Supplement: Estimating latent baseline-by-treatment interactions in statistical mediation analysis

1. Covariance expectations

Consider a two-wave mediation model representing a randomized pretest-posttest design.

$X \sim Bernoulli(p = 0.5), where\ \mu_x = 0.5, \sigma_x^2 = 0.25$
$M_1 \sim N(0, 1)$
$Y_1 \sim N(0, 1)$, and $Cov(M_1, Y_1) = .575$

$M_2 = i_3 + aX + s_m M_1 + s_{c1} Y_1 + h_1 X M_1 + h_2 X Y_1 + e_3$
$Y_2 = i_2 + c' X + b M_2 + s_y Y_1 + s_{c2} M_1 + h_3 X M_1 + h_4 X Y_1 + e_2$

The implied covariance matrix of this model is:

	X	*M1*	*Y1*	*XM1*	*XY1*
X	σ_x^2				
M1	$\sigma_{x,m1}$	σ_{m1}^2			
Y1	$\sigma_{x,y1}$	$\sigma_{m1,y1}$	σ_{y1}^2		
XM1	$\sigma_x^2 \mu_{m1}$ $+ \sigma_{x,m1} \mu_x$	$\sigma_{m1}^2 \mu_x$ $+ \sigma_{x,m1} \mu_{m1}$	$\sigma_{m1,y1} \mu_x$ $+ \sigma_{x,y1} \mu_{m1}$	$\sigma_{m1}^2 \mu_x^2 + \sigma_x^2 \mu_{m1}^2$ $+ 2\sigma_{x,m1} \mu_x \mu_{m1}$ $+ \sigma_x^2 \sigma_{m1}^2 - \sigma_{x,m1}^2$	
XY1	$\sigma_x^2 \mu_{y1}$ $+ \sigma_{x,y1} \mu_x$	$\sigma_{m1,y1} \mu_x$ $+ \sigma_{x,y1} \mu_{m1}$	$\sigma_{x,y1} \mu_{y1}$ $+ \sigma_{y1}^2 \mu_x$	$\sigma_x^2 \mu_{m1} \mu_{y1}$ $+ \sigma_{x,y1} \mu_x \mu_{m1}$ $+ \sigma_{x,m1} \mu_x \mu_{y1}$ $+ \sigma_{m1,y1} \mu_x^2$ $+ \sigma_x^2 \sigma_{m1,y1}$ $+ \sigma_{x,y1} \sigma_{x,m1}$	$\sigma_{y1}^2 \mu_x^2 + \sigma_x^2 \mu_{y1}^2$ $+ 2\sigma_{x,y1} \mu_x \mu_{y1} + \sigma_x^2 \sigma_{y1}^2$ $- \sigma_{x,y1}^2$
M2	$a\sigma_x^2$ $+ s_m \sigma_{x,m1}$ $+ s_{c1} \sigma_{x,y1}$ $+ h_1 \sigma_{x,xm1}$ $+ h_2 \sigma_{x,xy1}$	$a\sigma_{x,m1}$ $+ s_m \sigma_{m1}^2$ $+ s_{c1} \sigma_{m1,y1}$ $+ h_1 \sigma_{m1,xm1}$ $+ h_2 \sigma_{m1,xy1}$	$a\sigma_{x,y1}$ $+ s_m \sigma_{m1,y1}$ $+ s_{c1} \sigma_{y1}^2$ $+ h_1 \sigma_{y1,xm1}$ $+ h_2 \sigma_{y1,xy1}$	$a\sigma_{x,xm1}$ $+ s_m \sigma_{m1,xm1}$ $+ s_{c1} \sigma_{y1,xm1}$ $+ h_1 \sigma_{xm1}^2$ $+ h_2 \sigma_{xm1,xy1}$	$a\sigma_{x,xy1} + s_m \sigma_{m1,xy1}$ $+ s_{c1} \sigma_{y1,xy1} + h_1 \sigma_{xm1_{xy1}}$ $+ h_2 \sigma_{xy1}^2$
Y2	$c' \sigma_x^2$ $+ s_{c2} \sigma_{x,m1}$ $+ s_y \sigma_{x,y1}$ $+ b\sigma_{x,m2}$ $+ h_3 \sigma_{x,xm1}$ $+ h_4 \sigma_{x,xy1}$	$c' \sigma_{x.m1}$ $+ s_{c2} \sigma_{m1}^2$ $+ s_y \sigma_{m1,y1}$ $+ b\sigma_{m1,m2}$ $+ h_3 \sigma_{m1,xm1}$ $+ h_4 \sigma_{m1,xy1}$	$c' \sigma_{x,y1}$ $+ s_{c2} \sigma_{m1,y1}$ $+ s_y \sigma_{y1}^2$ $+ b\sigma_{y1,m2}$ $+ h_3 \sigma_{y1,xm1}$ $+ h_4 \sigma_{y1,xy1}$	$c' \sigma_{x.xm1}$ $+ s_{c2} \sigma_{m1,xm1}$ $+ s_y \sigma_{y1,xm1}$ $+ b\sigma_{xm1.m2}$ $+ h_3 \sigma_{,xm1}^2$ $+ h_4 \sigma_{xm1,y1}$	$c' \sigma_{x.xy1} + s_{c2} \sigma_{m1,xy1}$ $+ s_y \sigma_{y1,xy1} + b\sigma_{xy1.m2}$ $+ h_3 \sigma_{xm1,xy1} + h_4 \sigma_{xy1}^2$

	M2	Y2
M2	$a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + s_{c1}^2\sigma_{y1}^2 + h_1^2\sigma_{xm1}^2 + h_2^2\sigma_{xy1}^2 + 2as_m\sigma_{x,m1} + 2as_{c1}\sigma_{x,y1} + 2ah_1\sigma_{x,xm1} + 2ah_2\sigma_{x,xy1} + 2s_m s_{c1}\sigma_{m1,y1} + 2s_m h_1\sigma_{m1,xm1} + 2s_m h_2\sigma_{m1,xy1} + 2s_{c1}h_1\sigma_{y1,xm1} + 2s_{c1}h_2\sigma_{y1,xy1} + 2h_1h_2\sigma_{xm1,xy1} + \sigma_{e_3}^2$	
Y2	$ac'\sigma_x^2 + ab\sigma_{x,m2} + as_y\sigma_{x,y1} + as_{c2}\sigma_{x,m1} + ah_3\sigma_{x,xm1} + ah_4\sigma_{x,xy1} + s_m c'\sigma_{m1,x} + s_m b\sigma_{m1,m2} + s_m s_y\sigma_{m1,y1} + s_m s_{c2}\sigma_{m1}^2 + s_m h_3\sigma_{m1,xm1} + s_m h_4\sigma_{m1,xy1} + s_{c1}c'\sigma_{x,y1} + s_{c1}b\sigma_{y1,m2} + s_{c1}s_y\sigma_{y1}^2 + s_{c1}s_{c2}\sigma_{y1,m1} + s_{c1}h_3\sigma_{y1,xm1} + s_{c1}h_4\sigma_{y1,xy1} + h_1c'\sigma_{x,xm1} + h_1b\sigma_{m2,xm1} + h_1s_y\sigma_{y1,xm1} + h_1s_{c2}\sigma_{m1,xm1} + h_1h_3\sigma_{xm1}^2 + h_1h_4\sigma_{xm1,xy1} + h_2c'\sigma_{x,xy1} + h_2b\sigma_{m2,xy1} + h_2s_y\sigma_{y1,xy1} + h_2s_{c2}\sigma_{m1,xy1} + h_2h_3\sigma_{xm1,xy1} + h_2h_4\sigma_{xy1}^2 + b\sigma_{e_3}^2$	$c'^2\sigma_x^2 + b^2\sigma_{m2}^2 + s_{y1}^2\sigma_{y1}^2 + s_{c2}^2\sigma_{m1}^2 + h_3^2\sigma_{xm1}^2 + h_4^2\sigma_{xy1}^2 + 2bc'\sigma_{x,m2} + 2c's_y\sigma_{x,y1} + 2c's_{c2}\sigma_{x,m1} + 2c'h_3\sigma_{x,xm1} + 2c'h_4\sigma_{x,xy1} + 2bs_y\sigma_{m2,y1} + 2bs_{c2}\sigma_{m1,m2} + 2bh_3\sigma_{m2,xm1} + 2bh_4\sigma_{m2,xy1} + 2s_y s_{c2}\sigma_{m1,y1} + 2s_y h_3\sigma_{y1,xm1} + 2s_y h_4\sigma_{y1,xy1} + 2s_{c2}h_3\sigma_{m1,xm1} + 2s_{c2}h_4\sigma_{m1,xy1} + 2h_3h_4\sigma_{xm1,xy1} + \sigma_{e_2}^2$

Recall that E[XY] = Cov(X,Y) + E[X]*E[Y]. For a situation in which we mean-center X, M1, and Y1 so that μ_x, μ_{m1}, and μ_{y1} are zero, and X is randomized so that $Cov(X, m1) = Cov(X, y1) = 0$, the covariance algebra of the model reduces to:

	X	*M1*	*Y1*	*XM1*	*XY1*
X	σ_x^2				
M1	0	σ_{m1}^2			
Y1	0	0	σ_{y1}^2		
XM1	0	0	0	$\sigma_x^2\sigma_{m1}^2$	
XY1	0	0	0	$\sigma_x^2\sigma_{m1,y1}$	$\sigma_x^2\sigma_{y1}^2$
M2	$a\sigma_x^2$	$s_m\sigma_{m1}^2 + s_{c1}\sigma_{m1,y1}$	$s_m\sigma_{m1,y1} + s_{c1}\sigma_{y1}^2$	$h_1\sigma_{xm1}^2 + h_2\sigma_{xm1,xy1}$	$h_1\sigma_{xm1,xy1} + h_2\sigma_{xy1}^2$
Y2	$c'\sigma_x^2 + b\sigma_{x,m2}$	$s_{c2}\sigma_{m1}^2 + s_y\sigma_{m1,y1} + b\sigma_{m1,m2}$	$s_{c2}\sigma_{m1,y1} + s_y\sigma_{y1}^2 + b\sigma_{y1,m2}$	$b\sigma_{xm1.m2} + h_3\sigma_{xm1}^2 + h_4\sigma_{xm1,xy1}$	$b\sigma_{xy1,m2} + h_3\sigma_{xm1,xy1} + h_4\sigma_{xy1}^2$

