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**Development of speckle shearing interferometer error analysis
as an aperture function of wavefront divergence**

W S Wan Abdullah[†] and J N Petzing[‡]

[†] Malaysian Institute for Nuclear Technology Research

Malaysian Institute for Nuclear Technology Research (MINT), Bangi, 43000 Kajang,
Malaysia

[‡]Wolfson School of Mechanical & Manufacturing Engineering

Loughborough University, Loughborough, Leicestershire, LE11 3TU

Contact Details:

Dr Jon Petzing

Wolfson School of Mechanical & Manufacturing Engineering

Loughborough University

Loughborough

Leicestershire

LE11 3TU

Tel: 01509 227617

Fax: 01509 227648

E-mail: j.petzing@lboro.ac.uk

Abstract

Increasing the confidence in wholefield speckle based optical metrology transducers requires a detailed understanding of the error sources of the respective instruments. The analysis of error contributions to the optical phase output of a Michelson based speckle shearing interferometer have been modelled. Specific attention has been made to the effect of the aperture at the image plane, with respect to collimated and non-collimated object illumination. This modelling presents an advance on a previous modelled analytical relationship, which includes partial displacement derivative terms and components as a function of illumination geometries and importantly aperture effects. The work has identified a phase error contribution due to the aperture function of between 0.15% and 1.48%, dependant on the object distance, when considering a planar object undergoing predominantly surface to normal deformation.

Keywords: speckle, shearing, shearography, errors, repeatability

1.0 Introduction

The development of the speckle shearing interferometer [1] has been characterised by several common technological phases, which can be summarised as; invention, demonstration, application. The technique can be based on a number of differing optical designs, but all have a common ability of at least measuring the out-of-plane (normal to the object surface) first order partial displacement derivatives ($\partial w/\partial x$ and $\partial w/\partial y$). In certain cases, the optical design can be manipulated such that in-plane displacement derivatives such as $\partial u/\partial x$, and even second order partial derivatives ($\partial^2 w/\partial x^2$) can be measured (although not generally in a real-time discrete manner).

The development of the speckle shearing interferometer in recent years has been aided by the dramatic changes in laser technology, image processing hardware and software. This has led to increased commercialisation of instrumentation and rising interest from various industrial sectors, which view the instrument data as being suitable for quantified defect and even possibly elements of strain analysis.

Generating quantified data from such a transducer or measurement instrument is but one step towards eventually solving an engineering problem, because the user must have confidence in the measurement transducer. This confidence is generally gained by understanding and quantifying all of the sources of error and uncertainty associated with the instrument, and then applying a calibration strategy which identifies the levels of error and accuracy for the instrument, which allows the linking of the measurand to the primary standard (the metre in this case). In certain circumstances where an instrument

error analysis is not available, this measurement confidence may be gained from statements of instrument repeatability [2]

Significant work has been completed generally within speckle metrology to understand various issues of error and uncertainty, with several very recent publications now pushing the issues of instrumentation quality and data confidence [3,4]. Until recently, when considering speckle shearing interferometry specifically, analysis had typically been limited to issues concerning fringe visibility [5-8], and cross-sensitivity between displacement and displacement derivative components [9,10]. More recent published works have started to investigate other error sources, including the lateral shearing amount, displacement derivative order, sensitivity vector, rigid body motion and geometry effects [11,12].

Another issue, which has also been reported, is that of quantified errors being generated as a function of non-collimated illumination. A ray based theoretical and experimental analysis of a Michelson based speckle shearing interferometer [13,14] suggested that significant errors (up to 10%) could be inadvertently generated due to object illumination wavefront geometries. Vector based analysis supports these findings and has furthered aspects of this work [15]. Further work has now been completed in this area, which has enhanced the original theoretical model [13], and helps to refine the mathematical model of a simplified case for a speckle shearing interferometer. It is the intention of this paper to demonstrate the advances completed in this area.

2.0 Analysis of the theoretical system

Much of the initial development of speckle shearing interferometers for optical metrology applications, can originally be traced to a small number of key publications by a number of researchers [16,17]. Significant work [18,19] subsequently demonstrated the use of wedge based optics for the application of speckle shearing interferometers to deformation analysis. One of the core features of this work (and further work by other authors using alternative optical configurations including the Michelson based optics) is the mathematical description used for the analysis of the optical phase signals (Δ) produced by the interferometer, which was identified as being applicable to Michelson or wedge based speckle shearing interferometers:

$$\Delta = \frac{2\pi}{\lambda} \left[(1 + \cos \theta) \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x} \sin \theta \right] \delta x \quad (1)$$

where ' λ ' is the laser wavelength, ' θ ' is the angle of illumination, $\partial w / \partial x$ and $\partial u / \partial x$ represent out-of-plane and in-plane first order displacement derivatives, and δx is the extent of the lateral shear applied to the interferometer.

