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Elasto-multi-body dynamics of a multicylinder internal combustion engine

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Abstract: This paper presents a multi-body model of a four-cylinder, four-stroke diesel engine, incorporating component flexibility. The model also includes engine firing order and experimentally measured combustion time history. The paper presents numerical predictions for conical motion of the flywheel as a result of combined torsion–deflection modes of the flexible crankshaft system. The half-engine order responses induce complex three-dimensional whirling motion of the flywheel, which is responsible for repetitive shock loading of the drivetrain system through impact with the clutch system. This leads to an assortment of noise and vibration concerns, one of which is in-cycle vibration of the clutch system. Numerical prediction of this vibration response agrees well with experimental findings.

Keywords: internal combustion engine, multi-body dynamics, elastodynamics, torsion–deflection vibration

NOTATION

$\{a\}$	position vector of a local frame with respect to the global frame	m	multiple of k th harmonics of engine vibration
A	piston crown area	$m\beta$	mass fraction of burnt fuel
B	bore size	n	number of cylinders
c	journal clearance	p	gas force
C	constraint function	P	combustion pressure
$[D]$	damping matrix	q	generalized coordinates
$(e, \beta)^T$	local part frame of reference	Q	heat energy
F	force	R_j	radius of the journal
F_f	piston friction force	t	time
F_q	generalized forces in the Euler frame of reference	T	torque or cylinder temperature in equation (20)
G	modulus of rigidity	T_f	Petroff's friction torque
h_c	lower calorific value of fuel	T_g	gas temperature
h_t	instantaneous heat transfer coefficient	T_S	starter motor torque
I	second area moment of inertia	T_w	piston wall temperature
j	cylinder identity number	v	speed of entraining motion in the journal bearing
J	mass moment of inertia	v_p	piston translational velocity
K	kinetic energy	V	cylinder volume
l	journal bearing width		
L	beam length	γ	ratio of specific heat for trapped gas
		η	lubricant dynamic viscosity
		λ	Lagrangian multipliers
		ς	condensed form of the state variable
		τ	period of application of heat
		$\dot{\phi}_{cr}$	crankshaft angular velocity
		(ψ, θ, ϕ)	Euler angles
		ω	crankshaft radiancy (angular frequency)

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1 INTRODUCTION

Combined torsion and deflection modes for multicylinder engines can be obtained analytically, as described later (see also reference [1]). The analysis indicates non-diminishing responses for odd and half order engine harmonics, which is consistent with experimental evidence, for example that reported by Dixon *et al.* [2], showing that major contributions are observed at 2.5 and 3.5 engine order with four-cylinder in-line engines. This is in good agreement with the analytical calculations reported here, and can also be partially caused by combustion difference between cylinders, as suggested by Dixon *et al.* [2]. The analysis in this section indicates that the first mode for combined torsion and deflection in four-cylinder in-line engines can coincide with the 2.5 engine order in the speed range 3500–4000 r/min (the first mode being typically in the range 150–200 Hz). However, the second and higher combined torsion and deflection modes usually occur above 350 Hz. This means that the contribution at 3.5 engine order can only be significant as a source of excitation for crankshaft out-of-plane modes at engine speeds above 5500 r/min. Dixon *et al.* [2], however, have noted its significance at an engine speed of 2000 r/min. They have observed that the significant contribution at 3.5 engine order cannot be explained in terms of cylinder imbalance. Furthermore, Kinoshita *et al.* [3] have also shown that the first torsion–deflection out-of-plane of crank throw mode for their particular four-cylinder engine occurs at a frequency of 360 Hz.

The torsional vibration signature in a four-cylinder engine occurs at engine orders 2, 4, 6 with radial vibrations exhibiting contributions at the same frequencies, but also having odd and half engine order content. The even orders contribute to the bending mode spectrum due to combustion variation from cylinder to cylinder, while half and odd orders occur owing to combined torsion and bending. The 3.5 order appears as a dominant source in the speed range 5000–6000 r/min (a confirmation of the arguments presented here). The acceleration signal obtained radially at the crankshaft front-end by Kinoshita *et al.* [3] coincided with the half crankshaft rotation period (i.e. twice the engine order). Simultaneous measurements from the fourth main journal bearing caps indicated signal fluctuations at the first order of the combustion process (half engine order). This appears to corroborate the hypothesis that the fundamental forcing frequency for the impact dynamics problem highlighted above is the half engine order. This paper attempts to verify this claim through detailed elasto-multi-body dynamic analysis.