	M2	*Y2*
M2	$a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + s_{c1}^2\sigma_{y1}^2 + h_1^2\sigma_{xm1}^2 + h_2^2\sigma_{xy1}^2 + 2s_m s_{c1}\sigma_{m1,y1} + 2h_1h_2\sigma_{xm1,xy1} + \sigma_{e_3}^2$	
Y2	$ac'\sigma_x^2 + ab\sigma_{x,m2} + s_m b\sigma_{m1,m2} + s_m s_y\sigma_{m1,y1} + s_m s_{c2}\sigma_{m1}^2 + s_{c1}b\sigma_{y1,m2} + s_{c1}s_y\sigma_{y1}^2 + s_{c1}s_{c2}\sigma_{y1,m1} + h_1 b\sigma_{m2,xm1} + h_1h_3\sigma_{xm1}^2 + h_1h_4\sigma_{xm1,xy1} + h_2 b\sigma_{m2,xy1} + h_2h_3\sigma_{xm1,xy1} + h_2h_4\sigma_{xy1}^2 + b\sigma_{e_3}^2$	$c'^2\sigma_x^2 + b^2\sigma_{m2}^2 + s_{y1}^2\sigma_{y1}^2 + s_{c2}^2\sigma_{m1}^2 + h_3^2\sigma_{xm1}^2 + h_4^2\sigma_{xy1}^2 + 2bc'\sigma_{x,m2} + 2bs_y\sigma_{m2,y1} + 2bs_{c2}\sigma_{m1,m2} + 2bh_3\sigma_{m2,xm1} + 2bh_4\sigma_{m2,xy1} + 2s_y s_{c2}\sigma_{m1,y1} + 2h_3h_4\sigma_{xm1,xy1} + \sigma_{e_2}^2$

If we substitute everything in terms of exogenous variables, we get:

	X	*M1*	*Y1*	*XM1*	*XY1*
X	σ_x^2				
M1	0	σ_{m1}^2			
Y1	0	0	σ_{y1}^2		
XM1	0	0	0	$\sigma_x^2\sigma_{m1}^2$	
XY1	0	0	0	$\sigma_x^2\sigma_{m1,y1}$	$\sigma_x^2\sigma_{y1}^2$
M2	$a\sigma_x^2$	$s_m\sigma_{m1}^2$ $+ s_{c1}\sigma_{m1,y1}$	$s_m\sigma_{m1,y1}$ $+ s_{c1}\sigma_{y1}^2$	$h_1(\sigma_x^2\sigma_{m1}^2)$ $+ h_2(\sigma_x^2\sigma_{m1,y1})$	$h_1(\sigma_x^2\sigma_{m1,y1}) + h_2(\sigma_x^2\sigma_{y1}^2)$
Y2	$c'\sigma_x^2$ $+ b(a\sigma_x^2)$	$s_{c2}\sigma_{m1}^2$ $+ s_y\sigma_{m1,y1}$ $+ b\big(s_m\sigma_{m1}^2$ $+ s_{c1}\sigma_{m1,y1}\big)$	$s_{c2}\sigma_{m1,y1}$ $+ s_y\sigma_{y1}^2$ $+ b\big(s_m\sigma_{m1,y1}$ $+ s_{c1}\sigma_{y1}^2\big)$	$b\Big(h_1(\sigma_x^2\sigma_{m1}^2)$ $+ h_2\big(\sigma_x^2\sigma_{m1,y1}\big)\Big)$ $+ h_3(\sigma_x^2\sigma_{m1}^2)$ $+ h_4(\sigma_x^2\sigma_{m1,y1})$	$b\Big(h_1\big(\sigma_x^2\sigma_{m1,y1}\big)$ $+ h_2\big(\sigma_x^2\sigma_{y1}^2\big)\Big)$ $+ h_3\big(\sigma_x^2\sigma_{m1,y1}\big)$ $+ h_4\big(\sigma_x^2\sigma_{y1}^2\big)$

	M2	*Y2*
M2	$a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + s_{c1}^2\sigma_{y1}^2 + h_1^2(\sigma_x^2\sigma_{m1}^2)$ $+ h_2^2\big(\sigma_x^2\sigma_{y1}^2\big)$ $+ 2s_ms_{c1}\sigma_{m1,y1}$ $+ 2h_1h_2\big(\sigma_x^2\sigma_{m1,y1}\big) + \sigma_{e_3}^2$	
Y2	$ac'\sigma_x^2 + ab(a\sigma_x^2) + s_mb\big(s_m\sigma_{m1}^2 + s_{c1}\sigma_{m1,y1}\big)$ $+ s_ms_y\sigma_{m1,y1} + s_ms_{c2}\sigma_{m1}^2$ $+ s_{c1}b\big(s_m\sigma_{m1,y1}$ $+ s_{c1}\sigma_{y1}^2\big) + s_{c1}s_y\sigma_{y1}^2$ $+ s_{c1}s_{c2}\sigma_{y1,m1}$ $+ h_1b\Big(h_1(\sigma_x^2\sigma_{m1}^2)$ $+ h_2\big(\sigma_x^2\sigma_{m1,y1}\big)\Big)$ $+ h_1h_3(\sigma_x^2\sigma_{m1}^2)$ $+ h_1h_4(\sigma_x^2\sigma_{m1,y1})$ $+ h_2b\Big(h_1\big(\sigma_x^2\sigma_{m1,y1}\big)$ $+ h_2\big(\sigma_x^2\sigma_{y1}^2\big)\Big)$ $+ h_2h_3\big(\sigma_x^2\sigma_{m1,y1}\big)$ $+ h_2h_4\big(\sigma_x^2\sigma_{y1}^2\big) + b\sigma_{e_3}^2$	$c'^2\sigma_x^2$ $+ b^2\big(a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + s_{c1}^2\sigma_{y1}^2$ $+ h_1^2(\sigma_x^2\sigma_{m1}^2) + h_2^2\big(\sigma_x^2\sigma_{y1}^2\big)$ $+ 2s_ms_{c1}\sigma_{m1,y1}$ $+ 2h_1h_2\big(\sigma_x^2\sigma_{m1,y1}\big) + \sigma_{e_3}^2\big)$ $+ s_{y1}^2\sigma_{y1}^2 + s_{c2}^2\sigma_{m1}^2 + h_3^2(\sigma_x^2\sigma_{m1}^2)$ $+ h_4^2\big(\sigma_x^2\sigma_{y1}^2\big) + 2bc'(a\sigma_x^2)$ $+ 2bs_y\big(s_m\sigma_{m1,y1} + s_{c1}\sigma_{y1}^2\big)$ $+ 2bs_{c2}\big(s_m\sigma_{m1}^2 + s_{c1}\sigma_{m1,y1}\big)$ $+ 2bh_3\Big(h_1(\sigma_x^2\sigma_{m1}^2)$ $+ h_2\big(\sigma_x^2\sigma_{m1,y1}\big)\Big)$ $+ 2bh_4\Big(h_1(\sigma_x^2\sigma_{m1}^2)$ $+ h_2\big(\sigma_x^2\sigma_{m1,y1}\big)\Big)$ $+ 2s_ys_{c2}\sigma_{m1,y1} + 2h_3h_4\big(\sigma_x^2\sigma_{m1,y1}\big)$ $+ \sigma_{e_2}^2$

Now, we if we assume no cross-lags ($s_{c1} = s_{c2} = 0$), the covariance matrix reduces to:

	X	*M1*	*Y1*	*XM1*	*XY1*
X	σ_x^2				
M1	0	σ_{m1}^2			
Y1	0	0	σ_{y1}^2		
XM1	0	0	0	$\sigma_x^2\sigma_{m1}^2$	
XY1	0	0	0	$\sigma_x^2\sigma_{m1,y1}$	$\sigma_x^2\sigma_{y1}^2$
M2	$a\sigma_x^2$	$s_m\sigma_{m1}^2$	$s_m\sigma_{m1,y1}$	$h_1\sigma_{xm1}^2 + h_2\sigma_{xm1,xy1}$	$h_1\sigma_{xm1,xy1} + h_2\sigma_{xy1}^2$
Y2	$c'\sigma_x^2 + b\sigma_{x,m2}$	$s_y\sigma_{m1,y1} + b\sigma_{m1,m2}$	$s_y\sigma_{y1}^2 + b\sigma_{y1,m2}$	$b\sigma_{xm1.m2} + h_3\sigma_{xm1}^2 + h_4\sigma_{xm1,xy1}$	$b\sigma_{xy1,m2} + h_3\sigma_{xm1,xy1} + h_4\sigma_{xy1}^2$

	M2	*Y2*
M2	$a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + h_1^2\sigma_{xm1}^2 + h_2^2\sigma_{xy1}^2 + 2h_1h_2\sigma_{xm1,xy1} + \sigma_{e_3}^2$	
Y2	$ac'\sigma_x^2 + ab\sigma_{x,m2} + s_mb\sigma_{m1,m2} + s_ms_y\sigma_{m1,y1} + h_1b\sigma_{m2,xm1} + h_1h_3\sigma_{xm1}^2 + h_1h_4\sigma_{xm1,xy1} + h_2b\sigma_{m2,xy1} + h_2h_3\sigma_{xm1,xy1} + h_2h_4\sigma_{xy1}^2 + b\sigma_{e_3}^2$	$c'^2\sigma_x^2 + b^2\sigma_{m2}^2 + s_{y1}^2\sigma_{y1}^2 + h_3^2\sigma_{xm1}^2 + h_4^2\sigma_{xy1}^2 + 2bc'\sigma_{x,m2} + 2bs_y\sigma_{m2,y1} + 2bh_3\sigma_{m2,xm1} + 2bh_4\sigma_{m2,xy1} + 2h_3h_4\sigma_{xm1,xy1} + \sigma_{e_2}^2$

If we substitute terms for only exogenous variables, we get:

	X	*M1*	*Y1*	*XM1*	*XY1*
X	σ_x^2				
M1	0	σ_{m1}^2			
Y1	0	0	σ_{y1}^2		
XM1	0	0	0	$\sigma_x^2\sigma_{m1}^2$	
XY1	0	0	0	$\sigma_x^2\sigma_{m1,y1}$	$\sigma_x^2\sigma_{y1}^2$
M2	$a\sigma_x^2$	$s_m\sigma_{m1}^2$	$s_m\sigma_{m1,y1}$	$h_1(\sigma_x^2\sigma_{m1}^2) + h_2(\sigma_x^2\sigma_{m1,y1})$	$h_1(\sigma_x^2\sigma_{m1,y1}) + h_2(\sigma_x^2\sigma_{y1}^2)$
Y2	$c'\sigma_x^2 + b(a\sigma_x^2)$	$s_y\sigma_{m1,y1} + b(s_m\sigma_{m1}^2)$	$s_y\sigma_{y1}^2 + b(s_m\sigma_{m1,y1})$	$b\left(h_1(\sigma_x^2\sigma_{m1}^2) + h_2(\sigma_x^2\sigma_{m1,y1})\right) + h_3(\sigma_x^2\sigma_{m1}^2) + h_4(\sigma_x^2\sigma_{m1,y1})$	$b\left(h_1(\sigma_x^2\sigma_{m1}^2) + h_2(\sigma_x^2\sigma_{m1,y1})\right) + h_3(\sigma_x^2\sigma_{m1,y1}) + h_4(\sigma_x^2\sigma_{y1}^2)$