This one relationship forms the basis for many subsequent works and texts by many other authors, providing a first order approximation to the true optical phase value. The issue of suitability of equation 1 has previously been discussed by the current authors [13,14], where it is correctly identified that this is a first order approximation, because the theoretical formulation involves a Taylor series [18,19], which includes second, third and higher order derivatives. However, it has been shown that typically, these

higher order derivatives may only become significant if a large lateral shear is used (δx), and even then the second order partial derivative will only contribute a few percent to the overall numerical analysis of the fringe function.

An important issue which is not often highlighted in research or trade publications, is that equation 1 is based on an analysis of one point at the object plane, and is consequently only truly valid for that one point at the surface normal, and importantly, when using collimated object illumination. However, this relationship is commonly used for the general wavefront approximations, and calculation of optical deformation phase across the entire illumination wavefront.

Existing work [13,14], which has analysed the consequence of using non-collimated illumination for a speckle pattern interferometer, resulted in a theoretical extension to equation 1 which was experimentally verified and correlated. This new formulation (equation 2) suggested that two additional error terms (equation 3) should be included within the phase description, which would compensate for the use of non-collimated object illumination, on a planar object.

$$\Delta_x = \frac{2\pi}{\lambda} \left\{ (1 + \cos\theta) \frac{\partial w}{\partial x} + \sin\theta \frac{\partial u}{\partial x} + |A| \frac{\partial w}{\partial x} + |B| \frac{\partial u}{\partial x} \right\} \delta x \quad (2)$$

$$A = \tan \frac{\beta}{2} \sin \theta$$

(3)

$$B = \frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)}$$

As in many previous cases, the initial formulation of ideas was based on the use of equation 1. However, further analysis recently completed has suggested that whilst a good correlation was produced between theoretical model and experimental results [13], the development of the theory required further modification and optimisation to improve the quality of the modelling.

The refinement introduced here has been to analyse the consequence of modelling the effect of an aperture in front of the image plane. It is perhaps surprising to note that in many instances, the imaging system aperture has been left out of theoretical descriptions of speckle shearing theory, within many publications. This is understandable for two reasons. Firstly, if an approximation is used based on the line of sight to the object surface normal, and secondly, the majority of applications during the 1970's and 1980's were qualitative in nature, which coincides with many of the publications in the technique. Consequently, any modelling of the optical system should consider what happens when imaging away from the surface normal, especially when the extremities of the object are studied.

2.1 Modelling the influence of the image plane aperture

The introduction of the aperture at the image plane can be seen in Figure 2. Ray tracings have then been used to consider points at an arbitrary position on the planar (flat) object surface, and at the extremity of the object. In order to aid clarity, the detail shown at the object plane is expanded in Figure 3, specifically highlighting the geometries associated with the path GOT in Figure 2.

Figures 2 and 3 show that the sensitivity vectors at the edge of illuminated area, Q_0 and T_2 are given by $\hat{S}_1, \hat{S}_2$ and $\hat{S}_3$. Since the deformation is small, the values of u and w are also small. Furthermore, $D_1 \gg D_0$. Therefore the angles $\angle ST_2Q$ and $\angle SOT$ can be approximately equal and hence the distance $QQ_1 \approx QQ_2$. The imaging angle ς' is the angle at point T_2 on the illuminated surface relative to the line of the optical axis and the maximum value of ς' is ς , measured at the edge of illuminated object, and is given by:

$$\varsigma = \tan^{-1} \left[\frac{D}{2D_1} \right] \quad (4)$$

where D_1 is the distance from the center of the illuminated area to the center of the camera aperture and D is the maximum of the inspected diameter (measured from edge to edge of the illuminated area).