These discussions highlight the complexity of the combined effects of inertial force and torque excitations upon the formation of coupled out-of-plane torsion–deflection modes of the crankshaft system. The analytical approaches outlined below, although very useful in the understanding of the complex engine dynamics spectra (particularly in conjunction with experimental data), do

not include the effect of many sources of non-linearity. These sources of non-linearity include crankshaft elasticity, assembly constraints, journal bearing hydrodynamics and the combustion process itself. Furthermore, the assembly of parts incorporates constraints, representing the attachment of parts or restraints that inhibit certain undesired motions such as the inclusion of torsional dampers at the crankshaft front-end. These non-linear sources are necessarily precluded or linearized so that analytical solutions can be obtained [1]. The exclusion of non-linearities can lead to erroneous conclusions. These shortcomings can be remedied by multi-body dynamic formulation of the engine dynamic problem which can incorporate all sources of non-linearity in the system. The following sections describe a number of multi-body dynamic models.

The term ‘whoop’, or in-cycle vibration, is applied to a clutch pedal noise and vibration problem, which can be heard as noise in the driver’s footwell area and felt as vibration through the clutch pedal. This phenomenon occurs during the depression and release of the clutch pedal in a motor vehicle. The critical range for the release system of the clutch is at a frequency of 250–300 Hz, when the clutch pedal is being operated. Therefore, this is a dynamic or in-cycle concern. Observations have indicated that vehicles with diesel engines and mechanically actuated clutches are more prone to this phenomenon. It has been shown that whoop is excited by the elastodynamic response of the crankshaft, caused by the combustion of the engine cylinders [4, 5]. The frequency and the amplitude of the vibration depend on the design parameters of the clutch and of the clutch actuation system.

Although whoop has been an industry-wide problem since the early 1980s, it has only in recent times received the fundamental study that it clearly deserves [4–7]. The traditional approach in dealing with this problem has been a palliative rather than a preventive one. Although various palliative measures have been advocated, a typical solution has been the use of a ‘Diehl fix’. This is a mass and a rubber block damper attached to the clutch release lever, in effect shifting the natural frequency of its oscillations from the normal 250–270 Hz range to around 450–520 Hz. At these higher frequencies the amplitudes of oscillations are considerably reduced and the problem is shifted to higher engine speeds [5, 6].

2 ENGINE TORSIONAL VIBRATIONS

2.1 Torsional rigid modes

The torsional vibrations of a multicylinder engine are induced by power torque variations, which result from the magnitude of the gas force and the engine firing order. Although the analytical solution to this is generally understood, it is nevertheless included here as it forms a part of the validation process for the multi-body dynamic analysis later on.

Equation (1) expresses the torque fluctuations in a multi-cylinder four-stroke engine as a Fourier series based upon the k th harmonics of the engine cycle frequency. For an n -cylinder engine the combined or resultant torque fluctuation can be expressed as

$$T = \sum_{j=1}^n p_{jk} e^{i(k/2)(\omega t - \phi_j)} \quad (1)$$

k is a positive integer number and can be formulated as even and odd multiples of $m = 1, 2, 3, \dots$ in order to represent various whole and half-multiples of engine order in the exponential term in the above equation:

$$\begin{aligned} k &= 4m && \text{(even engine orders: } k = 4, 8, 12, 16, 20, \dots) \\ k &= 4m - 2 && \text{(odd engine orders: } k = 2, 6, 10, 14, 18, \dots) \\ k &= 4m - 1 && \text{(half engine orders: } k = 3, 7, 11, 15, 19, \dots) \\ k &= 4m - 3 && \text{(half engine orders: } k = 1, 5, 9, 13, 17, \dots) \end{aligned}$$

Expanding the resulting Fourier series for the above conditions separately, and regrouping the remaining terms into an exponential form again, the power torque fluctuations for a four-cylinder engine in the case of even, odd and half order engine harmonics can be obtained as [1]

$$\begin{aligned} T &= (p_{1k} + p_{2k} + p_{3k} + p_{4k}) e^{i(k/2)\omega t} && \text{for } k = 4m \\ T &= (p_{1k} - p_{2k} - p_{3k} + p_{4k}) e^{i(k/2)\omega t} && \text{for } k = 4m - 2 \\ T &= \{(p_{1k} - p_{4k}) - i(p_{2k} - p_{3k})\} e^{i(k/2)\omega t} && \text{for } k = 4m - 1 \\ T &= \{(p_{1k} - p_{4k}) + i(p_{2k} - p_{3k})\} e^{i(k/2)\omega t} && \text{for } k = 4m - 3 \end{aligned}$$

For no combustion variation between cylinders, the power torque fluctuations diminish, except for even engine order multiples. Thus

$$T = 4p_k e^{i(k/2)\omega t} \quad \text{for } k = 4m \quad (2)$$

These analytical calculations are obviously consistent with the experimental findings for four-cylinder, four-stroke engines [2, 3], giving engine orders 2, 4, 6 and 8 as the main contributory factors.

