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Deterministic and reliability-based design: veneer cover soil stability

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ABSTRACT: A design chart is a graphical tool that provides solutions to different scenarios of a system. In this paper, two types of design chart are developed based on deterministic and reliability-based analyses for determining the interface shear strength required for stability of a cover system to achieve a target safety factor of 1.5 and failure probability of 1×10^{-2} . The deterministic design chart assists in the selection of different types of geosynthetic for lining materials based on the required interface shear strength for stability, and the reliability-based design chart enhances decision-making by taking into account the uncertainties in the design parameters, such as the variability of interface shear strength parameters. Additionally, the latter chart can also be used to determine the optimum slope angle for a containment facility that will satisfy both the target factor of safety and acceptable failure probability. Examples are provided to illustrate the use of the design charts in estimating the minimum required interface shear strength and their allowable variability for a given veneer cover, and the optimum slope inclination corresponding to different interface shear strengths and their associated variability.

KEYWORDS: Geosynthetics, Reliability-based design, Veneer cover system

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1. INTRODUCTION

Veneer cover soil systems in containment facilities such as landfills, dams and liquid impoundments involve geosynthetic-soil layers. These can consist of either single or multiple layers of geosynthetics and soils. The main design criterion for veneer cover soil is to ensure that no slippage occurs between the layers during and following construction. Instability can be caused by the weight of the cover soil, equipment loadings, seepage forces within the cover soil, and/or seismic forces in seismically active areas. Numerous researchers have introduced and adopted different methods to improve stability, such as tapered cover soil, toe berms, reinforcement and modifying the geometry of the facility (Koerner and Hwu 1991; Koerner and Soong 2005). Stability assessment has included using simple limit equilibrium wedge methods for preliminary design and relatively complex finite element methods for forensic studies.

In this study, the limit equilibrium wedge method with modified formulations from Giroud *et al.* (1995) is used to produce design charts. A design chart provides an

explicit graphical solution, which can be useful for preliminary design. It depicts the behaviour of a system if a significant design parameter changes, and thus assists in determining the optimum design. A deterministic design chart is created as a first step, and this is developed to produce a reliability-based design chart that incorporates the uncertainties in the significant parameters controlling veneer cover soil stability. Examples are included to demonstrate application of the charts, and to highlight the significance of uncertainty in design.

2. MODIFIED LIMIT EQUILIBRIUM WEDGE METHOD

Figure 1 illustrates two wedges that are considered to act along a cover soil slope. These are used to formulate the factor of safety against sliding mode of failure. Unlike most researchers, who defined the factor of safety in the limit equilibrium two-wedge method as the ratio between the available and mobilised values of the strength parameters, Giroud *et al.* (1995) defined the factor of safety in

Figure 1. Schematic diagram of veneer cover soil showing base case values for design parameters used in examples

the two-wedge method as the ratio of the resistance over the driving forces. Their proposed formulations are adopted in this study because they are computationally simple and reduce geometric manipulation. However, their formulations have been modified so that the terms are expressed in terms of interface shear strengths rather than individual derived interface shear parameters of friction angle and adhesion. This modification allows more flexibility in choosing lining materials with different strength characteristics.

The modified formulations inherit limitations and assumptions similar to those asserted in Giroud *et al.* (1995). They include uniform thickness of the cover soil, dry conditions, which assume that the drainage system is working efficiently, and no other slippage except at the weakest interface. However, the approach developed in this study can be extended to take account of tapered cover soil and/or submergence conditions, but this involves creating additional design charts for specific cases.

2.1. Definitions and formulations

Symbols used in the formulations are defined in the schematic diagram of veneer cover soil shown in Figure 1. Other symbols used are defined in the Notations section. Formulations used to produce the design charts are derived using force diagrams similar to the ones given in Giroud *et al.* (1995). The geometry and forces acting on the active and passive wedges, which are denoted with the subscripts 'A' and 'P', respectively, are shown in Figures 2 and 3. The resultant force (E) acting between the wedges is assumed to act parallel to the slope inclination. All forces used in the formulations are force per unit length perpendicular to the plane of the considered figures and diagrams.