The change of optical path length due to object deformation from the expanding lens S to the point Q_1 and Q_2 at the image plane as shown in Figure 3 is given by:

$$\delta l = \delta \mathbf{l} \cdot \hat{\mathbf{n}} \quad (5)$$

where $\hat{\mathbf{n}}$ is a surface unit vector. Assuming that the surface displacement vector ($\delta \mathbf{l}$) is along the surface unit vector ($\hat{\mathbf{n}}$), equation 5 can be written in form of the geometrical difference of the wave propagation as:

$$\delta l = (SO + OQ + QQ_1) - (ST_2 + T_2Q + QQ_2) \quad (6)$$

Note that at this point, the presence of the aperture modifies the modelling of the pathlength, adding additional terms when compared to the model developed without the aperture [13]. Because the path length $SG \approx ST_2$, $QQ_1 \approx QQ_2$ ($D_1 \gg D_0$ and displacement u is small) and α is small, δl can be written as:

$$\begin{aligned} \delta l &= (SG + GJ + JO + OT + TQ + QQ_1) - (ST_2 + T_2T_1 + T_1Q + QQ_2) \\ \delta l &= (GJ + JO + OT) - T_2T_1 \end{aligned} \quad (7)$$

With the assumption that the triangle SGH is isosceles with angles $\angle SGH = \angle GHS = (90 - \beta/2)^\circ$. The path length GO is given by:

$$GO = GJ + JO \quad (8)$$

The angle $\angle GT_2J = (90 + \beta/2 - \theta)^\circ$ and using trigonometry identities:

$$\frac{GJ}{\sin\left(90 + \frac{\beta}{2} - \theta\right)} = \frac{JT_2}{\sin\left(90 + \frac{\beta}{2}\right)} \quad (9)$$

Which gives:

$$GJ = \frac{JT_2}{\sin\left(90 + \frac{\beta}{2}\right)} \sin\left(90 + \frac{\beta}{2} - \theta\right) \quad (10)$$

200

201 Since $OO_2 = w$, $OT = \frac{w}{\cos\zeta'}$, $TO_2 = w \tan \zeta'$ and $T_2T_1 = u \sin \zeta'$; JT_2 can be written
 202 as:

$$JT_2 = w - OO_1 \quad (11)$$

205

206 The path lengths JO and OO_1 are given by:

207

$$JO = \frac{u - w \tan \zeta'}{\sin(\theta - \beta)} \quad (12)$$

209

$$OO_1 = \frac{(u - w \tan \zeta') \cos(\theta - \beta)}{\sin(\theta - \beta)} \quad (13)$$

211

212 Using the relationships established in equations 9 to 13, equation 8 may be rewritten

213 as:

214

$$GO = \frac{\sin\left(90 + \frac{\beta}{2} - \theta\right)}{\sin\left(90 + \frac{\beta}{2}\right)} \left\{ w - \left(\frac{(u - w \tan \zeta') \cos(\theta - \beta)}{\sin(\theta - \beta)} \right) \right\} + \frac{u - w \tan \zeta}{\sin(\theta - \beta)} \quad (14)$$

216

217

Therefore the change of optical pathlength (δl) can be written as:

$$\delta l = \frac{\sin\left(90 + \frac{\beta}{2} - \theta\right)}{\sin\left(90 + \frac{\beta}{2}\right)} \left\{ w - \left(\frac{(u - w \tan \zeta') \cos(\theta - \beta)}{\sin(\theta - \beta)} \right) \right\} + \frac{u - w \tan \zeta}{\sin(\theta - \beta)} + \frac{w}{\cos \zeta'} - u \sin \zeta' \quad (15)$$

$$\delta l = \left\{ \frac{\cos \frac{\beta}{2} \cos \theta + \sin \frac{\beta}{2} \sin \theta}{\cos \frac{\beta}{2}} \right\} w + \frac{w}{\cos \zeta'} + u \left\{ \frac{1}{\sin(\theta - \beta)} - \frac{\cos\left(\frac{\beta}{2} - \theta\right)}{\cos \frac{\beta}{2}} \frac{\cos(\theta - \beta)}{\sin(\theta - \beta)} - \sin \zeta' \right\} + w \tan \zeta' \left\{ \left[\frac{\cos\left(\frac{\beta}{2} - \theta\right)}{\cos \frac{\beta}{2}} \frac{\cos(\theta - \beta)}{\sin(\theta - \beta)} \right] - \frac{1}{\sin(\theta - \beta)} \right\} \quad (16)$$