2.2 Torsional-deflection modes

The cylinder power torques act at different locations along the crankshaft. It is therefore clear that the cylinder block is

subjected to torsional deflection with each cylinder power stroke. In the simplest analysis of this effect, the cylinder block may be considered as a beam element. The torsional-deflection (i.e. the angle of twist), θ , for a given beam of length L is given by the general torsion formula as

$$\theta = \frac{TL}{GJ} \quad (3)$$

where G is the modulus of rigidity of the beam material, T is the power torque and J is the second area moment of inertia of the modelled beam.

In the same manner as in the previous case, the combined torsion-deflection of the cylinder block can be considered as having been induced by k harmonics of the fluctuating cylinder power torques as [1]

$$\theta_k = \sum_{j=1}^n \frac{T_{jk} L_j}{GJ} \quad (4)$$

where L_j denotes the distance from the centre of gravity of the j th cylinder to the point of application of the power torque of the corresponding cylinder along the crankshaft axis (see Fig. 1). For $\{L_j\} = B\{1.5, 0.5, -0.5, -1.5\}^T$, the combined angle of twist is obtained as

$$\begin{aligned} \theta_k &= \frac{B}{GJ} \{1.5p_{1k} e^{i(k/2)\omega t} + 0.5p_{2k} e^{i(k/2)(\omega t - 3\pi)} \\ &\quad - 0.5p_{3k} e^{i(k/2)(\omega t - \pi)} - 1.5p_{4k} e^{i(k/2)(\omega t - 2\pi)}\} \end{aligned} \quad (5)$$

Again, as in the previous case, the torsional deflection modes can be represented by k harmonics of power torque as [1]

$$\begin{aligned} \theta_k &= \frac{B}{GJ} \{1.5(p_{1k} - p_{4k}) + 0.5(p_{2k} - p_{3k})\} e^{i(k/2)\omega t} && \text{for } k = 4m \text{ (even orders)} \\ \theta_k &= \frac{B}{GJ} \{1.5(p_{1k} - p_{4k}) + 0.5(p_{3k} - p_{2k})\} e^{i(k/2)\omega t} && \text{for } k = 4m - 2 \text{ (odd orders)} \end{aligned}$$

Fig. 1 Crankshaft configuration

$$\theta_k = \frac{B}{GJ} \{1.5(p_{1k} + p_{4k}) + 0.5i(p_{2k} - p_{3k})\} e^{i(k/2)\omega t}$$

for $k = 4m - 3$ (half-orders)

$$\theta_k = \frac{B}{GJ} \{1.5(p_{1k} + p_{4k}) - 0.5i(p_{2k} + p_{3k})\} e^{i(k/2)\omega t}$$

for $k = 4m - 1$ (half-orders)

For no combustion variation between cylinders it can be observed that the first two expressions return diminished responses. Therefore, half engine order contributions remain only for $k = 4m - 1$ and $k = 4m - 3$. This results in engine 'roughness' output at $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$ and $\frac{7}{2}$ engine orders.

This simplified analytical treatment underpins the experimental findings in references [2] and [3].

3 MULTI-BODY ENGINE MODEL

The multi-body model is a constrained non-linear dynamics model incorporating the elastic behaviour of the crankshaft system as a series of elastic crankpins interspersed by rigid inertial elements represented by crank webs. The model is created using constrained Lagrangian dynamics in a generalized Eulerian body 3–1–3 frame of reference. If the generalized coordinates are denoted by $\{q_j\}_{j=1 \rightarrow 6} = \{x, y, z, \psi, \theta, \varphi\}^T$, then

$$\frac{d}{dt} \left(\frac{\partial K}{\partial \dot{q}_f} \right) - \frac{\partial K}{\partial q_j} - F_{q_j} + \sum_{k=1}^n \lambda_k \frac{\partial C_k}{\partial q_j} = 0 \quad (6)$$

The n constraint functions for the different joints in the engine model are represented by a combination of holonomic and non-holonomic functions as

$$\begin{bmatrix} C_k \\ \dot{q}_f \end{bmatrix} \frac{\partial C_k}{\partial q_j} = 0, \quad j = 1 \rightarrow 6, k = 1 \rightarrow n \quad (7)$$