The force diagram for the passive wedge is solved first to obtain E_p . The weight of the passive wedge and the contribution from cover soil cohesion are given as follows.

$$W_p = \frac{\gamma h^2}{2 \sin b \cos b} \quad (1)$$

$$C_p = \frac{ch}{\sin b} \quad (2)$$

Figure 2. Passive wedge: (a) geometry and force; (b) force diagram (after Giroud *et al.* 1995). Note: Force diagram is not closed, which indicates that resistance forces are greater than driving forces, thus yielding $FS > 1$

N_p in Equation 3 is obtained by solving the forces in the vertical direction.

$$N_p = W_p + E_p \sin b \quad (3)$$

Substituting W_p into and solving the forces acting in the horizontal direction yields Equation 4 for E_p .

$$E_p \cos b = W_p \tan \phi + E_p \sin b \tan \phi + C_p \quad (4)$$

E_p is then finally resolved to Equation 5 by substituting Equations 1 and 2 into Equation 4.

$$E_p = \frac{(ch/\sin b) + (\gamma_d h^2/2 \sin b \cos b) \tan \phi}{\cos b - \sin b \tan \phi} \quad (5)$$

Considering the active wedge force diagram (Figure 3), W_A and C_A are computed as follows.

$$W_A = \frac{\gamma_d H h}{\sin b} \quad (6)$$

$$C_A = \frac{\alpha H}{\sin b} \quad (7)$$

N_A in Equation 8 is then acquired by solving the force diagram in the direction perpendicular to the slope inclination.

Figure 3. Active wedge: (a) geometry and force; (b) force diagram (after Giroud *et al.* 1995)

$$N_A = W_A \cos b = \frac{\gamma_d H h}{\sin b} \cos b \quad (8)$$

Finally, the following factor of safety against sliding, FS, is formulated as the ratio between the resisting (F_R) and driving (F_D) forces acting on the active wedge in the direction parallel to the slope inclination.

$$FS = \frac{F_R}{F_D} = \frac{E_A + C_A + N_A \tan \delta + T_{gs}}{W_A \sin b} \quad (9)$$

Resolving for FS by substituting Equations 5 to 8 into Equation 9 yields

$$FS = \frac{\alpha + \gamma_d h \cos b \tan \delta}{\gamma_d h \sin b} + \frac{c + (\gamma_d h / 2 \cos b) \tan \phi}{\gamma_d H \sin b (\cos b - \sin b \tan \phi)} + \frac{T_{gs}}{\gamma_d H h} \quad (10)$$

Giving

$$FS = \frac{\tau_{IN}}{\gamma_d h \sin b} + \frac{\tau_{soil}}{\gamma_d H \sin b (\cos b - \sin b \tan \phi)} + \frac{T_{gs}}{\gamma_d H h} \quad (11)$$

where

$$\tau_{IN} = \alpha + \gamma_d h \cos b \tan \delta \quad (12)$$

$$\tau_{soil} = c + \frac{\gamma_d h}{2 \cos b} \tan \phi \quad (13)$$

In order to reduce the number of variables incorporated in the design charts, T_{gs} is not considered, which is a conservative assumption, and hence is on the safe side. Moreover, it is good practice to minimise tensile stresses in non-reinforcing geosynthetics such as geomembranes and geotextiles.

2.2. Modification to FS formulation for use in design charts

To avoid infinite values of calculated FS (e.g. $\phi = 60^\circ$ and $b = 30^\circ$), the denominator of the second term (D_2) in Equation 11 is maximised with slope inclination as follows, which results in a conservative FS.

$$\frac{dD_2}{db} = 0 \quad (14)$$

As a first step, D_2 is generalised to extricate the equation from the effect of cover soil cohesion by introduction of the R factor as follows.

$$D_2' = \gamma_d H \sin b \left(\cos b - R \frac{\sin^2 b}{\cos b} \right) \quad (15)$$

where $R = \tan \phi / \tan b$.