Through a summarised process of trigonometric manipulation, the expression for δl can be developed:

$$\delta l = \left(\frac{1}{\cos \zeta'} + \cos \theta \right) w + w \tan \frac{\beta}{2} \sin \theta + w \tan \zeta' \left\{ \frac{\cos^2 \theta \cos \beta}{\sin(\theta - \beta)} + \frac{[\sin \theta \cos \beta - \sin(\theta - \beta)] \sin \theta}{\sin(\theta - \beta)} \right\} + w \tan \zeta' \left\{ \frac{\tan \frac{\beta}{2} \sin \theta}{\tan(\theta - \beta)} - \frac{1}{\sin(\theta - \beta)} \right\} + \left\{ \frac{1}{\sin(\theta - \beta)} - \frac{\cos^2 \theta \cos \beta}{\sin(\theta - \beta)} - \frac{[\sin \theta \cos \beta - \sin(\theta - \beta)] \sin \theta}{\sin(\theta - \beta)} - \frac{\tan \frac{\beta}{2} \sin \theta}{\tan(\theta - \beta)} - \sin \zeta' \right\} u \quad (17)$$

$$\begin{aligned}
232 \quad \delta l = & \left(\frac{1}{\cos \zeta'} + \cos \theta \right) w + w \tan \frac{\beta}{2} \sin \theta + w \tan \zeta' \left\{ \frac{\cos \beta + \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta) - 1}{\sin(\theta - \beta)} - \sin \theta \right\} \\
233 \quad & + u \sin \theta + \left\{ \frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)} - \sin \zeta' \right\} u
\end{aligned} \tag{18}$$

234

235 Consequently, equation 15 can be written as follows:

236

$$237 \quad \delta l = (1 + \cos \theta)w + u \sin \theta + wA + uB + wA' + uB' \tag{19}$$

238 where

$$239 \quad A = \tan \frac{\beta}{2} \sin \theta + \tan \zeta' \left\{ \frac{\cos \beta + \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta) - 1}{\sin(\theta - \beta)} - \sin \theta \right\}$$

$$240 \quad A' = \frac{1 - \cos \zeta'}{\cos \zeta'}$$

$$241 \quad B = \frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)}$$

$$242 \quad B' = -\sin \zeta'$$

243

244 Equation 19 represents the change of optical path length due to object deformation. It
 245 can be seen that the factors A , B , A' and B' are the additional factors that are contributed
 246 by the divergent non-collimated illumination wavefront and the imaging angle / aperture
 247 function (varying across the illuminated surface). Compared to the original formulation
 248 of the speckle shearing interferometer theory [13], A' and B' are the new factors
 249 introduced as a function of the aperture at the image plane. These additional factors are
 250 related to the general OOP and IP deformations in the object deformation function that
 251 gives the influence of illumination wavefront curvature and the imaging angle to the

phase change measurement. They can produce a positive or negative contributions to the fringe function, depending on the direction of illumination. However, the above factors are cancelled out at a point on the optical axis where the values of inclination angle (β) and imaging angle (ζ') are both equal to zero.

3.0 Applying lateral shear to the interferometer

The first stage of the analysis has only considered the geometry associated with the object surface deformation and the consequent change of path length. The second stage of the analysis is to consider the correlation interferometric function of the shearing interferometer, by laterally shearing the image of the object. This is based on the assumptions that the direction of lateral shearing is in x -direction and the amount of shearing is δx , with the optical configuration being based on the Michelson design, as shown in Figure 1.

The speckle from a point $N(x, y)$ of the first mirror of the interferometer, interferes with a speckle from a neighbouring point $N'(x+\delta x, y)$ of the second mirror. When the object is deformed, the displacement of the point $N(x, y)$ will be (u, v, w) and the displacement of the point $N'(x+\delta x, y)$ will be $(u+\delta u, v+\delta v, w+\delta w)$ on the image plane. The change in path length of light scattered from the mirror N' is δl_2 and can be developed from equation 19:

$$\delta l_2 = (1 + \cos \theta)(w + \delta w) + \sin \theta(u + \delta u) + A(w + \delta w) + B(u + \delta u) + A'(w + \delta w) + B'(u + \delta u) \quad (20)$$

With the assumption that the relative light path length change due to deformation is within the pixel size of the CCD camera (any issues of speckle decorrelation are therefore assumed to be minimised), the relative light path length change can be written as:

$$\Delta L = \delta l_2 - \delta l \quad (21)$$

$$\Delta L = (1 + \cos \theta) \delta w + \sin \theta \delta u + A \delta w + B \delta u + A' \delta w + B' \delta u \quad (22)$$