The compliance of the flexible members are given by stiffness and damping matrices of the form defined by Bernoulli's three-dimensional beam elements:

$$\{F, T\}^T = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_{zz}}{L^3} & 0 & 0 & 0 & \frac{-6EI_{zz}}{L^2} \\ 0 & 0 & \frac{12EI_{yy}}{L^3} & 0 & \frac{6EI_{yy}}{L^2} & 0 \\ 0 & 0 & 0 & \frac{GI_{xx}}{L} & 0 & 0 \\ 0 & 0 & \frac{6EI_{yy}}{L^2} & 0 & \frac{4EI_{yy}}{L} & 0 \\ 0 & \frac{-6EI_{zz}}{L^2} & 0 & 0 & 0 & \frac{4EI_{zz}}{L} \end{bmatrix} \times \{e, \beta\}^T - [D] \left\{ \frac{de}{dt}, \frac{d\beta}{dt} \right\} \quad (8)$$

where the first matrix is the stiffness matrix, $[D]$ is the structural damping matrix, given usually as a percentage of the former (in the range 1–5 per cent for the crankshaft system), and $\{e, \beta\}^T$ represents the local part frame of reference. This is an approximate approach, based on Eulerian beam formulation. A more accurate representation of elastic parts would be by mass/inertial and stiffness/damping matrices through finite element modal analysis, using superelement capability. This approach is not deemed necessary for the purpose of discretization of simple continuous structures such as crankpins.

3.1 Generalized forces

These elastic restoring forces and other applied forces are transformed to Euler's generalized coordinates by the application of Bernoulli's principle of virtual work. For the applied forces

$$\{\delta W\} = \{\delta q_i\}^T \{F\} + \{\delta \beta\}^T \{e\} [T_{ig}] \{F\} \quad (9)$$

where

$$[T_{ij}] = \begin{bmatrix} \cos \psi \cos \varphi - \sin \psi \cos \theta \sin \varphi & & \\ & -\cos \psi \sin \varphi - \sin \psi \cos \theta \cos \varphi & \sin \psi \sin \theta \\ \sin \psi \cos \varphi + \cos \psi \cos \theta \sin \varphi & & \\ & -\sin \psi \sin \varphi + \cos \psi \cos \theta \cos \varphi & -\cos \psi \sin \theta \\ \sin \theta \sin \varphi & \sin \theta \cos \varphi & \cos \theta \end{bmatrix}$$

and $j = g$ denotes the ground fixed frame of reference.

The coefficient of $\{\delta q_i\}^T$ gives the generalized force, while the second term provides the torque about the centre of mass of the part with the infinitesimal rotation $\{\delta \beta\}^T$ expressed in terms of the local part frame of reference. The torque contribution [i.e. the second term in equation (9)] can be given in terms of the global Euler frame of reference as $[E]^T \{e\} [T_{ig}] \{F\}$, where $[E]$ is the transformation from the Euler axis frame to the local part frame:

$$[E] = \begin{bmatrix} \sin \theta \sin \varphi & 0 & \cos \varphi \\ \sin \theta \cos \varphi & 0 & -\sin \varphi \\ \cos \theta & 1 & 0 \end{bmatrix}$$

For applied torques, the virtual work is given as

$$\{\delta W\} = \{\delta q_r\}^T [E]^T \{e\} \{T\} \quad (10)$$

and the coefficient of $\{\delta q_r\}^T$ gives the generalized torque contributions, noting that $\{\delta \beta\} = [E] \{\delta q_r\}$. The

elastic forces, as set out in equation (8), are also transformed, using

$$\begin{aligned}\{\mathbf{F}_{q_j}\}_{j=1 \rightarrow 3} &= \{\mathbf{F}\} \quad \text{and} \\ \{\mathbf{F}_{q_j}\}_{j=4 \rightarrow 6} &= [\mathbf{E}]^T([\mathbf{T}_{ig}]\{\mathbf{T}\} + \{\mathbf{e}\}[\mathbf{T}_{ig}]\{\mathbf{F}\})\end{aligned}$$

3.2 Constraining reactions

The generalized forces are the applied, restoring and dissipative forces, such as those introduced by the compliant members in the previous section. The constraining elements in a multi-body system also introduce reactions that are included in Lagrangian dynamics by the last term on the left-hand side of equation (6).