	M2	Y2
M2	$a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + h_1^2(\sigma_x^2\sigma_{m1}^2) + h_2^2(\sigma_x^2\sigma_{y1}^2) + 2h_1h_2(\sigma_x^2\sigma_{m1,y1}) + \sigma_{e_3}^2$	
Y2	$ac'\sigma_x^2 + ab(a\sigma_x^2) + s_m b(s_m\sigma_{m1}^2) + s_m s_y \sigma_{m1,y1} + h_1 b\left(h_1(\sigma_x^2\sigma_{m1}^2) + h_2\left(\sigma_x^2\sigma_{m1,y1}\right)\right) + h_1h_3(\sigma_x^2\sigma_{m1}^2) + h_1h_4(\sigma_x^2\sigma_{m1,y1}) + h_2 b\left(h_1(\sigma_x^2\sigma_{m1}^2) + h_2\left(\sigma_x^2\sigma_{m1,y1}\right)\right) + h_2h_3(\sigma_x^2\sigma_{m1,y1}) + h_2h_4(\sigma_x^2\sigma_{y1}^2) + b\sigma_{e_3}^2$	$c'^2\sigma_x^2 + b^2\left(a^2\sigma_x^2 + s_m^2\sigma_{m1}^2 + h_1^2(\sigma_x^2\sigma_{m1}^2) + h_2^2\left(\sigma_x^2\sigma_{y1}^2\right) + 2h_1h_2\left(\sigma_x^2\sigma_{m1,y1}\right) + \sigma_{e_3}^2\right) + s_{y1}^2\sigma_{y1}^2 + h_3^2(\sigma_x^2\sigma_{m1}^2) + h_4^2\left(\sigma_x^2\sigma_{y1}^2\right) + 2bc'(a\sigma_x^2) + 2bs_y\left(s_m\sigma_{m1,y1}\right) + 2bh_3\left(h_1(\sigma_x^2\sigma_{m1}^2) + h_2\left(\sigma_x^2\sigma_{m1,y1}\right)\right) + 2bh_4\left(h_1\left(\sigma_x^2\sigma_{m1,y1}\right) + h_2\left(\sigma_x^2\sigma_{y1}^2\right)\right) + 2h_3h_4\left(\sigma_x^2\sigma_{m1,y1}\right) + \sigma_{e_2}^2$

2. Additional Tables

Table 1. *Bias estimates and interval coverage of the parameters from the mediation model with latent BTIs with discrete, nonnormal indicators*

	Relative Bias						*Standardized Bias*					
	a-path	**b-path**	**c'-path**	**sm-path**	**sy-path**	**ab**	**cy-path**	**h1**	**h2**	**cm-path**	**h3**	**h4**
Summed	-0.100	-0.041	-0.119	-0.118	-0.163	-0.138	-0.007	-0.043	-0.068	0.008	-0.094	-0.113
Factor	-0.071	-0.050	-0.088	-0.123	-0.167	-0.121	0.002	-0.059	-0.075	0.008	-0.104	-0.132
UPI	0.008	-0.103	0.028	-0.143	-0.179	-0.098	0.165	-0.020	-0.032	0.242	-0.073	-0.064
LMS	0.008	-0.105	0.028	-0.141	-0.178	-0.101	0.147	0.012	-0.111	0.240	-0.102	-0.109
Bayes	0.008	-0.108	0.022	-0.130	-0.173	-0.106	0.133	0.017	-0.099	0.202	-0.089	-0.094
	Raw Bias											
Summed	-0.099	-0.026	-0.142	-0.116	-0.137	-0.062	-0.006	-0.006	-0.013	-0.009	-0.037	-0.022
Factor	-0.025	-0.019	-0.025	-0.122	-0.141	-0.013	-0.009	-0.010	-0.018	-0.005	-0.023	-0.025
UPI	0.002	-0.033	0.008	-0.141	-0.151	-0.011	0.012	-0.003	-0.007	0.026	-0.014	-0.011
LMS	0.002	-0.034	0.008	-0.139	-0.150	-0.011	0.010	0.004	-0.022	0.026	-0.018	-0.017
Bayes	0.002	-0.034	0.006	-0.128	-0.146	-0.012	0.008	0.006	-0.021	0.025	-0.018	-0.016
	Interval Coverage											
Summed	0.939	0.947	0.937	0.853	0.731	0.933	0.947	0.954	0.951	0.950	0.951	0.948
Factor	0.942	0.947	0.942	0.830	0.702	0.934	0.949	0.954	0.953	0.952	0.952	0.947
UPI	0.944	0.907	0.939	0.721	0.552	0.932	0.944	0.946	0.947	0.939	0.945	0.943
LMS	0.944	0.906	0.939	0.735	0.561	0.931	0.946	0.944	0.946	0.941	0.945	0.939
Bayes	0.962	0.942	0.966	0.783	0.649	0.952	0.961	0.965	0.959	0.972	0.968	0.967

Note: There were seven categories with thresholds at -1.43,-.43, .38, .94, 1.44, 2.53. These were taken from Rhemtulla et al. (2012). True values of the a-path were .285 and .39, which are roughly both small effects.

Table 2. *Statistical power of the mediation model parameters across methods for conditions with discrete, nonnormal indicators*

	a-path (s)	a-path (~s)	b-path (s)	b-path (m)	c'-path (s)	h1 (s)	h2 (s)	h3 (s)	h4 (s)	ab (s,s)	ab (m,s)	ab (s,m)	ab (m,m)
N = 250													
Summed	0.422	0.637	0.244	0.899	0.427	0.155	0.158	0.153	0.168	0.089	0.130	0.360	0.554
Factor	0.435	0.652	0.250	0.909	0.453	0.156	0.167	0.157	0.181	0.092	0.139	0.377	0.575
UPI	0.492	0.717	0.327	0.989	0.553	0.219	0.219	0.215	0.264	0.142	0.209	0.475	0.698
LMS	0.492	0.720	0.327	0.989	0.550	0.242	0.206	0.212	0.258	0.141	0.209	0.473	0.704
Bayes	0.419	0.659	0.252	0.978	0.457	0.192	0.184	0.155	0.190	0.082	0.134	0.396	0.628
N = 500													
Summed	0.722	0.920	0.504	0.996	0.764	0.299	0.304	0.321	0.362	0.330	0.447	0.723	0.914
Factor	0.728	0.924	0.514	0.997	0.774	0.306	0.317	0.335	0.379	0.341	0.456	0.733	0.919
UPI	0.769	0.945	0.602	1.000	0.829	0.382	0.400	0.409	0.475	0.430	0.553	0.774	0.944
LMS	0.766	0.946	0.600	1.000	0.829	0.437	0.381	0.419	0.477	0.430	0.549	0.772	0.945
Bayes	0.726	0.932	0.528	1.000	0.775	0.367	0.350	0.333	0.388	0.343	0.471	0.736	0.931

Note: Power estimates for the stability of M and Y are not show because the power was almost 1 across conditions. (s) is for small effect size and (m) is for medium effect size. There were seven categories with thresholds at -1.43,-.43, .38, .94, 1.44, 2.53. These were taken from Rhemtulla et al. (2012). True values of the a-path were .285 and .39, which are roughly both small effects.

Table 3. *Type 1 error for parameters from the mediation model across methods in conditions with discrete, nonnormal indicators*

	cy-path	cm-path	h1	h2	h3	h4
	N = 250					
Summed	0.053	0.047	0.040	0.046	0.042	0.040
Factor	0.049	0.046	0.037	0.042	0.039	0.038
UPI	0.056	0.057	0.053	0.052	0.047	0.050
LMS	0.052	0.053	0.056	0.052	0.048	0.052
Bayes	0.038	0.025	0.036	0.040	0.028	0.028
	N = 500					
Summed	0.053	0.053	0.050	0.047	0.050	0.056
Factor	0.052	0.051	0.050	0.047	0.049	0.055
UPI	0.056	0.065	0.056	0.053	0.053	0.063
LMS	0.055	0.063	0.068	0.050	0.052	0.067
Bayes	0.040	0.031	0.043	0.038	0.031	0.038

Note: There were seven categories with thresholds at -1.43,-.43, .38, .94, 1.44, 2.53. These were taken from Rhemtulla et al. (2012). True values of the a-path were .285 and .39, which are roughly both small effects.

3. Additional Figure

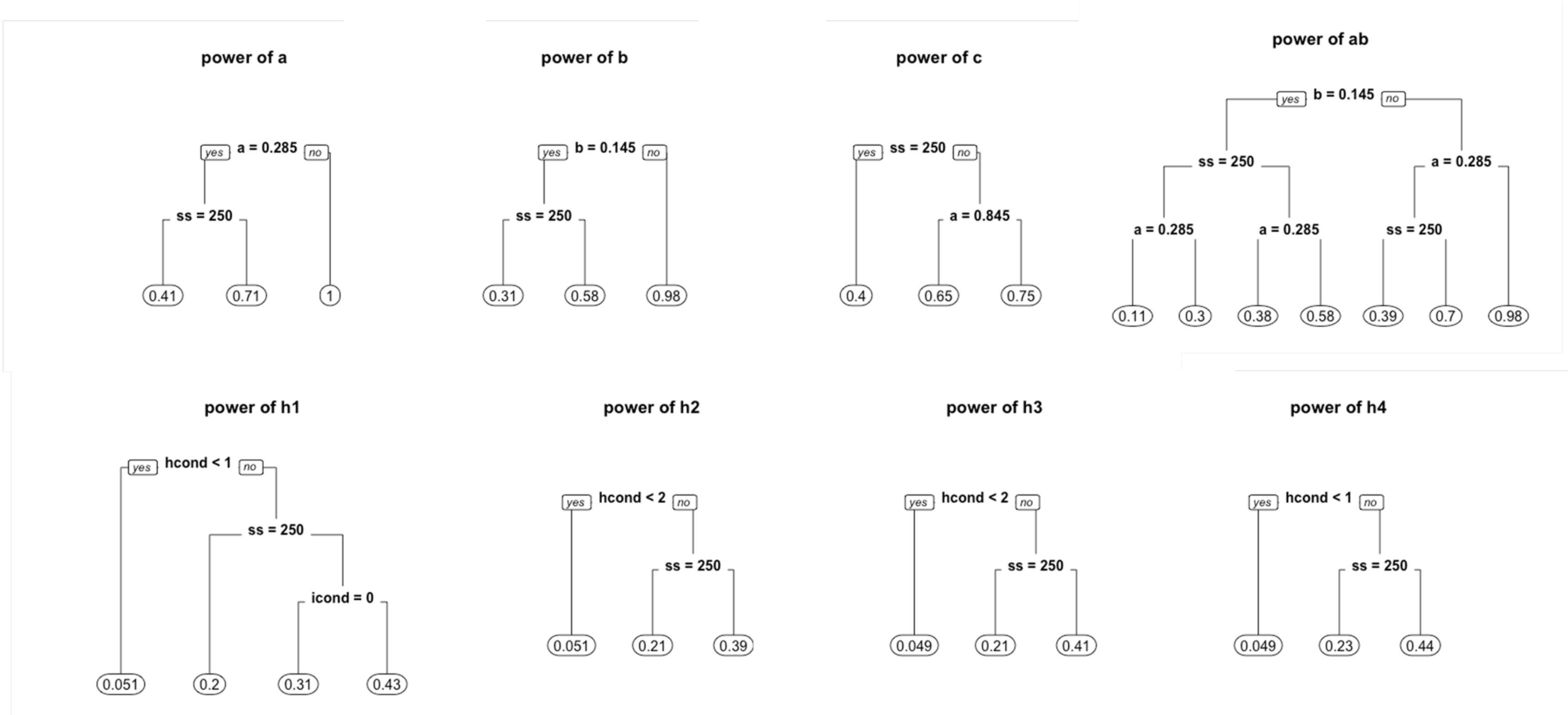
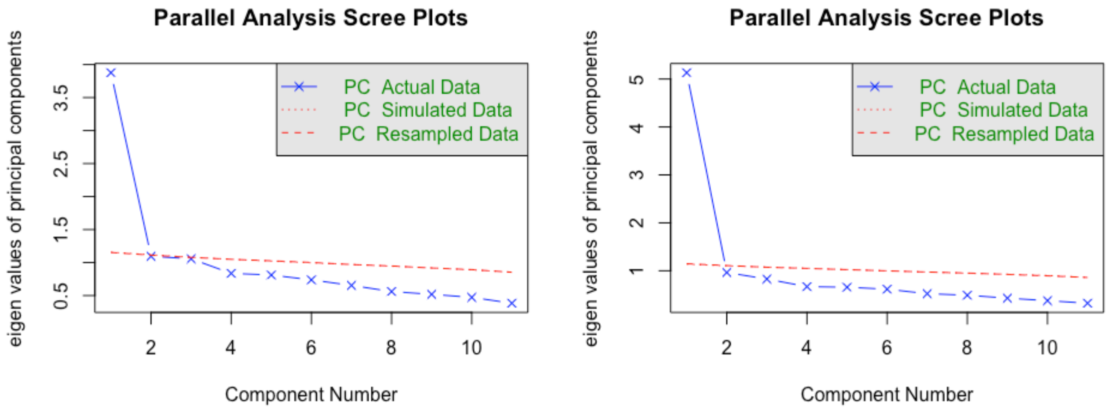
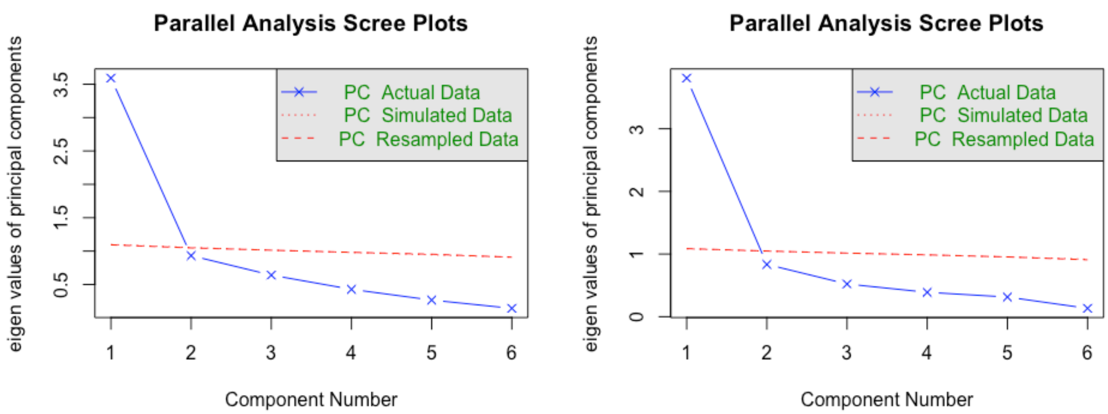


