an *approximation*. However, this behaviour is then completely inverted for frequencies (approximately) greater than 1600 rad/sec, that is, the damping effect is reduced by increasing the controllable orifice area. The afore-mentioned characteristics reveals the complex and highly nonlinear properties of the SAVLD.

Fig. 5. Tuning results for LHR.

B. Closed-loop results

Closed-loop simulations were carried out for the force tracking NDI controller using the more realistic damper model, $\mathcal{M}_1$. For these simulations, the piston displacement is given by

$$d(t) = 0.01 \sin(2\pi R t)$$

with $R = 3.5$ Hz. Preliminary simulations of the vibration controller when taking the damper as an ideal force actuator (the desired damper force is the same as the delivered damper force) provided that the harmonic content at 4R was higher than at 5R and 6R. Therefore for the simulations we use a reference force as that shown in the middle subplot of Fig. 5. For simplicity, the tuning computations are carried out only over the controller parameter k_N . A suitable value for ϵ_N was found to be $\epsilon_N = 1e5$ and k_F was set to unity. The selection of the weights was made rather ideally. For LHR, W becomes an ideal low pass filter with a bandwidth of $2R$. For LHR&HHT, $W = 1$ and for HHT, W was chosen as an ideal band-pass filter, rejecting frequencies outside the interval $[3R, 7R]$ Hz. Similarly, W_u was varied, depending on the objective; numerical values of its parameters (see (25)) are given in Table I.

Figure 6 summarises the numerical results obtained by the simulations: (left-hand side subplot) the Fourier coefficient of the fundamental harmonic (R Hz) in the force output is given along (right-hand side subplot) with the percentage ratios of the various harmonic reference signals to the level present in the force output - a value close to 100% means better tracking.

Parameter	LHR	LHR & HHT	HHT
k_N	1.02e-4	2.44e-5	8.37e-6
$\max(A)[\text{mm}^2]$	199.94	88.14	26.57
γ	2e3	1e10	1e10

TABLE I
WEIGHT AND CONTROLLER PARAMETERS FOR DIFFERENT MODES

Fig. 6. Achieved levels of disturbance rejection and harmonic tracking.

The performance function J as a function of the parameter k_N is displayed for every operating mode in Fig. 7. Results when “optimal” values of k_N are used are shown in Figures 5, 8 and 9. The subplot in the middle illustrates the reference force r and the force delivered by the damper, y , together with a scaled version of the disturbance $\hat{d}(t)$. The plot at the top displays the controlled orifice area $A(u)$ which represents a scaled version of the input actions. The bottom graph shows the Fast Fourier Transform (FFT) of the reference and the output force signals to indicate levels of frequency domain tracking.

Observations from these results are as follows:

- The NDI-based controller performs reasonably well in all working modes. As expected the level of fundamental in the force output is lowest in the LHR mode, with some harmonic tracking capabilities shown. Conversely, the HHT mode saw the lowest level of disturbance rejection albeit with much improved harmonic tracking capabilities (up to 81 % at 5R). Not surprisingly, the LHR & HHT mode, yields performance somewhere in between: better LHR capabilities but poorer HHT capabilities than the HHT mode alone.
- Overall, tracking of the higher harmonic content is acceptably good - the lowest and largest tracking ratios are 0.46 and 0.86, respectively. However the performance can be easily deteriorated by minor changes in the reference. Recall the damper can only deliver a force which has the same sign of the disturbance. This fact imposes a major limitation on the achievable performance. For instance, it would be extremely difficult (if not impossible) to achieve any tracking at all of a reference force which is in anti-phase with the piston displacement $\hat{d}$.
- LHR is the mode which is more demanding in terms of control input usage. We observe that in this scenario, the system becomes slightly saturated. Conversely, HHT leads to a lower control gain hence requiring approximately up to 13% of the largest achievable control input.

It is important to note that “perfect” tracking and disturbance rejection can never be achieved with this semi-active damper: the damping level can only be altered, but it always

Fig. 7. Performance function $J(k_N)$.

Fig. 8. Tuning results for LHR and HHT.

must be positive. It is also interesting to note that although the NDI controller was designed using the approximate representation, $\mathcal{M}_4$, it appeared to function satisfactorily using the model $\mathcal{M}_1$ coupled with the relief valve dynamics. Finally, the ultimate criterion upon which the operating mode of the SAVLD controller is decided should be the overall vibration reduction - this is the main discussion of the following section.

Fig. 9. Tuning results for HHT.