The reaction forces and moments introduced by the imposed constraints are obtained in the same manner in terms of the Lagrangian multipliers, λ_k , where the following relation for infinitesimal changes should hold:

$$\lambda_k \delta C_k = 0 \quad (11)$$

in which C_k is a holonomic constraint function. These are the primitive functions that ensure positional or orientation conditions. Certain combinations of these form a physical joint. The most common types are the at-point or point coincident constraint, the in-plane and in-line joint primitives, perpendicular axes and prescribed angular orientation such as the imposition of parallel axes condition. Coupling constraints may also be imposed, relating the position (by a holonomic constraint) or velocity (by a non-holonomic constraint) of parts with respect to each other. Since a combination of at-point and perpendicular or parallel axes are employed in the formulation of various joints in the multi-body engine model, as an example the formulated reactive forces for a revolute pair (being a combination of the former two cases) is given here. Using Bernoulli's virtual work principle for an at-point constraint, it can be shown that

$$\begin{aligned}\delta W &= (\{\delta \mathbf{q}_i\}_i^T + \{\delta \mathbf{q}_r\}_i^T [\mathbf{E}]_i^T \{\mathbf{e}_i\} [\mathbf{T}_{ig}] - \{\delta \mathbf{q}_j\}_j^T \\ &\quad + \{\delta \mathbf{q}_r\}_j^T [\mathbf{E}]_j^T \{\mathbf{e}_j\} [\mathbf{T}_{jg}]) \{\lambda\}\end{aligned} \quad (12)$$

Therefore, the generalized force for both parts i and j are given as the coefficients of the terms $\{\delta \mathbf{q}_i\}_i^T$, in other words $\{\lambda\}$. The generalized constraining torque is provided by the coefficient of the terms $\{\delta \mathbf{q}_r\}_i^T$ or $[\mathbf{E}]_{ij}^T \{\mathbf{e}_{ij}\} [\mathbf{T}_{ij}] \{\lambda\}$.

For the perpendicular axes, there are clearly no generalized constraining forces, and the generalized constraining moments on parts i and j are given as $[\mathbf{E}]_{ij}^T \{\hat{\mathbf{a}}_{ij}\} [\mathbf{T}_{ij}] \{\mathbf{a}_{ji}\} \{\lambda\}$. Note that for codirected z axes at a revolute joint the vectors $\{\mathbf{a}_i\}$ and $\{\mathbf{a}_j\}$ yield the conditions $\mathbf{z}_i \cdot \mathbf{x}_j = \mathbf{z}_i \cdot \mathbf{y}_j = 0$, which restates axis orthogonality constraints. In addition to these two constraint functions, the relative motion of two joint markers are constrained, and thus $\{\mathbf{a}_i\} - \{\mathbf{a}_j\} = 0$. This relationship renders three additional algebraic constraint

functions in respect of motions along the directions x , y and z .

3.3 Equations of motion

The formulated generalized forces in case of body forces, applied forces and the constraining reactions can be implemented in equation (6). The differential-algebraic equation set can now be represented as follows:

$$\begin{bmatrix} \left[\frac{1}{\delta t} \frac{\partial K}{\partial \dot{\mathbf{q}}_j} + \frac{\partial K}{\partial \mathbf{q}_j} \right] & \begin{bmatrix} \left[\frac{\partial C_k}{\partial \mathbf{q}_j} \right]^T & 0 \\ 0 & \left[\frac{\partial C_k}{\partial \mathbf{q}_j} \right]^T \end{bmatrix} \\ \left[\frac{\partial C_k}{\partial \mathbf{q}_j}, \dot{\mathbf{q}} \frac{\partial}{\partial \mathbf{q}_j} \left(\frac{\partial C_k}{\partial \mathbf{q}_j} \right) \right] & [0] \end{bmatrix} \times \{\delta \mathbf{q}_j, \delta \lambda_k\}^T = \{\mathbf{F}_{q_j}\} \quad (13)$$

3.4 Method of solution

In general, the rigid body equations of motion for each part within the engine model can be rewritten as

$$F_j = f\left(\frac{\partial^2 q}{\partial t^2}, \frac{\partial q}{\partial t}, q, \lambda, t\right) = 0, \quad j = 1, \dots, 6 \quad (14)$$

$$F_k = \left(T_k, \frac{\partial q_k}{\partial t}, q_k\right) = 0, \quad k = 1, \dots, 6 \quad (15)$$

Because the translational components of momentum are obtained directly from the translational velocities, the six equations generated from F_k can be reduced to three by introducing the components of angular momentum in the form

$$F_r = \left(T_r, \frac{\partial q_r}{\partial t}, q_r\right), \quad k = 4, 5, 6 \quad (16)$$

where $\{\mathbf{q}_r\} = \{\psi, \theta, \varphi\}^T$.

Equations of motion are reduced to first order by the substitution $\vartheta = \dot{\mathbf{q}}_k$, and thus they can be represented in a condensed format as $F(\zeta, \varsigma, \lambda, t) = 0$, where $\varsigma = (\vartheta, q, \dot{T}_r)^T$. The algebraic scalar functions for holonomic constraints are also included in the condensed format as

$$C(\varsigma) = 0 \quad (17)$$

Equations (16) and (17) are solved using a combination of Cholesky factorization, Newton–Raphson iterations and a predictor-corrector scheme, including step-by-step integration for state variable derivatives. The procedure is highlighted in references [1] and [8].