Figure S1. Trees predicting power from simulation conditions for conditions with normally-distributed indicators. Note: hcond = 0 is h1-h4 = 0, hcond = 1 is h1 & h4 = .3 and h2 & h3 = 0, hcond = 2 is h1-h4 = .3, icond = 0 is 4 items for M and 8 items for Y, icond = 1 is 6 items for M and Y, ss is sample size.

4. Psychometric Analyses of the Illustration Data



Depression at baseline and posttest, respectively



Job search self-efficacy at baseline and posttet, respectively

Psychometric analyses for the factor analyses models per scale per timepoint, along with testing for longitudinal scalar invariance. Per our analyses, it seems like longitudinal scalar invariance models fit generally well to serve in our illustration.

DEP at T0 CFA

```
## lavaan 0.6-7 ended normally after 25 iterations
##
##   Estimator                                         DWLS
##   Optimization method                             NLMINB
##   Number of free parameters                           22
##
##                                                     Used       Total
##   Number of observations                            1170        1285
##
## Model Test User Model:
##
##   Test statistic                                 135.642
##   Degrees of freedom                                  44
##   P-value (Chi-square)                             0.000
##
## Model Test Baseline Model:
##
##   Test statistic                                4942.611
##   Degrees of freedom                                  55
##   P-value                                          0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                      0.981
##   Tucker-Lewis Index (TLI)                         0.977
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                            0.042
##   90 Percent confidence interval - lower           0.034
##   90 Percent confidence interval - upper           0.050
##   P-value RMSEA <= 0.05                            0.941
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                             0.049
##
## Parameter Estimates:
##
##   Standard errors                               Standard
##   Information                                   Expected
##   Information saturated (h1) model          Unstructured
##
```

```
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   f1 =~
##     DEP01             0.378    0.018   21.440    0.000    0.378    0.369
##     DEP02             0.681    0.022   31.556    0.000    0.681    0.587
##     DEP03             0.737    0.021   35.467    0.000    0.737    0.667
##     DEP04             0.468    0.018   26.208    0.000    0.468    0.492
##     DEP05             0.632    0.021   30.205    0.000    0.632    0.565
##     DEP06             0.612    0.019   31.722    0.000    0.612    0.598
##     DEP07             0.608    0.018   33.882    0.000    0.608    0.661
##     DEP08             0.774    0.020   37.813    0.000    0.774    0.763
##     DEP09             0.076    0.008    9.596    0.000    0.076    0.176
##     DEP010            0.245    0.012   19.835    0.000    0.245    0.343
##     DEP011            0.379    0.016   23.191    0.000    0.379    0.430
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .DEP01             0.905    0.043   21.226    0.000    0.905    0.864
##    .DEP02             0.882    0.051   17.207    0.000    0.882    0.656
##    .DEP03             0.679    0.051   13.214    0.000    0.679    0.555
##    .DEP04             0.686    0.052   13.183    0.000    0.686    0.758
##    .DEP05             0.853    0.055   15.643    0.000    0.853    0.681
##    .DEP06             0.674    0.052   13.032    0.000    0.674    0.643
##    .DEP07             0.478    0.041   11.652    0.000    0.478    0.564
##    .DEP08             0.429    0.049    8.800    0.000    0.429    0.417
##    .DEP09             0.181    0.030    5.949    0.000    0.181    0.969
##    .DEP010            0.449    0.042   10.695    0.000    0.449    0.882
##    .DEP011            0.633    0.050   12.638    0.000    0.633    0.815
##     f1                1.000                               1.000    1.000
##
## R-Square:
##                    Estimate
##     DEP01             0.136
##     DEP02             0.344
##     DEP03             0.445
##     DEP04             0.242
##     DEP05             0.319
##     DEP06             0.357
##     DEP07             0.436
##     DEP08             0.583
##     DEP09             0.031
##     DEP010            0.118
##     DEP011            0.185
```

```
modindices(fit.dep0, sort=TRUE)
```

```
##       lhs op    rhs     mi    epc sepc.lv sepc.all sepc.nox
## 24  DEP01 ~~  DEP02 25.447  0.201   0.201    0.225    0.225
## 78 DEP010 ~~ DEP011 19.190  0.122   0.122    0.229    0.229
```

```
## 40  DEP02 ~~  DEP09  8.793 -0.046  -0.046   -0.115   -0.115
```

DEP at T2 CFA

```
## lavaan 0.6-7 ended normally after 25 iterations
##
##   Estimator                                         DWLS
##   Optimization method                             NLMINB
##   Number of free parameters                           22
##
##                                                     Used       Total
##   Number of observations                            1254        1285
##
## Model Test User Model:
##
##   Test statistic                                  69.296
##   Degrees of freedom                                  44
##   P-value (Chi-square)                             0.009
##
## Model Test Baseline Model:
##
##   Test statistic                                6848.752
##   Degrees of freedom                                  55
##   P-value                                          0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                      0.996
##   Tucker-Lewis Index (TLI)                         0.995
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                            0.021
##   90 Percent confidence interval - lower           0.011
##   90 Percent confidence interval - upper           0.031
##   P-value RMSEA <= 0.05                            1.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                             0.041
##
## Parameter Estimates:
##
##   Standard errors                               Standard
##   Information                                   Expected
##   Information saturated (h1) model          Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
```

```
##   f1 =~
##     DEP21             0.486    0.017   29.366    0.000    0.486    0.536
##     DEP22             0.772    0.020   39.049    0.000    0.772    0.713
##     DEP23             0.683    0.019   35.448    0.000    0.683    0.705
##     DEP24             0.555    0.019   29.262    0.000    0.555    0.583
##     DEP25             0.703    0.019   37.004    0.000    0.703    0.675
##     DEP26             0.796    0.021   37.961    0.000    0.796    0.690
##     DEP27             0.730    0.019   38.483    0.000    0.730    0.719
##     DEP28             0.870    0.021   42.136    0.000    0.870    0.814
##     DEP29             0.233    0.013   18.425    0.000    0.233    0.439
##     DEP210            0.413    0.016   26.198    0.000    0.413    0.517
##     DEP211            0.664    0.020   33.400    0.000    0.664    0.616
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .DEP21             0.585    0.048   12.304    0.000    0.585    0.712
##    .DEP22             0.578    0.055   10.461    0.000    0.578    0.492
##    .DEP23             0.472    0.055    8.539    0.000    0.472    0.503
##    .DEP24             0.599    0.058   10.375    0.000    0.599    0.660
##    .DEP25             0.591    0.052   11.272    0.000    0.591    0.544
##    .DEP26             0.697    0.063   11.117    0.000    0.697    0.524
##    .DEP27             0.498    0.054    9.301    0.000    0.498    0.483
##    .DEP28             0.386    0.058    6.673    0.000    0.386    0.338
##    .DEP29             0.228    0.041    5.618    0.000    0.228    0.807
##    .DEP210            0.467    0.048    9.754    0.000    0.467    0.732
##    .DEP211            0.723    0.061   11.791    0.000    0.723    0.621
##     f1                1.000                               1.000    1.000
##
## R-Square:
##                    Estimate
##     DEP21             0.288
##     DEP22             0.508
##     DEP23             0.497
##     DEP24             0.340
##     DEP25             0.456
##     DEP26             0.476
##     DEP27             0.517
##     DEP28             0.662
##     DEP29             0.193
##     DEP210            0.268
##     DEP211            0.379
```