If the optical phase change, Δ_x , is:

$$\Delta_x = \mathbf{k}_x \cdot \Delta \mathbf{L} \quad (23)$$

where $\mathbf{k}_x$ is the wave propagation vector, $\mathbf{k}_x = 2\pi/\lambda \mathbf{n}_x$ and $\Delta \mathbf{L}$ is the displacement vector (for this analysis, the wave propagation vector is assumed along the displacement vector), then equation 22 can be written as:

$$\Delta_x = \frac{2\pi}{\lambda} \Delta L \quad (24)$$

$$\Delta_x = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \delta w + \sin \theta \delta u + A \delta w + B \delta u + A' \delta w + B' \delta u \right\} \quad (25)$$

$$\frac{\Delta_x}{\delta x} = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\delta w}{\delta x} + \sin \theta \frac{\delta u}{\delta x} + A \frac{\delta w}{\delta x} + B \frac{\delta u}{\delta x} + A' \frac{\delta w}{\delta x} + B' \frac{\delta u}{\delta x} \right\} \quad (26)$$

where δx is the amount of shearing in horizontal direction. If δx is small, equation 26 can be approximated as:

$$\frac{\Delta_x}{\delta x} = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} + A \frac{\partial w}{\partial x} + B \frac{\partial u}{\partial x} + A' \frac{\partial w}{\partial x} + B' \frac{\partial u}{\partial x} \right\} \quad (27)$$

which may be written in the familiar format as:

$$\Delta_x = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} + A \frac{\partial w}{\partial x} + B \frac{\partial u}{\partial x} + A' \frac{\partial w}{\partial x} + B' \frac{\partial u}{\partial x} \right\} \delta x \quad (28)$$

where

$$A = \tan \frac{\beta}{2} \sin \theta + \tan \zeta' \left\{ \frac{\cos \beta + \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta) - 1}{\sin(\theta - \beta)} - \sin \theta \right\}$$

$$A' = \frac{1 - \cos \zeta'}{\cos \zeta'}$$

$$B = \frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)}$$

$$B' = -\sin \zeta'$$

The functions A , B , A' and B' in equation 28 are the OOP and IP additional sensitivity factors propagated by the curvature wavefront and aperture geometrical factors, their values dependant on the magnitudes of inclination angle (β), imaging angle (ζ') and the illumination angle (θ). A similar relationship exists for the vertical sheared component (δy).

4.0 Analysis of the theoretical model with respect to collimated and non-collimated illumination

It is important at this point to test the model described in equation 28 with respect to the primary criteria of the analysis, namely differences caused by the use of collimated and non-collimated illumination. The maximum inclination angle β depends on two variable parameters, the illuminated object diameter (D) and the distance from expanding lens to the object surface ($R(L)$):

$$\beta = \sin^{-1} \left[\frac{D}{2R(L)} \right] \quad (29)$$

For a given laser power the diameter of the inspected object is determined by the illuminated area on the object surface, which depends on the camera lens and the power of the expansion lens. In normal routine inspection, both of these parameters are determined based on the coverage area of the inspected object. Figure 4 represents the theoretical relationship of the inclination angle (β) with the diameter of illuminated object for $R(L)$ at 600mm. It can be seen that the inclination angle or the degree of curvature of the illumination wavefront on the object surface is linearly dependant on the illuminated area.

Equation 28 initially represents the theoretical phase function of the shearing interferometer for a non-collimated illumination wavefront, which is clearly the generalised case. There are two main components that influence the phase difference in the curvature phase function, which are contributed from the derivative out-of-plane

(OOP) and the derivative in-plane (IP) components. These two components can however be analyzed individually or as one function. The approach of the OOP case is to analyze the functions of A , A' , B and B' in the equation 28. This requires consideration of the relationship based on the absolute values of the above parameters:

$$\Delta_x = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} + |A| \frac{\partial w}{\partial x} + |B| \frac{\partial u}{\partial x} + |A'| \frac{\partial w}{\partial x} + |B'| \frac{\partial u}{\partial x} \right\} \delta x \quad (30)$$

Equation 30 shows that Δ_x is a function of $\partial w/\partial x$ and $\partial u/\partial x$, (the first partial displacement derivative components), whilst $(1 + \cos \theta)$ and $\sin \theta$ are the sensitivity factors. The functions A and B can be treated as divergence sensitivity factors. The sensitivity factors are functions of the position of the light source, the camera and the point on the object (Figure 1).