3.5 Description of the model

The present multi-body model of the internal combustion engine is shown in Fig. 2 for a 1.81 L four-cylinder, four-stroke diesel engine. The model comprises an assembly of inertial elements: flywheel, piston assembly, connecting rods, crankshaft, cam gear and the starter motor. It also includes journal bearings that provide resistive torque as well as support for the assembly. The model includes component flexibility for the crankshaft system—non-linear elastic crankpins, with lumped mass webs, as previously described. The journal bearing supports are considered to be thin shell bearings, in keeping with their current use in modern motor vehicles. The description of the formulated thin shell bearing is beyond the scope of this paper. Readers are referred to reference [10]. This necessitates transient elastohydrodynamic analysis of pressure distribution and lubricant film thickness. The cylinder combustion force is calculated from the cylinder combustion gas pressure, which has been measured experimentally by Ford Motor Company (see Fig. 3) and is included in the model.

The assembly of parts has been achieved by the use of holonomic constraint functions. The starter motor torque is used for the same engine, with a maximum value of -300 N m , and operates for 0.1 s (the negative sign indicating a clockwise sense of application). Therefore, the starter motor torque can be represented by a function such as [11]

$$T_s = \begin{cases} -M(300 + \dot{\phi}_{\text{cr}}) & \text{at } 0 \leq t \leq 0.1 \\ 0 & \text{at } t > 0 \end{cases} \quad (18)$$

where M is a constant.

The resistive torque is also a measured quantity, because, in addition to the tractive contributions at the journal bearings, there are other contributory factors such as dry friction in the crankshaft assembly that are difficult to quantify by analytical means. The contribution due to hydrodynamic traction can be obtained using Petroff's bearing relationship, which for a π film is given as [12]

$$T_f = \frac{\pi \eta v \ell R_J^2}{c} \quad (19)$$

All the rigid body inertial properties of the parts (mass and mass moments of inertia) are given in Table 1. The constraints between various components denoted as part I and part J are given in Table 2.

A key input into the engine model is the combustion force. This was achieved by including the measured cylinder pressure for a complete cycle into each of the modelled cylinders and as a spline function in terms of the crankangle position. This is shown in Fig. 3. The combustion pressure can also be obtained numerically using the first law of thermodynamics for the trapped air-fuel mass. The gas mixture can be treated as ideal with air properties. The rate of change in pressure is given as [11, 13]

$$\begin{aligned} \frac{dP}{dt} &= \frac{\gamma - 1}{V} \frac{dQ}{d\tau} - \frac{\gamma}{V} P \frac{dV}{dt} - \frac{\gamma - 1}{V} \frac{dQ}{dT} \\ \frac{dQ}{d\tau} &= h_c \frac{d(m\beta)}{dt} \\ \frac{dQ}{dT} &= h_t A (T_g - T_w) \end{aligned} \quad (20)$$

The mass fraction of the fuel burnt is calculated as a Wiebe function [14], and the instantaneous heat transfer coefficient

Fig. 2 Engine and clutch system multi-body model

Fig. 3 Measured combustion pressure

can be estimated by Woschini's correlation [15]. The instantaneous piston force acts on the piston crown area and is obtained by the simultaneous solution of differential equations (20) and the integration of the pressure distribution there.

A peak combustion pressure of 3.35 MPa occurs at a crank angle position of approximately 13° after top dead centre (TDC), which corresponds to the crank angle of 0° in the figure. As the same spline is used for all the cylinders, no combustion variation between cylinders is allowed in this analysis. This means that the results of the numerical predictions can be verified against the aforementioned closed-form analytical solutions while remaining quite realistic for modern motor vehicle performance.

The piston friction force acts between the piston compression ring and the cylinder wall and is formulated in terms of the instantaneous cylinder pressure in each stroke of the piston. The value of the friction force is then approximated using experimentally obtained cylinder pressure as a function of crankshaft angular velocity and the instantaneous

location of the contact area given in terms of the piston translational velocity. Thus, an expression of the following form is used:

$$F_f = 0.25Af(\dot{\phi}_{cr})\frac{2}{\pi}\tan^{-1}(100v_p) \quad (21)$$

3.6 Simulation results

A simulation run of the multi-body model was undertaken for a period of 0.5 s with 1500 integration time steps on a Pentium II machine. The CPU time was approximately 2 min.