VI. SIMULATION RESULTS: THE NDI CONTROLLER AND THE VIBRATION CONTROL SCHEME

This section shows the results of combining the inner-loop SAVLD controller together with an outer-loop ACSR vibration control scheme developed by AgustaWestland. The ACSR controller scheme was constructed using the Higher Harmonic Control concept [15]. In such a control approach, the controlled system is treated in the frequency domain and a linear static model is constructed on-line to correlate a truncated number of harmonics of the control actions with a certain number of harmonics in the vibrations. Taking advantage of the periodicity of the process, the control scheme tries then to reconstruct a force signal to induce destructive interference. Strictly speaking, the ACSR global control scheme is not a feedback-based architecture, but it has, however, proved successful at steady-state flight conditions [15], [8], [21].

In addition, the SAVLD actuator simulation model, $\mathcal{M}_1$ was implemented within the Coupled Rotor Fuselage Model (CFRM) simulation model also developed by AgustaWestland. Roughly speaking the CFRM is a high fidelity nonlinear, time periodic model of the helicopter and rotor fuselage. The CRMF model was set up to represent the dynamics of the five-bladed AW101 helicopter. Thus the complete simulation consisted of the inner-loop NDI control law, the outer-loop ACSR algorithm, the SAVLD model $\mathcal{M}_1$ and the nonlinear time varying model of the helicopter dynamics. This simulation is thought to be a reasonably realistic model of the helicopter behaviour.

The performance of the local controller is shown in Fig-

Fig. 10. Vibration results at 60 knots.

Fig. 11. NDI controller performance at 60 knots.

Fig. 12. Vibration results at 120 knots.

Fig. 13. NDI controller performance at 120 knots.

ures 10-13 for the case where it is optimally tuned for HHT and steady flight conditions with cruise velocities of 60 and 120 knots. The global control scheme becomes activated approximately in the time interval $[8.2, 15.1]$ seconds. By comparison of the peak values of the vibration signals it is evident that vibration reduction is achieved when the control is activated, see Figure 10 for instance. Similar simulations were repeated for the different settings of the local controller: the results are summarised in Figure 14 which shows the ratio (in percentage) between the (peak) vibration signals when the control is on and off - a value of 100% corresponds to no reduction at all in the undesirable motion.

From the simulations outcomes, the following observations follow:

- Simulation outcomes corroborate that with the achieved level of local performance, vibration is reduced.
- Setting the local controller to operate in HHT mode for energy savings provides the best performance in terms of vibration reduction when compared to the other two identified modes: LHR and LHR&HHT. We note a vibration reduction as low as 50% at 120 knots in one of the accelerometers.
- Vibrations are diminished in locations where the accelerometers 1,2 and 4 are located for all operating modes and for all considered velocities in the range between 60 and 120 knots.
- At 120 knots a slight increase of the undesirable motion (about a 10% increase) is registered in accelerometer 3 when the vibration control scheme becomes activated. This is not considered as a major detriment and such an increase is overshadowed by the significant reduction levels registered in the other accelerometers.
- The SAVLD controller performs as expected from the analysis and simulations in the previous section. Variations in the controllable orifice area do not reach the control signal limits when tuned in HHT.

Fig. 14. Overall vibration reduction results.

VII. CONCLUDING REMARKS

We have discussed, in this paper, a NDI-based force tracking system using an SAVLD and which is applicable for helicopter vibration control. Three major observations from this work are listed below:

- An NDI-control strategy developed for the control problem of force tracking with a SAVLD can offer sufficient performance and enable vibration reduction in helicopter at steady-state forward flight conditions.
- Due to the non-linearity in the actuator, conflicting frequency domain specifications define three different operating modes for the local controller. Reference tracking and disturbance rejection demands in different frequency regions must be traded off.
- Interestingly, the best vibration reduction results were obtained when the SAVLD controller was tuned in the mode known as HHT which is less demanding in terms of energy considerations. HHT sets the NDI controller to focus the control efforts on tracking the higher harmonic content of the reference force and ignore up to some extent the influence of the disturbance signal (lower harmonic content). The results were obtained by simulation tools for steady flight conditions at low and high speeds.

Overall, the system performs well for the considered set of conditions and the nominal nonlinear analysis also suggests that there are no major stability issues at the local level. However, the performance may easily deteriorate because of physical limitations in the SAVLD. In addition, recall that the presented vibration reduction scheme is only applicable to steady-state forward flight due to the nature of the HHC algorithms and the limitations of the coupled rotor-fuselage model. The performance of the damper is also crucial for load reductions in the flight envelopes when high damping is required. In such cases, it is assumed the bypass valve of the damper is constantly adjusted for each flight condition in order to avoid any resonance or instability issues of the coupled rotor-fuselage system. In addition, as noted in the work of

Botasso et al [3], it is possible to achieve vibration reduction while achieving a non-critical loss of lag damping and hence operate under recommended safety conditions. These aspects are the subject of ongoing research.

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