More specifically, the values of A and B are the functions of object illumination angle θ and the inclination angle β (the curvature of illumination wavefront). However the values of A' and B' are independent of the divergence illumination and only depend on the point of the illuminated surface relative to the point on the optical axis. The values of θ , ζ' and β are assumed to be independent to one another, as given by equations 4 and 29, where ζ and β are independently defined. For a fixed imaging angle ζ' , the maximum value of A and B for any value of θ and β is unity, since the maximum value of β is 90° .

It is therefore important to consider equation 28, both in terms of non-collimated and collimated illumination geometries, because in certain cases, the mathematical

modelling of the speckle shearing interferometer will be greatly simplified, whilst in other cases, the four error term contributions (A , B , A' , B') will be significant. The testing of equation 28 is treated with respect to a point in three different positions; on the optical axis (object surface normal – Figure 2), at some arbitrary point away from the optical axis, and at the extremities of the object.

4.1 Interferometer using non-collimated illumination

- Assuming the optical phase function is at a point on the optical axis (Figure 1). The imaging angle (ζ') and inclination angle (β) are zero, hence; $A = 0$, $B = 0$, $A' = 0$ and $B' = 0$. Therefore equation 28 reduces to the form shown in equation 1.

- The optical phase function is at a point other than on the optical axis. Hence $\zeta' \neq 0$ and $\beta \neq 0$, and equation 28, when fully substituted, becomes:

$$\Delta_p = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} \right\} \delta x$$

$$+ \frac{2\pi}{\lambda} \left\{ \left[\frac{1 - \cos \zeta'}{\cos \zeta'} \right] \frac{\partial w}{\partial x} + \left[\tan \frac{\beta}{2} \sin \theta + \tan \zeta' \left\{ \frac{\cos \beta + \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta) - 1}{\sin(\theta - \beta)} - \sin \theta \right\} \right] \frac{\partial w}{\partial x} \right\} \delta x$$

$$+ \frac{2\pi}{\lambda} \left\{ \left[\frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)} \right] \frac{\partial u}{\partial x} - \sin \zeta' \frac{\partial u}{\partial x} \right\} \delta x \quad (31)$$

- The optical phase function is at the edge of illuminated area. At this point $\zeta' = \zeta \neq 0$ and $\beta \neq 0$, hence equation 28 can be written as:

$$\begin{aligned} \Delta_{Pe} = & \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} \right\} \delta x \\ & + \frac{2\pi}{\lambda} \left\{ \left[\frac{1 - \cos \zeta}{\cos \zeta} \right] \frac{\partial w}{\partial x} + \left[\tan \frac{\beta}{2} \sin \theta + \tan \zeta \left\{ \frac{\cos \beta + \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta) - 1}{\sin(\theta - \beta)} - \sin \theta \right\} \right] \frac{\partial w}{\partial x} \right\} \delta x \\ & + \frac{2\pi}{\lambda} \left\{ \left[\frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)} \right] \frac{\partial u}{\partial x} - \sin \zeta \frac{\partial u}{\partial x} \right\} \delta x \quad (32) \end{aligned}$$

4.2 Interferometer using collimated illumination

- Again, assuming the optical phase function is at a point on the optical axis, then as in the non-collimated case, this simplifies to the form of equation 1.
- The optical phase function is at a point other than on the optical axis. The imaging angle $\varsigma' \neq 0$ and $\beta = 0$, therefore the parameters A , A' , B and B' will be simplified to:

$$A = -\tan \varsigma' \sin \theta$$

$$A' = \frac{1 - \cos \varsigma'}{\cos \varsigma'}$$

$$B = 0$$

$$B' = -\sin \varsigma'$$

And equation 28 will become:

$$\Delta_{cP} = \frac{2\pi}{\lambda} \left\{ (1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} - \sin \varsigma' \frac{\partial u}{\partial x} - \tan \varsigma' \sin \theta \frac{\partial w}{\partial x} + \left(\frac{1 - \cos \varsigma'}{\cos \varsigma'} \right) \frac{\partial w}{\partial x} \right\} \delta x$$

(33)

- Finally, if the optical phase function is at the edge of the illuminated area, $\varsigma' = \varsigma \neq 0$, and $\beta = 0$, and equation 28 will be transformed to:

5.0 Considering the optical phase model as an error function

From Equations 32 and 34, the relative maximum phase difference at the edge of the illuminated area due to the divergent illumination wavefront can be defined, as the difference of the maximum relative phase measured by divergent illumination beam, to the maximum relative phase measured by collimated illumination beam:

$$M_r = \frac{\Delta_{Pe} - \Delta_{ce}}{\Delta_{ce}}$$

Via substitution:

$$M_r = \frac{\left| \tan \frac{\beta}{2} \sin \theta \right| \frac{\partial w}{\partial x} + \left| \tan \varsigma \left\{ \frac{\cos \beta + \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta) - 1}{\sin(\theta - \beta)} \right\} \right| \frac{\partial w}{\partial x} + \left| \left\{ \frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \theta \cos(\theta - \beta)}{\sin(\theta - \beta)} \right\} \right| \frac{\partial u}{\partial x}}{(1 + \cos \theta) \frac{\partial w}{\partial x} + \sin \theta \frac{\partial u}{\partial x} - \sin \varsigma \frac{\partial u}{\partial x} - \tan \varsigma \sin \theta \frac{\partial w}{\partial x} + \left\{ \frac{1 - \cos \varsigma}{\cos \varsigma} \right\} \frac{\partial w}{\partial x}} \quad (35)$$

The maximum relative phase change difference value can be defined accurately if the derivative value terms $\partial w/\partial x$ and $\partial u/\partial x$ are known. However the individual derivative factors $\partial w/\partial x$ and $\partial u/\partial x$ in equation 35 are difficult to calculate simultaneously since there are typically no details of in-plane contribution of out-of-plane test object even at normal illumination angle (parallel to camera axis).

A straightforward method can be practically imposed to overcome the above difficulties, by measuring the maximum phase data using divergent illumination and the maximum phase data using collimated illumination, but maintaining all other interferometer variables as constants, as previously reported [13]. The difference of

these two optical illumination geometries can be considered as the relative maximum phase change difference at the edge of illuminated area in the measurement analysis.

The maximum phase difference trend can be predicted by individual inspection of divergence sensitivity factor (A , B , A' and B') values from equation 32. First consider the extreme case of the individual value of A , B , A' and B' when the angle between the illumination wavefront and the interferometer camera axis θ is zero and ninety degrees respectively. For the non-collimated case, with $\theta = 0^\circ$, the factors in equation 32 become:

$$A = -\tan \varsigma \left[\frac{\cos \beta - 1}{\sin \beta} \right] = -\tan \varsigma \tan \frac{\beta}{2}$$

$$B = \frac{1 - \cos \beta}{-\sin \beta} = -\tan \frac{\beta}{2}$$

$$A' = \frac{1 - \cos \varsigma}{\cos \varsigma} = \tan \frac{\varsigma}{2} \tan \varsigma$$

$$B' = -\sin \varsigma$$

And can be written as:

$$\Delta_x = \frac{2\pi\delta x}{\lambda} \left\{ 2 \frac{\partial w}{\partial x} + \left| \tan \frac{\beta}{2} \right| \left| \frac{\partial u}{\partial x} \right| + \left| \tan \varsigma \tan \frac{\beta}{2} \right| \left| \frac{\partial w}{\partial x} \right| + \left| \tan \frac{\varsigma}{2} \tan \varsigma \right| \left| \frac{\partial u}{\partial x} \right| + |\sin \varsigma| \left| \frac{\partial u}{\partial x} \right| \right\} \quad (37)$$

In case of collimated illumination equation 38 will be simplified to:

$$\Delta_{cx} = \frac{2\pi\delta x}{\lambda} \left\{ 2 \frac{\partial w}{\partial x} + \left| \tan \frac{\varsigma}{2} \tan \varsigma \right| \frac{\partial u}{\partial x} + |\sin \varsigma| \frac{\partial u}{\partial x} \right\} \quad (38)$$

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474

475 If $\theta = \pm 90^\circ$, the factors in equation 32 can be simplified as:

476

$$A = \pm \tan \frac{\beta}{2} \pm \tan \varsigma$$

$$B = \frac{1 - \cos \beta - \tan \frac{\beta}{2} \sin \beta}{\cos \beta} = 0$$

$$A' = \frac{1 - \cos \varsigma}{\cos \varsigma} = \tan \frac{\varsigma}{2} \tan \varsigma$$

$$B' = -\sin \varsigma$$

481

482 Resulting in the phase term Δ_x being described as:

483

$$\Delta_x = \frac{2\pi\delta x}{\lambda} \left\{ \frac{\partial w}{\partial x} \pm \frac{\partial u}{\partial x} + \left(\tan \frac{\beta}{2} \mp \tan \varsigma \right) \left| \frac{\partial w}{\partial x} + \tan \frac{\varsigma}{2} \tan \varsigma \frac{\partial u}{\partial x} + |\sin \varsigma| \frac{\partial u}{\partial x} \right| \right\} \quad (39)$$