Table 1 Inertial properties in the engine model

Part name	Mass (kg)	I_{xx} (kg mm ²)	I_{yy} (kg mm ²)	I_{zz} (kg mm ²)
Pistons	0.5–0.8	500–900	500–900	450–870
Conrods	0.8–1.0	2e3–5e3	400–650	2e3–5e3
Crankpins	0.2–0.5	75–90	75–90	100–150
Camgear	0.03–0.07	1.0–6.0	1.0–6.0	3.5–8.5
Journals	0.5–0.7	150–175	150–175	200–275
Webs	0.5–0.9	450–700	90–200	450–900
Flywheel	7.0–10.0	2e4–5e4	2e4–5e4	6e4–8e4

Table 2 Constraints used in the multi-body model

I part	J part	Constraint type	DOF removed
Piston 1	Conrod 1	Revolute	5
Piston 2	Conrod 2	Revolute	5
Piston 3	Conrod 3	Revolute	5
Piston 4	Conrod 4	Revolute	5
Piston 1	Ground	Translational	5
Piston 2	Ground	Translational	5
Piston 3	Ground	Translational	5
Piston 4	Ground	Translational	5
Conrod 1	Crankpin 1	In-line	2
Conrod 2	Crankpin 2	In-line	2
Conrod 3	Crankpin 3	In-line	2
Conrod 4	Crankpin 4	In-line	2
Camgear	Ground	Revolute	5
Crank	Ground	Revolute	5
Camgear	Crank	Coupler	1

The engine conditions are governed by the above specified cylinder combustion gas force, culminating in a nominal crankshaft speed of 3500 r/min. The actual average angular speed of the crankshaft is 3660 r/min (see Fig. 4), indicating an amplitude oscillation of 127 r/min, which is quite significant. These oscillations are due to the various harmonics of the engine order vibrations. It can be observed that the crankshaft angular velocity, given in deg/s in the figure, rises from a stationary value to the steady state conditions after a period of 0.4 s. The first 0.1 s corresponds to the angular motion of the crankshaft, initiated by the starter motor torque, before the combustion process commences. Under the start-up conditions, the crankshaft must rotate fast enough to force an air–fuel mixture to enter the cylinders. The starter motor torque, operating in this case for 0.1 s, initiates this requirement.

The frequency composition of the torsional oscillations in Fig. 4 is obtained by fast Fourier transformation of the history after the commencement of the combustion process. This is shown in Fig. 5. In the spectrum, the most significant contribution can be seen at engine order 2 with lesser contributions at the engine orders 4 and 6. This trend agrees with the previously mentioned analytical results and the experimental investigations in references [2] and [3]. The significance of engine order 2 torsional oscillations of the crankshaft system for a four-stroke, four-cylinder engine is in fact well established.

The spectrum shown in Fig. 5 corresponds to the rigid body dynamics of the crankshaft system. To observe the torsion–deflection vibration due to component flexibility, earlier referred to as engine ‘roughness’, the conical whirling motion of the flywheel should be monitored when the elasticity of the crankshaft system is included in the model. The application of gas force to the elastic crankshaft system

results in the phenomenon referred to as ‘spreading of the crank webs’ (see Fig. 6). To include component flexibility, equation (8) is incorporated in the analysis for all the crankpins of the crankshaft. The motion occurs on account of the flexibility of the crankshaft system and the distortion of thin shell engine bearings. The latter is also modelled in some detail in the engine model by the inclusion of the elastohydrodynamic lubricant reaction when the centre of the journal orbits around the centre of the bearing shell. Mathematical description of the engine bearing model is beyond the scope of this paper, and for further information readers are referred to references [9] and [10].

The complex conical whirl of the crankshaft, schematically represented in Fig. 7, leads to impact loading of the drivetrain system through the engagement/disengagement of the clutch system. This effect has been observed by experimental investigation of the flywheel motion using non-contact sensing with equipitched eddy current probes positioned in proximity of the flywheel (shown in Fig. 7) [7].

As can be seen, the action of the flywheel may be regarded as a combination of its rotation about the axis of the crankshaft and a rocking motion about the lateral transverse plane. The effect of this motion can best be observed by its ‘nodding’ component, the to and fro motion towards the clutch system (see Fig. 8). The measurements from the experimental set-up are shown in Fig. 9 as axial movements of the flywheel versus pedal travel. One can observe that during clutch travel all three probes measure displacement of the flywheel. This is better indicated in Fig. 10, also showing a rise in clutch pedal acceleration, accompanied with noise in the driver’s footwell area, monitored by a microphone pick-up. This noise and vibration phenomenon is termed ‘in-cycle clutch pedal vibration’, so-called because the effect is noted during the

Fig. 4 Crankshaft torsional oscillation history

Fig. 5 Frequency composition of torsional vibration

clutch actuation process. This concern is commonly referred to as 'whoop' in industry, owing to the nature of the noise.