The following approximation is obtained by differentiating D_2' of Equation 15 with slope inclination and solving Equation 14.

$$b = \sin^{-1} \left\{ \left[\frac{(3R + 3) - \sqrt{(3R + 3)^2 - 4(2R + 2)}}{2(2R + 2)} \right]^{0.5} \right\} \quad (16)$$

A chart of maximised values for the denominator of the second term in Equation 11 is illustrated in Figure 4. In a sensitivity analysis, the modification yields differences of less than 0.05 when compared with FS values calculated using the original formulation. However, the difference of FS between the exact solution and using the modification is insignificant only for ϕ and c not greater than 40° and 5 kPa, respectively, slope inclinations between 15° and 32° , γ_d between 17 and 20 kN/m³, and at any landfill height and cover soil thickness. When the maximised value for the denominator of second term in Equation 11 is used, any values of the parameters that fall outside these ranges may result in lower FS values than using the exact

Figure 4. Maximum values for denominator of term 2 in Equation 11

solution, and therefore these factors of safety will be conservative.

3. DEVELOPMENT OF DETERMINISTIC DESIGN CHART

3.1. Sensitivity analysis for deterministic design chart

To produce a deterministic design chart, a sensitivity study is conducted to evaluate the most significant parameters for the veneer stability model. The sensitivity of the input parameters is investigated using Tornado charts. A Tornado chart depicts the percentage of change in the performance measure ($\partial\mu_Y/\mu_Y$) such as FS, for a percentage change in input parameter ($\partial\mu_{X_i}/\mu_{X_i}$) about its mean value as follows:

$$S_i = \frac{\partial\mu_Y/\mu_Y}{\partial\mu_{X_i}/\mu_{X_i}} = \frac{\partial Y}{\partial X_i} \bigg|_{\mu_X} \frac{\mu_{X_i}}{\mu_Y} \quad (17)$$

Positive S_i values indicate a beneficial contribution in increasing the performance measure with increasing value of the input parameter, and negative values imply the opposite. Figure 5a shows a Tornado chart, using the partial derivatives shown in the Appendix, evaluated at the base case values stated in Table 1. Figure 5b demonstrates

Figure 5. Tornado charts to illustrate sensitivity of: (a) input parameters to change in FS; (b) contribution of input parameter uncertainty to uncertainty in FS

the percentage contribution of uncertainty in input parameters to uncertainty in FS, computed using the following equation.

$$\Delta\sigma_{FS}^2 = \frac{S_i^2 V_{X_i}^2}{\sum_{all i} S_i^2 V_{X_i}^2} \times 100\% \quad (18)$$

where V_{X_i} is the coefficient of variation for an input parameter.

Additionally, each design parameter is varied in its normal range stated in Table 1, with other parameters kept at the base case values. The parameters that change FS by less than 5% of the target FS of 1.5 within their normal

Table 1. Design values for deterministic parametric study (bold values are varied typical range)

Parameters	H (m)	b (degrees)	γ_d (kN/m ³)	ϕ (degrees)	c (kPa)	h (m)	δ (degrees)	α (kPa)
Base value	30	26.6	18	30	0	1	22.8	5
H	20–50	26.6	18	30	0	1	22.8	5
b	30	15–35	18	30	0	1	22.8	5
γ_d	30	26.6	16–20	30	0	1	22.8	5
ϕ	30	26.6	18	25–45	0	1	22.8	5
c	30	26.6	18	30	0–10	1	22.8	5
h	30	26.6	18	30	0	0.2–2	22.8	5
δ	30	26.6	18	30	0	1	15–35	5
α	30	26.6	18	30	0	1	22.8	0–10

ranges (i.e. $1.425 < FS < 1.575$) are considered to be insignificant. Based on the Tornado charts and additional sensitivity analysis outputs depicted in Figure 6, it is found that the critical parameters are interface shear strength parameters α and δ , slope inclination b and cover soil thickness h . Insignificant parameters include the friction angle ϕ and cohesion c of the cover soil, the height of the landfill facility, H , and the dry unit weight of cover soil, γ_d . To create the design charts in subsequent analyses, insignificant parameters are kept at their constant mean base case values. It has to be highlighted that the insignificant parameters have little influence on the FS because the geometry of the landfill of the study is a dry uniform slope. Other types of geometry, such as a tapered slope and saturation conditions, may yield different outcomes.