485

486 If collimated illumination is used, Equation 39 can be written as:

487

$$\Delta_{cx} = \frac{2\pi\delta x}{\lambda} \left\{ \frac{\partial w}{\partial x} \pm \frac{\partial u}{\partial x} + |\tan \varsigma| \frac{\partial w}{\partial x} + \left| \tan \frac{\varsigma}{2} \tan \varsigma \right| \frac{\partial u}{\partial x} + |\sin \varsigma| \frac{\partial u}{\partial x} \right\} \quad (40)$$

489

490 The maximum imaging angle (ς) will depend on the size of illuminated area and the

491 distance from the illuminated surface to the image plane. The individual phase

contributions have been modelled and are shown in Figure 5, based on an experimental configuration of a 100mm illuminated diameter (planar flat object) with a source to object distance of 600mm (source to image plane distance of 980mm) and an illumination angle 45° . Under these conditions, the relative maximum phase change contribution due to the aperture contribution using the above criteria is seen to be 0.25%. this is in comparison to the 6.05% maximum relative phase change difference which is caused by the divergent aspect of the modelling for the same interferometer parameters. This part of the model was previously verified experimentally [13].

Clearly 0.25% is a small contribution to the instrument error budget. However, if the object distance decreases (object to image plane), then this element increases to 1.48% at 100mm. Conversely, if the object distance increases to 1000mm, then this error contribution reduces to 0.14%. In comparison, the modelled divergent component of the error term at 200mm object distance is 18.4%, although this has not been experimentally verified, with experimentation [13] limited to 400mm object distance. These results are valid for the assumption that the object surface undergoes a motion dominated in the out-of-plane or surface normal direction. Further analysis is required when considering objects which display predominantly in-plane motion.

6.0 Conclusions

The growing importance of developing confidence in optical metrology data, requires a better understanding of the sources of error within the instruments. Speckle shearing interferometers fall into this category, because there is a significant industrial demand for this type of transducer to produce quantitative data. As a prerequisite to defining traceability routes to the primary standards, and even calibration artefacts, a detailed error and uncertainty analysis is required for the interferometer. Some aspects of error analysis have previously been completed in literature for various speckle based techniques, but other elements of the speckle shearing interferometer have as yet remained untouched.

An originally proposed model which described optical phase errors as a function of non-collimated object illumination for a speckle shearing interferometer, has been optimised and up-dated to include phase error terms caused by the aperture at the image plane. The development of the theoretical model has been tested by analysing different points on the object surface under collimated and non-collimated illumination conditions, assuming that the surface exhibits predominantly out-of-plane deformation characteristics. The modelling results have been further considered in the context of existing validating experimental data.

The results show that the aperture generates a small but at times significant contribution (0.15% at 1000mm – 1.48% at 200mm) to the optical phase term, which is dependant on the object distance, although other contributions expressed in the model are larger. Furthermore, the new terms aid the correlation between model and experimental data,

identified in previous published work. This work is currently being extended to understand explicit issues of in-plane deformation terms and errors.

The new phase relationship for the speckle shearing interferometer could also be regarded as the general equation for object deformation that includes all geometrical parameters involved in the measurement system. However, it should be clearly identified that this model is based on the analysis of flat planar surfaces or structures. If objects with three dimensional relief are to be investigated, then local variations of transducer sensitivity as a function of object relief, would require additional compensating terms in the optical phase description.

Furthermore, whilst the initial elements of this model are pertinent to other wholefield speckle techniques (Electronic Speckle Pattern Interferometry for instance), the final elements of the model are specific to the speckle shearing interferometer due to the lateral shearing component. It should also be noted that this model has been developed in isolation from other known error sources, such as the effect of higher order partial derivatives, and issues such as knowing (or not knowing) the exact positions of the primary optical elements of the instrument.

However, the value of this model is that it identifies significant error terms as a function of the aperture and the divergent illumination criteria, which will contribute to the whole error budget of the instrumentation. It is recognised that under certain conditions, these contributions are small (large object distances), but if the parameters change, then so do the error contributions.

The model also provides further basis for developing error mapping routines within the typical image processing software used for correlation fringe manipulation, and which would compensate for specific instrumentation variables. And finally, the model helps to define aspects of the full error analysis of the instrumentation, this being a prerequisite for achieving the true metrological calibration-traceability of the technique.

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569

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