SE at T1 CFA

```
## lavaan 0.6-7 ended normally after 19 iterations
##
##   Estimator                                         DWLS
##   Optimization method                             NLMINB
##   Number of free parameters                           12
##
```

```
##                                                   Used       Total
##   Number of observations                          1261        1285
##
## Model Test User Model:
##
##   Test statistic                               165.218
##   Degrees of freedom                                 9
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              3887.411
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.960
##   Tucker-Lewis Index (TLI)                       0.933
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.117
##   90 Percent confidence interval - lower         0.102
##   90 Percent confidence interval - upper         0.133
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.094
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model        Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   f1 =~
##     SE11              0.616    0.020   30.989    0.000    0.616    0.633
##     SE12              0.784    0.023   34.448    0.000    0.784    0.745
##     SE13              0.809    0.024   34.006    0.000    0.809    0.736
##     SE14              0.724    0.023   31.144    0.000    0.724    0.695
##     SE15              0.851    0.025   34.713    0.000    0.851    0.749
##     SE16              0.814    0.024   33.370    0.000    0.814    0.739
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .SE11              0.569    0.047   11.998    0.000    0.569    0.600
```

```
##    .SE12               0.492    0.054    9.078    0.000    0.492    0.445
##    .SE13               0.554    0.058    9.553    0.000    0.554    0.458
##    .SE14               0.561    0.059    9.551    0.000    0.561    0.517
##    .SE15               0.565    0.061    9.290    0.000    0.565    0.438
##    .SE16               0.550    0.061    9.062    0.000    0.550    0.453
##     f1                1.000                               1.000    1.000
##
## R-Square:
##                    Estimate
##     SE11              0.400
##     SE12              0.555
##     SE13              0.542
##     SE14              0.483
##     SE15              0.562
##     SE16              0.547
```

```
modindices(fit.se1, sort=TRUE, minimum.value = 10)
```

```
##     lhs op  rhs      mi    epc sepc.lv sepc.all sepc.nox
## 19 SE12 ~~ SE13 137.089  0.598   0.598    1.145    1.145
## 28 SE15 ~~ SE16  44.674  0.368   0.368    0.660    0.660
## 16 SE11 ~~ SE14  19.082  0.188   0.188    0.332    0.332
## 22 SE12 ~~ SE16  18.498 -0.214  -0.214   -0.411   -0.411
## 25 SE13 ~~ SE16  17.300 -0.216  -0.216   -0.391   -0.391
```

```
se1.cfa.cor<- 'f1 =~ SE11 + SE12 + SE13 + SE14+ SE15 + SE16
SE12~~SE13'

fit.se1.cor <- cfa(se1.cfa.cor, data=se1, std.lv=TRUE, estimator = 'DWLS')
summary(fit.se1.cor, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
```

```
## lavaan 0.6-7 ended normally after 23 iterations
##
##   Estimator                                         DWLS
##   Optimization method                             NLMINB
##   Number of free parameters                           13
##
##                                                     Used       Total
##   Number of observations                            1261        1285
##
## Model Test User Model:
##
##   Test statistic                                  27.852
##   Degrees of freedom                                   8
##   P-value (Chi-square)                             0.001
##
## Model Test Baseline Model:
##
##   Test statistic                                3887.411
##   Degrees of freedom                                  15
##   P-value                                          0.000
```

```
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.995
##   Tucker-Lewis Index (TLI)                       0.990
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.044
##   90 Percent confidence interval - lower         0.027
##   90 Percent confidence interval - upper         0.063
##   P-value RMSEA <= 0.05                          0.665
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.038
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   f1 =~
##     SE11              0.644    0.021   30.727    0.000    0.644    0.661
##     SE12              0.645    0.024   27.092    0.000    0.645    0.613
##     SE13              0.662    0.025   26.471    0.000    0.662    0.602
##     SE14              0.759    0.025   30.973    0.000    0.759    0.729
##     SE15              0.898    0.026   34.283    0.000    0.898    0.791
##     SE16              0.868    0.026   33.233    0.000    0.868    0.788
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .SE12 ~~
##    .SE13              0.564    0.046   12.258    0.000    0.564    0.773
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .SE11              0.534    0.049   10.953    0.000    0.534    0.563
##    .SE12              0.691    0.051   13.529    0.000    0.691    0.624
##    .SE13              0.771    0.055   14.144    0.000    0.771    0.638
##    .SE14              0.508    0.061    8.349    0.000    0.508    0.469
##    .SE15              0.482    0.065    7.465    0.000    0.482    0.374
##    .SE16              0.459    0.064    7.125    0.000    0.459    0.379
##     f1                1.000                               1.000    1.000
##
## R-Square:
##                    Estimate
```

```
##     SE11              0.437
##     SE12              0.376
##     SE13              0.362
##     SE14              0.531
##     SE15              0.626
##     SE16              0.621

lavTestLRT(fit.se1.cor, fit.se1)

## Chi-Squared Difference Test
##
##             Df AIC BIC   Chisq Chisq diff Df diff Pr(>Chisq)
## fit.se1.cor  8           27.852
## fit.se1      9          165.218     137.37       1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

modindices(fit.se1.cor, sort=TRUE, minimum.value = 10)

##     lhs op  rhs    mi   epc sepc.lv sepc.all sepc.nox
## 28 SE15 ~~ SE16 19.02 0.264   0.264    0.562    0.562

se1.cfa.cor2<- 'f1 =~ SE11 + SE12 + SE13 + SE14+ SE15 + SE16
SE12~~SE13
SE15~~SE16'

fit.se1.cor2 <- cfa(se1.cfa.cor2, data=se1, std.lv=TRUE, estimator = 'DWLS')
summary(fit.se1.cor2, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)

## lavaan 0.6-7 ended normally after 27 iterations
##
##   Estimator                                       DWLS
##   Optimization method                           NLMINB
##   Number of free parameters                         14
##
##                                                   Used       Total
##   Number of observations                          1261        1285
##
## Model Test User Model:
##
##   Test statistic                                 8.865
##   Degrees of freedom                                 7
##   P-value (Chi-square)                           0.262
##
## Model Test Baseline Model:
##
##   Test statistic                              3887.411
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
```

```
##
##   Comparative Fit Index (CFI)                    1.000
##   Tucker-Lewis Index (TLI)                       0.999
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.015
##   90 Percent confidence interval - lower         0.000
##   90 Percent confidence interval - upper         0.039
##   P-value RMSEA <= 0.05                          0.994
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.022
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model        Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   f1 =~
##     SE11              0.663    0.022   30.304    0.000    0.663    0.680
##     SE12              0.668    0.025   26.359    0.000    0.668    0.635
##     SE13              0.685    0.027   25.779    0.000    0.685    0.623
##     SE14              0.780    0.026   30.472    0.000    0.780    0.749
##     SE15              0.825    0.030   27.605    0.000    0.825    0.726
##     SE16              0.791    0.030   26.204    0.000    0.791    0.718
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .SE12 ~~
##    .SE13              0.533    0.048   11.188    0.000    0.533    0.763
##  .SE15 ~~
##    .SE16              0.253    0.056    4.513    0.000    0.253    0.424
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .SE11              0.510    0.050   10.213    0.000    0.510    0.537
##    .SE12              0.660    0.053   12.432    0.000    0.660    0.596
##    .SE13              0.740    0.057   13.072    0.000    0.740    0.612
##    .SE14              0.476    0.063    7.612    0.000    0.476    0.439
##    .SE15              0.609    0.066    9.199    0.000    0.609    0.473
##    .SE16              0.587    0.066    8.861    0.000    0.587    0.484
##     f1                1.000                                1.000    1.000
##
## R-Square:
##                    Estimate
```

```
##     SE11              0.463
##     SE12              0.404
##     SE13              0.388
##     SE14              0.561
##     SE15              0.527
##     SE16              0.516
```

```
lavTestLRT(fit.se1.cor2, fit.se1.cor)
```

```
## Chi-Squared Difference Test
##
##               Df AIC BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit.se1.cor2  7           8.865
## fit.se1.cor   8          27.852     18.987       1  1.316e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

SE at T2 CFA

```
## lavaan 0.6-7 ended normally after 19 iterations
##
##   Estimator                                       DWLS
##   Optimization method                           NLMINB
##   Number of free parameters                         12
##
##                                                   Used       Total
##   Number of observations                          1261        1285
##
## Model Test User Model:
##
##   Test statistic                               165.218
##   Degrees of freedom                                 9
##   P-value (Chi-square)                           0.000
##
## Model Test Baseline Model:
##
##   Test statistic                              3887.411
##   Degrees of freedom                                15
##   P-value                                        0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                    0.960
##   Tucker-Lewis Index (TLI)                       0.933
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                          0.117
##   90 Percent confidence interval - lower         0.102
##   90 Percent confidence interval - upper         0.133
```

```
##   P-value RMSEA <= 0.05                          0.000
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                           0.094
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   f1 =~
##     SE11              0.616    0.020   30.989    0.000    0.616    0.633
##     SE12              0.784    0.023   34.448    0.000    0.784    0.745
##     SE13              0.809    0.024   34.006    0.000    0.809    0.736
##     SE14              0.724    0.023   31.144    0.000    0.724    0.695
##     SE15              0.851    0.025   34.713    0.000    0.851    0.749
##     SE16              0.814    0.024   33.370    0.000    0.814    0.739
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .SE11              0.569    0.047   11.998    0.000    0.569    0.600
##    .SE12              0.492    0.054    9.078    0.000    0.492    0.445
##    .SE13              0.554    0.058    9.553    0.000    0.554    0.458
##    .SE14              0.561    0.059    9.551    0.000    0.561    0.517
##    .SE15              0.565    0.061    9.290    0.000    0.565    0.438
##    .SE16              0.550    0.061    9.062    0.000    0.550    0.453
##     f1                1.000                               1.000    1.000
##
## R-Square:
##                    Estimate
##     SE11              0.400
##     SE12              0.555
##     SE13              0.542
##     SE14              0.483
##     SE15              0.562
##     SE16              0.547
```

```
modindices(fit.se2, sort=TRUE, minimum.value = 10)
```

```
##     lhs op  rhs     mi    epc sepc.lv sepc.all sepc.nox
## 19 SE22 ~~ SE23 69.530  0.380   0.380    1.067    1.067
## 28 SE25 ~~ SE26 17.537  0.189   0.189    0.454    0.454
## 22 SE22 ~~ SE26 12.503 -0.140  -0.140   -0.373   -0.373
```

```
se2.cfa.cor<- 'f1 =~ SE21 + SE22 + SE23 + SE24+ SE25 + SE26
SE22~~SE23'
```

```r
fit.se2.cor <- cfa(se2.cfa.cor, data=se2, std.lv=TRUE, estimator = 'DWLS')
summary(fit.se2.cor, fit.measures=TRUE, standardized=TRUE, rsquare=TRUE)
```

```
## lavaan 0.6-7 ended normally after 20 iterations
##
##   Estimator                                         DWLS
##   Optimization method                             NLMINB
##   Number of free parameters                           13
##
##                                                     Used       Total
##   Number of observations                            1281        1285
##
## Model Test User Model:
##
##   Test statistic                                  17.259
##   Degrees of freedom                                   8
##   P-value (Chi-square)                             0.028
##
## Model Test Baseline Model:
##
##   Test statistic                                3352.659
##   Degrees of freedom                                  15
##   P-value                                          0.000
##
## User Model versus Baseline Model:
##
##   Comparative Fit Index (CFI)                      0.997
##   Tucker-Lewis Index (TLI)                         0.995
##
## Root Mean Square Error of Approximation:
##
##   RMSEA                                            0.030
##   90 Percent confidence interval - lower           0.010
##   90 Percent confidence interval - upper           0.050
##   P-value RMSEA <= 0.05                            0.953
##
## Standardized Root Mean Square Residual:
##
##   SRMR                                             0.034
##
## Parameter Estimates:
##
##   Standard errors                               Standard
##   Information                                   Expected
##   Information saturated (h1) model          Unstructured
##
## Latent Variables:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##   f1 =~
```