3.2. Deterministic design chart

Figure 7 depicts deterministic design charts of veneer cover stability for a landfill height of 30 m and cover soil thickness h of 0.5 m and 1 m, respectively. The curves in the design chart are created to achieve FS of 1.5 for any combination of maximum slope inclination or minimum required interface shear strength at the weakest interface. Applications of the deterministic design chart are illustrated below. The deterministic design chart enables the selection of different types of geosynthetic as lining materials based on the required interface shear strength for stability. Therefore suitable and economical geosynthetic materials can be selected to form a liner that will satisfy the minimum requirement of safety against sliding. Additionally, landfill capacity can be optimised by adopting the highest possible slope angle that satisfies safety

Figure 6. Sensitivity study of (a) positive and (b) detrimental input parameters on FS

Figure 7. Deterministic design charts for sliding stability with: (a) $h = 0.5$ m; (b) $h = 1.0$ m

against sliding, given that the site and lining materials have been selected. However, one major limitation of the deterministic design chart is that it does not consider the uncertainty associated with each design parameter, and therefore the resulting FS of 1.5 is ambiguous.

3.3. Application and examples

Example 1: Selection of liner materials

Given the veneer cover configuration in Figure 1, R is calculated as 1.15, which results in the required interface shear strength τ_{IN} of 11.8 kPa using Figure 7b. Examples of interface shear strength parameters that satisfy τ_{IN} of 11.8 kPa to achieve a factor of safety of 1.5 are stated in Table 2. Therefore any geosynthetic and soil materials that have the combination of interface shear parameters stated

Table 2. Interface shear strength parameters to achieve $\tau_{IN} = 11.8$ kPa

α (kPa)	δ (degrees)
0	36.2
2	31.2
5	22.8
10	6.3

in Table 2 are suitable as lining materials, to satisfy stability against sliding.

Example 2: Optimising landfill capacity

A few iterations are required in order to obtain the maximum allowable slope angle for the landfill that satisfies FS of 1.5 against sliding. The iteration steps are as follows.

1. Assume an initial value for τ_{IN} .
2. Acquire R from Figure 7 and calculate b .
3. Calculate τ_{IN} using Equation 12.
4. Compare τ_{IN} calculated in step 3 with the assumed value of τ_{IN} in step 1 and repeat the process until the two values converge.

Given a similar cover soil as in Example 1, and given that the weakest interface in the lining system has α and δ of 5 kPa and 20° respectively, assume the initial value for τ_{IN} is 10 kPa. R is obtained as 1.4 using Figure 7, and subsequently b is calculated as 22.4° . τ_{IN} is then calculated using Equation 12, and the new value is 11.06 kPa. This value is not equal to the initial assumed τ_{IN} , which is 10 kPa. The process is repeated until τ_{IN} from consecutive iterations converges. Three iterations are needed for this example, as demonstrated in Table 3. Therefore the maximum allowable slope angle for the landfill to ensure safety against sliding is 24.5° .

4. DEVELOPMENT OF RELIABILITY-BASED DESIGN CHART

Most uncertainty and variability are dealt with statistically, which involves estimation of expected values and standard deviations of the design parameters taken as random variables. In probabilistic or reliability methods, uncertainties reflected in the input parameters are evaluated statistically to produce corresponding uncertainties in the

Table 3. Iteration process to obtain maximum slope angle given that $\alpha = 5$ kPa and $\delta = 20^\circ$

Iteration	Initial τ_{IN} (kPa)	R	b (degrees)	τ_{IN} (kPa)
1	10 ^a	1.40	22.4	11.06
2	11.06	1.25	24.8	10.95
3	10.95	1.27	24.5	10.96

^a Assumed.

performance function such as factor of safety. The value of an input parameter or random variable is represented using a probability distribution. A probability distribution function states all the possible values that a random variable can take and their corresponding probability of occurrence. Statistical moments of a variable, namely mean and standard deviation, are usually required to define a distribution or a probability density function.