A thorough investigation of the whoop problem has shown that its root cause is the torsion-deflection modes of the crankshaft system, with the spectrum of vibration of the clutch system axial oscillations being dominated by both half engine order torsion-deflection modes and the usual even order torsional contributions [5, 6]. The fundamental contributory source is the half engine order, being the main combustion force signature for the four-stroke combustion process. Since all the multiples of the half engine order remain in elastodynamics of the crankshaft system, as indicated by the aforementioned analytical method, the coincidence of one of the harmonics with the clutch release lever natural frequency gives rise to the whoop problem [5, 6].

The validity of the simulation results can be ascertained against the experimental evidence for whoop, as well as against the aforementioned analytical solutions. Figure 11 shows the flywheel nodding motion during the simulation period. It can be observed that a maximum amplitude of 0.14 mm is obtained for axial displacement under steady state conditions. This compares favourably with the experimental results in Fig. 9, which shows an amplitude of 0.12 mm (the full-scale vertical axis being ± 0.15 mm).

The spectrum of vibration is shown in Fig. 12. Note that both the even order torsional vibrations at engine orders 2, 4 and 6 and the engine 'roughness' at the combustion frequency and all its multiples appear in the spectrum. This result is therefore consistent with the previously stated analytical calculations. The comparison of this spec-

Fig. 6 Spreading of the crank webs

Fig. 7 Torsion-deflection oscillations of the flywheel

Fig. 8 To and fro motion of the flywheel (nodding motion)

trum with that for a rigid crankshaft system shown in Fig. 5 indicates the significant contributions made by engine roughness resulting from the modern trend in the use of materials of lighter construction in the presence of increasing combustion forces.

The natural frequency of oscillations of the clutch release lever has been shown to occur in the range 250–270 Hz [4–6], depending upon the position of the lever during the clutch pedal travel. Therefore, at different engine speeds resonant conditions are induced by coincidence or proximity to one or a number of contributory sources in Fig. 12. Figure 13 shows the whoop response frequency at different engine speeds. This is a carrier frequency obtained by

Fig. 10 Clutch whoop phenomenon

monitoring vibrations of the clutch system from the clutch cable at the pedal side and from the clutch lever. These signals are modulated by cross-correlation and a subsequent cepstral analysis. The signal processing procedure is highlighted by Kelly [4]. It can be observed that the carrier frequency remains unchanged for all engine speeds, confirming the whoop frequency to be the natural frequency

Axial pedal movement versus pedal travel at 3500r/min

Fig. 9 Measured flywheel "nodding" motion

Fig. 11 Flywheel nodding motion

of the clutch lever. Clearly, resonant conditions would be expected when one of the spectral contributions due to the crankshaft system vibrations coincides or occurs in the proximity of the clutch lever natural frequency. For the conditions reported in this paper, this would be expected to occur because of proximity to the contributions at 7.5, 8 and 8.5 engine orders. Although these are insignificant compared with other contributions in Fig. 12, their presence should lead to whoop.

A multi-body clutch model, described in references [5] and [6], was coupled with the engine model described here

Fig. 12 Spectrum of flywheel nodding oscillations

Carrier frequencies at different engine speeds

Fig. 13 Whoop response frequency at different engine speeds

Fig. 14 Flywheel impact force during clutch actuation

Fig. 15 Clutch vibration spectrum, with the in-cycle response

(see Fig. 2). An impact force is introduced by the flywheel nodding motion into the clutch system during clutch actuation. This impact force is obtained from the multi-body engine model and is transmitted into the clutch model (see Fig. 14). The resulting axial oscillations of the clamped clutch system yield the spectrum shown in Fig. 15. The whoop response, coinciding with the natural frequency of release lever oscillations, is indicated in the figure. All other contributions occur at the multiples of the half engine order. The contribution at 140–150 Hz has been observed to be the dominant noise source in the driver's footwell area [16].

4 CONCLUSIONS

This paper has shown the relationship between component flexibility and engine roughness response. This has been proposed by a number of authors by observation of experimental results [2,3], but has not hitherto been shown in a fundamental analysis as reported here. The results of multi-body dynamic analysis are also consistent with the analytical solutions. Furthermore, a practical powertrain noise and vibration problem, whoop, has been investigated by the

incorporation of a clutch system model. The numerical predictions agree with the experimental findings, giving further credence to the modelling methodology employed in this paper.

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