```
##     SE21              0.658    0.022   30.085    0.000    0.658    0.763
##     SE22              0.618    0.024   25.922    0.000    0.618    0.669
##     SE23              0.625    0.024   26.046    0.000    0.625    0.662
##     SE24              0.643    0.022   28.841    0.000    0.643    0.730
##     SE25              0.812    0.025   32.179    0.000    0.812    0.801
##     SE26              0.689    0.023   30.120    0.000    0.689    0.751
##
## Covariances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##  .SE22 ~~
##    .SE23              0.365    0.042    8.655    0.000    0.365    0.751
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)   Std.lv  Std.all
##    .SE21              0.311    0.046    6.782    0.000    0.311    0.418
##    .SE22              0.473    0.048    9.849    0.000    0.473    0.553
##    .SE23              0.500    0.048   10.390    0.000    0.500    0.561
##    .SE24              0.362    0.051    7.052    0.000    0.362    0.467
##    .SE25              0.367    0.057    6.418    0.000    0.367    0.358
##    .SE26              0.366    0.050    7.283    0.000    0.366    0.435
##     f1                1.000                               1.000    1.000
##
## R-Square:
##                    Estimate
##     SE21              0.582
##     SE22              0.447
##     SE23              0.439
##     SE24              0.533
##     SE25              0.642
##     SE26              0.565
```

```
lavTestLRT(fit.se2.cor, fit.se2)
```

```
## Chi-Squared Difference Test
##
##               Df AIC BIC  Chisq Chisq diff Df diff Pr(>Chisq)
## fit.se2.cor  8          17.259
## fit.se2      9          87.041     69.782       1  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
modindices(fit.se2.cor, sort=TRUE, minimum.value = 10)
```

```
## [1] lhs      op       rhs      mi       epc      sepc.lv  sepc.all sepc.no
x
## <0 rows> (or 0-length row.names)
```

Longitudinal Invariance Modeling

		χ^2	df	CFI	RMSEA	SRMR	$\Delta\ \chi^2$	Δ df
Depression	Configural	327.577	197	0.991	0.024	0.038		
	Metric	549.159	207	0.978	0.038	0.050	221.59	10
	Scalar	845.993	217	0.959	0.050	0.059	296.82	10
Job Search Self-efficacy	Configural	320.873	47	0.975	0.068	0.066		
	Metric	341.341	52	0.973	0.067	0.068	20.468	5
	Scalar	345.132	57	0.973	0.063	0.068	3.792	5

5. Code to estimate the two-wave mediation model with latent BTIs

Estimating latent baseline-by-treatment interactions (BTIs) in statistical mediation analysis: R/Mplus Code

Below, we provide code to conduct five different approaches to estimate latent BTI effects in the two-wave mediation model. Standard errors for each of the paths could be estimated using the 95% percentile bootstrap confidence intervals, if desired.

First, we start by summarizing the dataset from the illustration, and then code is provided.

Jobs II dataset and preliminary analyses

Below is the jobs ii dataset that we use for this example. Note that cTx is the treatment/control indicator, which is centered. SE1 and SE2 is job search self-efficacy [the mediator] at time 1 and time 2, and DEP0 and DEP2 is depression symptoms [the outcome] at time 1 and time 2.

```
setwd('~/Downloads')
datt22=read.csv('jobsii_supp.csv')
head(datt22)

##            cTx SE11 SE12 SE13 SE14 SE15 SE16 DEP01 DEP02 DEP03 DEP04 DEP05 DEP06
## 1  0.3081705    5    5    5    5    4    4     1     4     2     2     2     1
## 2  0.3081705    5    4    4    5    5    5     2     1     1     1     1     1
## 3  0.3081705    3    4    3    3    3    3     1     3     2     1     2     1
## 4  0.3081705    2    3    4    4    3    4     4     4     2     2     2     2
## 5 -0.6918295    4    5    5    5    4    4     2     2     1     1     3     1
## 6  0.3081705    5    5    5    5    5    5     2     2     2     2     2     1
##   DEP07 DEP08 DEP09 DEP010 DEP011 SE21 SE22 S23 SE24 SE25 SE26 DEP21 DEP22
## 1     3     3     1      1      1    5    5   5    5    4    5     3     2
## 2     1     1     1      1      2    5    5   5    5    5    5     1     1
## 3     1     1     1      1      1    4    4   4    4    4    3     1     3
## 4     2     2     1      1      1    4    4   4    5    5    5     1     2
## 5     1     2     1      1      1    4    3   4    4    3    4     1     2
## 6     2     2     1      1      1    5    5   5    5    5    5     1     2
##   DEP23 DEP24 DEP25 DEP26 DEP27 DEP28 DEP2 DEP210 DEP211
## 1     1     1     2     1     3     2    1      1      2
## 2     1     1     1     1     1     1    1      1      1
## 3     2     1     3     1     2     2    1      4      2
## 4     2     3     3     5     2     2    2      1      1
## 5     1     1     3     2     2     2    1      1      1
## 6     1     1     1     1     1     1    1      1      1
```

As a preliminary analysis, we conduct longitudinal CFAs and use the output later.

```
library(lavaan)

#Longitudinal factor model for the mediator
m1='
m1=~NA*SE11+a*SE11+b*SE12+c*SE13+d*SE14+e*SE15+f*SE16
m2=~NA*SE21+a*SE21+b*SE22+c*S23+d*SE24+e*SE25+f*SE26
m2~m1
```

```
SE11~~SE21
SE12~~SE22
SE13~~S23
SE14~~SE24
SE15~~SE25
SE16~~SE26

SE12~~SE13 #correlated residual
SE22~~S23 #correlated residual
m1~~1*m1
m2~~m2
'

f1=cfa(m1,data=datt22,std.lv=T)

coef(f1)

##           a           b           c           d           e           f           a
##       0.661       0.675       0.686       0.739       0.910       0.854       0.661
##           b           c           d           e           f       m2~m1 SE11~~SE21
##       0.675       0.686       0.739       0.910       0.854       0.514       0.110
## SE12~~SE22   SE13~~S23 SE14~~SE24 SE15~~SE25 SE16~~SE26 SE12~~SE13   SE22~~S23
##       0.003       0.031       0.141       0.051       0.040       0.560       0.372
##      m2~~m2 SE11~~SE11 SE12~~SE12 SE13~~SE13 SE14~~SE14 SE15~~SE15 SE16~~SE16
##       0.535       0.539       0.682       0.754       0.531       0.453       0.416
## SE21~~SE21 SE22~~SE22   S23~~S23 SE24~~SE24 SE25~~SE25 SE26~~SE26
##       0.355       0.480       0.499       0.359       0.373       0.310


#Longitudinal factor model for the outcome

d1='
de1=~a*DEP01+b*DEP02+c*DEP03+d*DEP04+e*DEP05+f*DEP06+g*DEP07+h*DEP08+i*DEP09+j*DE
P010+k*DEP011
de2=~a*DEP21+b*DEP22+c*DEP23+d*DEP24+e*DEP25+f*DEP26+g*DEP27+h*DEP28+i*DEP2+j*DEP
210+k*DEP211
de1~~de2
DEP01~~DEP21
DEP02~~DEP22
DEP03~~DEP23
DEP04~~DEP24
DEP05~~DEP25
DEP06~~DEP26
DEP07~~DEP27
DEP08~~DEP28
DEP09~~DEP2
DEP010~~DEP210
DEP011~~DEP211
'
f2=cfa(d1,data=datt22,std.lv=T)
```

```
coef(f2) #estimated coefficients

##              a              b              c              d              e
##          0.445          0.701          0.680          0.510          0.677
##              f              g              h              i              j
##          0.712          0.675          0.837          0.161          0.322
##              k              a              b              c              d
##          0.507          0.445          0.701          0.680          0.510
##              e              f              g              h              i
##          0.677          0.712          0.675          0.837          0.161
##              j              k        de1~~de2   DEP01~~DEP21   DEP02~~DEP22
##          0.322          0.507          0.473          0.178          0.118
##   DEP03~~DEP23   DEP04~~DEP24   DEP05~~DEP25   DEP06~~DEP26   DEP07~~DEP27
##          0.121          0.270          0.235          0.226          0.097
##   DEP08~~DEP28    DEP09~~DEP2 DEP010~~DEP210 DEP011~~DEP211   DEP01~~DEP01
##          0.066          0.045          0.122          0.218          0.899
##   DEP02~~DEP02   DEP03~~DEP03   DEP04~~DEP04   DEP05~~DEP05   DEP06~~DEP06
##          0.914          0.723          0.664          0.837          0.660
##   DEP07~~DEP07   DEP08~~DEP08   DEP09~~DEP09 DEP010~~DEP010 DEP011~~DEP011
##          0.456          0.413          0.182          0.442          0.628
##   DEP21~~DEP21   DEP22~~DEP22   DEP23~~DEP23   DEP24~~DEP24   DEP25~~DEP25
##          0.570          0.590          0.466          0.602          0.562
##   DEP26~~DEP26   DEP27~~DEP27   DEP28~~DEP28     DEP2~~DEP2 DEP210~~DEP210
##          0.677          0.511          0.379          0.227          0.458
## DEP211~~DEP211
##          0.766
```

Scoring M and estimating observed XM interaction

Summed scores, correcting for unreliability

```
library(lavaan)
library(psych)

#estimate summed scores, centered on baseline measure
cSE1=rowSums(datt22[,2:7])-mean(rowSums(datt22[,2:7]))
cDEP1=rowSums(datt22[,8:18])-mean(rowSums(datt22[,8:18]))
cSE2=rowSums(datt22[,19:24])-mean(rowSums(datt22[,2:7]))
cDEP2=rowSums(datt22[,25:35])-mean(rowSums(datt22[,8:18]))

#create dataset with only summed scores
datt3=cbind(
  datt22[,1],cSE1,cDEP1,cSE2,cDEP2,datt22[,1]*cSE1,datt22[,1]*cDEP1)
colnames(datt3)=c('cTx','cSE1','cDEP1','cSE2','cDEP2','XM1','XY1')


covsum=cov(datt3)
round(covsum,3)

##          cTx   cSE1  cDEP1   cSE2  cDEP2    XM1    XY1
## cTx    0.213 -0.171 -0.043  0.152 -0.177  0.066  0.016
## cSE1  -0.171 24.609 -5.516 11.323 -5.762  1.038  0.946
## cDEP1 -0.043 -5.516 37.883 -4.837 20.441  0.946 -0.599
## cSE2   0.152 11.323 -4.837 19.790 -7.616 -0.062  0.633
## cDEP2 -0.177 -5.762 20.441 -7.616 53.312  0.618 -0.977
## XM1    0.066  1.038  0.946 -0.062  0.618  4.819 -1.546
## XY1    0.016  0.946 -0.599  0.633 -0.977 -1.546  8.305

#correct covariance matrix
covsum[2,2]=covsum[2,2]-sum(coef(f1)[23:28]) #correct varM1
covsum[4,4]=covsum[4,4]-sum(coef(f1)[29:34]) #correct varM2
covsum[3,3]=covsum[3,3]-sum(coef(f2)[35:45]) #correct varY1
covsum[5,5]=covsum[5,5]-sum(coef(f2)[46:56]) #correct varY2
covsum[6,6]=covsum[1,1]*covsum[2,2] #corrected varXM1
covsum[7,7]=covsum[1,1]*covsum[3,3] #corrected varXY1

#correct covM1M2
covsum[2,4]=covsum[4,2]=covsum[2,4]-sum(coef(f1)[14:19])
#correct covY1Y2
covsum[3,5]=covsum[3,5]=covsum[3,5]-sum(coef(f2)[24:34])


#fit model to covariance matrix
sc_model='
cSE2~cTx+cSE1+cDEP1+XM1+XY1
cDEP2~cTx+cSE2+cSE1+cDEP1+XM1+XY1'
```

```r
scfit=cfa(sc_model,sample.cov=covsum,sample.nobs=1126)
summary(scfit)
```

```
## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        13
##
##   Number of observations                          1126
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   cSE2 ~
##     cTx               1.162    0.219    5.318    0.000
##     cSE1              0.518    0.023   22.816    0.000
##     cDEP1            -0.058    0.019   -3.143    0.002
##     XM1              -0.144    0.050   -2.879    0.004
##     XY1              -0.020    0.041   -0.492    0.623
##   cDEP2 ~
##     cTx              -0.463    0.377   -1.227    0.220
##     cSE2             -0.281    0.051   -5.527    0.000
##     cSE1              0.035    0.047    0.745    0.456
##     cDEP1             0.619    0.032   19.541    0.000
##     XM1              -0.023    0.085   -0.270    0.787
##     XY1              -0.074    0.070   -1.061    0.289
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .cSE2             11.282    0.475   23.728    0.000
##    .cDEP2            32.743    1.380   23.728    0.000
```

Factor scores for M

```
library(lavaan)

#obtain factor scores from CFA output
k2=lavPredict(f1,method='bartlett',fsm=T)
tsmpM=attr(k2,'fsm')[[1]] #scoring matrix
head(k2) #factor scores

##              m1         m2
## [1,]  0.9838734  0.9372603
## [2,]  1.4733252  1.2291032
## [3,] -0.7934914 -0.3144488
## [4,] -0.4553417  0.8488736
## [5,]  0.7621510 -0.4138175
## [6,]  1.6454972  1.2510743

k3=lavPredict(f2,method='bartlett',fsm=T)
tsmpY=attr(k3,'fsm')[[1]] #scoring matrix
head(k3) #factor scores

##              de1         de2
## [1,]  0.32997209 -0.03963784
## [2,] -1.27753092 -1.33811621
## [3,] -0.96301956  0.31103609
## [4,]  0.06217877  0.69253732
## [5,] -0.73778490 -0.26187261
## [6,] -0.35835906 -1.17793331

#estimate covariance matrix of factor scores
facdat=cbind(
  datt22[,1],k2[,1],k3[,1],k2[,2],k3[,2],datt22[,1]*k2[,1],
             datt22[,1]*k3[,1])
covfac=cov(facdat)
colnames(covfac)=rownames(covfac)=c('cTx','cSE1','cDEP1',
                                    'cSE2','cDEP2','XM1','XY1')
round(covfac,3)

##          cTx   cSE1  cDEP1   cSE2  cDEP2    XM1    XY1
## cTx    0.213 -0.037 -0.010  0.035 -0.027  0.014  0.004
## cSE1  -0.037  1.162 -0.215  0.537 -0.220  0.043  0.036
## cDEP1 -0.010 -0.215  0.961 -0.172  0.506  0.036 -0.022
## cSE2   0.035  0.537 -0.172  0.917 -0.271 -0.015  0.022
## cDEP2 -0.027 -0.220  0.506 -0.271  1.314  0.027 -0.023
## XM1    0.014  0.043  0.036 -0.015  0.027  0.230 -0.060
## XY1    0.004  0.036 -0.022  0.022 -0.023 -0.060  0.213

#obtain error covariance matrix for M
errM=matrix(0,nrow=12,ncol=12)
diag(errM)=coef(f1)[23:34]
diag(errM[1:6,7:12])=coef(f1)[14:19]
diag(errM[7:12,1:6])=coef(f1)[14:19]
```

```r
#obtain error covariance matrix for Y
errY=matrix(0,nrow=22,ncol=22)
diag(errY)=coef(f2)[35:56]
diag(errY[1:11,12:22])=coef(f2)[24:34]
diag(errY[12:22,1:11])=coef(f2)[24:34]


#correct covariance matrix among M1 and M2
covfac[c(2,4),c(2,4)] = covfac[c(2,4),c(2,4)]-tsmpM%*%errM%*%t(tsmpM)

#correct covariance matrix among Y1 and Y2
covfac[c(3,5),c(3,5)] = covfac[c(3,5),c(3,5)]-tsmpY%*%errY%*%t(tsmpY)

#corrected variances of XM1 and XM2
covfac[6,6]=covfac[1,1]*covfac[2,2]
covfac[7,7]=covfac[1,1]*covfac[3,3]

#estimate model
fc_model='
cSE2~cTx+cSE1+cDEP1+XM1+XY1
cDEP2~cTx+cSE2+cSE1+cDEP1+XM1+XY1'

fcfit=cfa(fc_model,sample.cov=covfac,sample.nobs=1126)
summary(fcfit)

## lavaan 0.6-12 ended normally after 1 iterations
##
##   Estimator                                         ML
##   Optimization method                           NLMINB
##   Number of model parameters                        13
##
##   Number of observations                          1126
##
## Model Test User Model:
##
##   Test statistic                                 0.000
##   Degrees of freedom                                 0
##
## Parameter Estimates:
##
##   Standard errors                             Standard
##   Information                                 Expected
##   Information saturated (h1) model          Structured
##
## Regressions:
##                    Estimate  Std.Err  z-value  P(>|z|)
##   cSE2 ~
##     cTx               0.267    0.047    5.701    0.000
##     cSE1              0.518    0.022   23.163    0.000
```

```
##     cDEP1             -0.065    0.025   -2.634    0.008
##     XM1               -0.197    0.049   -3.992    0.000
##     XY1               -0.063    0.055   -1.144    0.253
##   cDEP2 ~
##     cTx               -0.063    0.062   -1.029    0.304
##     cSE2              -0.224    0.039   -5.782    0.000
##     cSE1               0.009    0.035    0.246    0.805
##     cDEP1              0.535    0.032   16.662    0.000
##     XM1                0.012    0.065    0.187    0.851
##     XY1               -0.032    0.071   -0.445    0.656
##
## Variances:
##                    Estimate  Std.Err  z-value  P(>|z|)
##    .cSE2               0.516    0.022   23.728    0.000
##    .cDEP2              0.869    0.037   23.728    0.000
```

Structural models to accommodate a latent XM interaction M

Unconstrained indicator approach

The UPI approach can be estimated in any structural equation modeling. Below we provide code to estimate the UPI approach in Mplus.

```
TITLE:    Unconstrained product indicator for jobs ii data
DATA: FILE IS datt22.dat; !items centered at baseline
VARIABLE: NAMES ARE cTx se11-se16 depo1-depo11 se21-se26 dep1-dep11;
USEVARIABLE are cTx se11-se16 depo1-depo11 se21-se26 dep1-dep11
xm1-xm6 xy1-xy11;
DEFINE: !define product variables and include in USEVARIABLES above
xm1 = se11*cTx;
xm2 = se12*cTx;
xm3 = se13*cTx;
xm4 = se14*cTx;
xm5 = se15*cTx;
xm6 = se16*cTx;

xy1 = depo1*cTx;
xy2 = depo2*cTx;
xy3 = depo3*cTx;
xy4 = depo4*cTx;
xy5 = depo5*cTx;
xy6 = depo6*cTx;
xy7 = depo7*cTx;
xy8 = depo8*cTx;
xy9 = depo9*cTx;
xy10 = depo10*cTx;
xy11 = depo11*cTx;

ANALYSIS: estimator = ML; !bootstrap=500;
MODEL:

se1 by se11* (a1) !specify invariance across time
se12 (b1)
se13 (c1)
se14 (d1)
se15 (e1)
se16 (f1);
de1 by depo1* (a2)
depo2 (b2)
depo3 (c2)
depo4 (d2)
depo5 (e2)
depo6 (f2)
depo7 (g2)
depo8 (h2)
depo9 (i2)
```

```
depo10 (j2)
depo11 (k2);

se2 by se21* (a1)
se22 (b1)
se23 (c1)
se24 (d1)
se25 (e1)
se26 (f1);

de2 by dep1* (a2)
dep2 (b2)
dep3 (c2)
dep4 (d2)
dep5 (e2)
dep6 (f2)
dep7 (g2)
dep8 (h2)
dep9 (i2)
dep10 (j2)
dep11 (k2);

se11 with se21; !correlated residuals across time
se12 with se22;
se13 with se23;
se14 with se24;
se15 with se25;
se16 with se26;

se12 with se13; !correlated residual within time
se22 with se23; !correlated residual within time

  depo1 with dep1; !correlated residuals across time
  depo2 with dep2;
  depo3 with dep3;
  depo4 with dep4;
  depo5 with dep5;
  depo6 with dep6;
  depo7 with dep7;
  depo8 with dep8;
  depo9 with dep9;
  depo10 with dep10;
  depo11 with dep11;

xm by xm1* xm2-xm6; !define latent interaction term XM1
xm2 with xm3; !correlated residual within time
xm@.213; !constrain to var(x)*var(m1)
xy by xy1* xy2-xy11; !define latent interaction term XY1
xy@.213; !constrain to var(x)*var(y1)
```

```
[se11] (a3); [se21] (a3); !longitudinal invariance on intercepts
[se12] (b3); [se22] (b3);
[se13] (c3); [se23] (c3);
[se14] (d3); [se24] (d3);
[se15] (e3); [se25] (e3);
[se16] (f3); [se26] (f3);

[depo1@0] (a4);   [dep1@0] (a4); !longitudinal invariance intercepts
[depo2] (b4);   [dep2] (b4);
[depo3] (c4);   [dep3] (c4);
[depo4] (d4);   [dep4] (d4);
[depo5] (e4);   [dep5] (e4);
[depo6] (f4);   [dep6] (f4);
[depo7] (g4);   [dep7] (g4);
[depo8] (h4);   [dep8] (h4);
[depo9] (i4);   [dep9] (i4);
[depo10] (j4);  [dep10] (j4);
[depo11] (k4);  [dep11] (k4);


se2 on cTx (apath) !regression for M2
se1
de1
xm (h1path)
xy;

de2 on cTx !regression for Y2
se2 (bpath)
se1 de1 xm xy;

xm with se1@0; !constraints where interactions do not correlate
xm with de1@0;
xy with se1@0;
xy with de1@0;
xm with cTx@0;
xy with cTx@0;


[se1@0]; [de1@0]; [se2]; [de2]; !(conditional) means
se1@1;  de1@1; se2; de2; !(residual) variances


OUTPUT: tech1 modindices(10); !cinterval(bootstrap);
```

Latent moderated structural equations

Below we provide Mplus code to estimate latent BTI effects in the two wave model.

```
TITLE:  LMS examples for jobs ii data
DATA: FILE IS datt22.dat; !items centered at baseline
VARIABLE: NAMES ARE cTx se11-se16 depo1-depo11 se21-se26 dep1-dep11;
USEVARIABLE are cTx se11-se16 depo1-depo11 se21-se26 dep1-dep11;
ANALYSIS: Type = random; estimator = ML; algorithm = integration;

MODEL:
se1 by se11* (a1) !specify invariance across time
se12 (b1)
se13 (c1)
se14 (d1)
se15 (e1)
se16 (f1);
de1 by depo1* (a2)
depo2 (b2)
depo3 (c2)
depo4 (d2)
depo5 (e2)
depo6 (f2)
depo7 (g2)
depo8 (h2)
depo9 (i2)
depo10 (j2)
depo11 (k2);
depo11 (l2);
se2 by se21* (a1)
se22 (b1)
se23 (c1)
se24 (d1)
se25 (e1)
se26 (f1);
de2 by dep1* (a2)
dep2 (b2)
dep3 (c2)
dep4 (d2)
dep5 (e2)
dep6 (f2)
dep7 (g2)
dep8 (h2)
dep9 (i2)
dep10 (j2)
dep11 (k2);

[se11@0] (a3); [se21@0] (a3); !longitudinal invariance on intercepts
[se12] (b3); [se22] (b3);
[se13] (c3); [se23] (c3);
```

```
[se14] (d3); [se24] (d3);
[se15] (e3); [se25] (e3);
[se16] (f3); [se26] (f3);

[depo1@0] (a4);   [dep1@0] (a4); !longitudinal invariance on intercepts
[depo2] (b4);   [dep2] (b4);
[depo3] (c4);   [dep3] (c4);
[depo4] (d4);   [dep4] (d4);
[depo5] (e4);   [dep5] (e4);
[depo6] (f4);   [dep6] (f4);
[depo7] (g4);   [dep7] (g4);
[depo8] (h4);   [dep8] (h4);
[depo9] (i4);   [dep9] (i4);
[depo10] (j4);  [dep10] (j4);
[depo11] (k4);  [dep11] (k4);

se11 with se21; !correlated residuals across time
se12 with se22;
se13 with se23;
se14 with se24;
se15 with se25;
se16 with se26;
se12 with se13; !correlated residual within time
se22 with se23; !correlated residual within time

depo1 with dep1; !correlated residuals across time
depo2 with dep2;
depo3 with dep3;
depo4 with dep4;
depo5 with dep5;
depo6 with dep6;
depo7 with dep7;
depo8 with dep8;
depo9 with dep9;
depo10 with dep10;
depo11 with dep11;

xm1 | se1 XWITH cTx; ! specify latent interaction
xy1 | de1 XWITH cTx; ! specify latent interaction

se2 on cTx (apath) !regression for M2
se1
de1
xm1 (h1path)
xy1;

de2 on cTx !regression for Y2
se2 (bpath)
se1 de1 xm1 xy1;
```

```
[se1]; [de1]; [se2]; [de2]; !(conditional) means
se1@1;  de1@1; se2@1; de2@1; !(residual) variances

de1 with se1; !covariance for baseline measures


OUTPUT: tech1;
```

Bayesian mediation

Below we provide R code that interacts with JAGS estimate latent BTI effects in the two-wave model using MCMC with a Gibbs sampler in JAGS.

```
library(R2jags) #JAGS should be installed in computer
bcafa=function() { #specify model via function

  ######################################################################
  # Specify latent var model for M1 and M2
  ######################################################################
  for (i in 1:n){
    #two latent variables: construct + specific factor for corr residuals
    #invariance via using same loading for time 1 and time 2
    mu1[i,1] <- tau[1] + nu[i,9]*lambda[1] + nu[i,1]*beta[1]
    m1[i,1] ~ dnorm(mu1[i,1], inv.psi1[1]) #normal distribution
    mu2[i,1] <-  tau[1] + ksi2[i]*lambda[1]+ nu[i,1]*beta[1]
    m2[i,1] ~ dnorm(mu2[i,1], inv.psi2[1])


    #additional latent variable for item 2/3 for within time correlated residual
    mu1[i,2] <- tau[2] + nu[i,9]*lambda[2] + nu[i,2]*beta[2] + nu[i,7]*beta[7]
    m1[i,2] ~ dnorm(mu1[i,2], inv.psi1[2])
    mu2[i,2] <- tau[2] + ksi2[i]*lambda[2] + nu[i,2]*beta[2] + nu[i,8]*beta[8]
    m2[i,2] ~ dnorm(mu2[i,2], inv.psi2[2])

    mu1[i,3] <- tau[3] + nu[i,9]*lambda[3] + nu[i,3]*beta[3] + nu[i,7]*beta[7]
    m1[i,3] ~ dnorm(mu1[i,3], inv.psi1[3])
    mu2[i,3] <- tau[3] + ksi2[i]*lambda[3] + nu[i,3]*beta[3] + nu[i,8]*beta[8]
    m2[i,3] ~ dnorm(mu2[i,3], inv.psi2[3])

    mu1[i,4] <- tau[4] + nu[i,9]*lambda[4] + nu[i,4]*beta[4]
    m1[i,4] ~ dnorm(mu1[i,4], inv.psi1[4])
    mu2[i,4] <- tau[4] + ksi2[i]*lambda[4] + nu[i,4]*beta[4]
    m2[i,4] ~ dnorm(mu2[i,4], inv.psi2[4])

    mu1[i,5] <- tau[5] + nu[i,9]*lambda[5] + nu[i,5]*beta[5]
    m1[i,5] ~ dnorm(mu1[i,5], inv.psi1[5])
    mu2[i,5] <- tau[5] + ksi2[i]*lambda[5] + nu[i,5]*beta[5]
    m2[i,5] ~ dnorm(mu2[i,5], inv.psi2[5])

    mu1[i,6] <- tau[6] + nu[i,9]*lambda[6] + nu[i,6]*beta[6]
    m1[i,6] ~ dnorm(mu1[i,6], inv.psi1[6])
    mu2[i,6] <- tau[6] + ksi2[i]*lambda[6] + nu[i,6]*beta[6]
    m2[i,6] ~ dnorm(mu2[i,6], inv.psi2[6])
  }

  ######################################################################
  # Specify latent var model for Y1 and Y2
  ######################################################################
  for (i in 1:n){
    for(jj in 1:JJ){
```

```
    yu1[i,jj] <- tau.y[jj]+mu.y1[i]*lambda.y[jj] + nu.y[i,jj]*beta.y[jj]
    y1[i,jj] ~ dnorm(yu1[i,jj], inv.psi1.y[jj])
    yu2[i,jj] <-tau.y[jj]+ksi.y2[i]*lambda.y[jj] + nu.y[i,jj]*beta.y[jj]
    y2[i,jj] ~ dnorm(yu2[i,jj], inv.psi2.y[jj])
  }
}

######################################################################
# Specify the (prior) distribution for M1 and M2
######################################################################
for(j in 1:J){

  inv.psi1[j] ~ dgamma(1,1) # Precisions for observables
  inv.psi2[j] ~ dgamma(1,1) # Precisions for observables
  psi1[j] <- 1/inv.psi1[j]   # Variances for observables
  psi2[j] <- 1/inv.psi2[j]   # Variances for observables

}

for(j in 1:J){

  lambda[j] ~ dnorm(1,.5);T(0,) #half normal distribution for loadings
}

tau[1] <- 0 #first intercept to 0
for(j in 2:J){
  tau[j] ~ dnorm(0,.5)
}

for(k in 1:8){
  beta[k] ~ dnorm(1,.5);T(0,) #half normal distribution for loadings
}

mu.m ~ dnorm(0,1) #mean of m
nu.mean=c(0,0,0,0,0,0,0,0,mu.m) #mean vector
Imat=kk
for(i in 1:n){
  nu[i,1:9] ~ dmnorm.vcov(nu.mean,Imat) #multivariate prior
}
######################################################################
# Specify the (prior) distribution for Y1 and Y2
######################################################################

for(j in 1:JJ){
  inv.psi1.y[j] ~ dgamma(1,1) # Precisions for observables
  inv.psi2.y[j] ~ dgamma(1,1) # Precisions for observables
  psi1.y[j] <- 1/inv.psi1.y[j]   # Variances for observables
  psi2.y[j] <- 1/inv.psi2.y[j]   # Variances for observables

}
```

```
  for(j in 1:JJ){
    lambda.y[j] ~ dnorm(1,.5);T(0,) #half normal distribution for loadings
    beta.y[j] ~ dnorm(1,.5);T(0,)
  }
  tau.y[1] <- 0
  for(j in 2:JJ){
    tau.y[j] ~ dnorm(0,.5) # normal distribution for intercepts
  }

  nu.mean.y=c(0,0,0,0,0,0,0,0,0,0,0)
  Imat2=mm
  for(i in 1:n){
    nu.y[i,1:11] ~ dmnorm.vcov(nu.mean.y,Imat2) #multivariate priors
  }

  ######################################################################
  # Structural pars
  ######################################################################
  int1~dnorm(0,.001)
  int2~dnorm(0,.001)
  apath~dnorm(0,.001)
  bpath~dnorm(0,.001)
  cpath~dnorm(0,.001)
  smpath~dnorm(0,.001)
  sypath~dnorm(0,.001)
  cmpath~dnorm(0,.001)
  cypath~dnorm(0,.001)
  h1path~dnorm(0,.001)
  h2path~dnorm(0,.001)
  h3path~dnorm(0,.001)
  h4path~dnorm(0,.001)

  int0~dnorm(0,.001)  #intercept in regression Y1 on M1
  cov1~dnorm(0,.001)  #reg coefficient in regression Y1 on M1

  for (i in 1:n){
    mu.y[i]<-int0+cov1*nu[i,9]
    mu.y1[i] ~ dnorm(mu.y[i],1)
    kappa[i]<-int1+apath*x[i]+smpath*nu[i,9]+cypath*mu.y1[i]+
      h1path*nu[i,9]*x[i]+h2path*mu.y1[i]*x[i]
    ksi2[i] ~ dnorm(kappa[i],1)  # distribution for the latent variables
    kappa.y[i]<-int2+cpath*x[i]+bpath*ksi2[i]+sypath*mu.y1[i]+cmpath*nu[i,9]+
      h3path*nu[i,9]*x[i]+h4path*mu.y1[i]*x[i]
    ksi.y2[i] ~ dnorm(kappa.y[i],1)  # distribution for the latent variables

  }

} # closes the model
```

```r
#data and specification of values for loops
jagsdat=list(m1=datt22[,2:7],m2=datt22[,19:24],y1=datt22[,8:18],
y2=datt22[,25:35], n=nrow(datt22),J=6,JJ=11,kk=diag(9),mm=diag(11),x=datt22[,1])

#parameters to track
bpar <- c("tau", "lambda",'beta','psi1','psi2','int1','int2','smpath','sypath',
'cmpath','cypath','tau.e','tau.e2',"tau.y", "lambda.y",'beta.y','psi1.y',
'psi2.y', 'apath','bpath','cpath','h1path','h2path','h3path','h4path',
          'mu.m','cov1','int0')

#run model with the following specifications
#parallelization with three chains
#total iterations of 12000
#burn first 2000 iterations per chain
jagcfa4 <- jags.parallel(data = jagsdat,
                         parameters.to.save = bpar, n.chains=3,
                         n.iter = 12000, n.burnin = 2000, model.file = bcafa)
```