The reliability-based design chart presented in this paper considers uncertainty only in values of the design parameters. Besides the errors that might occur during laboratory testing, uncertainty also occurs in deciding the mobilised interface shear strength to adopt for design. Table 4 states the coefficient of variations of interface shear strength parameters between a textured HDPE geomembrane (TGM-NWGT) against a nonwoven geotextile, and between the textured geomembrane against Mercia Mudstone (TGM-fines), obtained from a laboratory repeatability test programme (Sia and Dixon 2007). Variations of the interface shear strength parameters are small compared with a compiled global database from a literature review, in which the variation reached up to 40%. Additional information on the repeatability test programme and the global database are presented by Sia and Dixon (2007); based on the reported variability, the interface shear strength parameters considered for the reliability-based design chart are varied from 5% up to 40%.

Before producing a reliability-based design chart, a sensitivity study has been conducted to find out which uncertain inputs will significantly affect the performance measure of reliability (e.g. reliability index or probability of failure), and hence which should be varied within their typical range.

4.1. Additional performance measure

Probability of failure (P_f) in reliability analysis is analogous to factor of safety in a deterministic approach. Instead of a ratio of 'failed' trials (i.e. FS less than 1.0) to the total number of trials, the probability of failure (P_f) is calculated as the area underneath the distribution of reliability index, for which FS is less than 1.0 and is assumed normally distributed. Reliability index and probability of failure are defined in Equations 19 and 20, respectively.

Table 4. Coefficient of variation V for TGM-NWGT and TGM-Fines interfaces for applied normal stresses not greater than 50 kPa (rounded up to nearest 1%)

	$V[\delta]$ (%)	$V[\alpha]$ (%)
TGM-NWGT		
Peak	8	13
Large displacement	7	15
TGM-Fines		
Peak	10	19
Large displacement	10	25

$$\beta = \frac{\mu_{FS} - 1}{\sigma_{FS}} \quad (19)$$

$$P_f \cong \Phi(-\beta) \quad (20)$$

μ_{FS} and σ_{FS} are the mean and standard deviation of factor of safety, which is assumed to be normally distributed, and Φ is a standard normal distribution function. In sensitivity analysis μ_{FS} is calculated using Equation 11, and evaluated for a base case value of each input parameter; σ_{FS} is computed using a Taylor series approximation:

$$\sigma_{FS}^2 = \sum \left[\sigma_{X_i}^2 \left(\frac{\partial FS}{\partial X_i} \right)^2 \right] \quad (21)$$

Using the base case values shown in Figure 1, μ_{FS} and σ_{FS} are computed as 1.5 and 0.16 (i.e. $V[FS] = 10.7\%$), respectively. For a target FS of 1.5 and its uncertainty in terms of coefficient of variation (V) of 10%, Figure 8 demonstrates that the assumption of normality for FS is

Figure 8. Comparison between normal and log-normal distributed FS

on the safe side, as this assumption would result in a higher failure probability for similar FS and degree of uncertainty.

4.2. Sensitivity analysis for reliability-based design chart

Similar to the deterministic analysis, Tornado charts are used to investigate the sensitivity of mean and uncertainty of input parameters to the reliability index, β (P_f was not used, because it does not have a closed-form solution). For accuracy, the partial derivatives of β with input parameters and their uncertainty in terms of standard deviation for Equation 17 are tabulated using Matlab version 6.5. The sensitivity measure S_i is evaluated at the base case values given in Tables 1 and 5 for the mean and their uncertainty values, respectively. The equations accompanying the Tornado charts in Figures 10 and 11 are formulas used to compute the sensitivity values.

Figure 9 illustrates the percentage change in design parameters and their corresponding uncertainty to the percentage change in β . Based on Figure 9b, all uncertainty in the design parameters has a detrimental effect on the reliability index, as dictated by the negative values. Figure 10a demonstrates the percentage change in uncertainty of FS for a percentage change in the uncertainty of an input parameter, and Figure 10b shows the contribution of uncertainty in design parameters to uncertainty in the computed β . The ranking of significant parameters based on the change in β , FS or their corresponding uncertainty is observed to be dependent on both the mean and the degree of uncertainty in input parameters (in terms of standard deviation). However, it is found consistently that slope inclination, interface friction and adhesion, as well as their uncertainty, are the major design parameters to consider for the development of reliability-based design charts. However, the coefficient of variation for slope inclination was kept constant at 5%, since its construction can be controlled (contractor allowable tolerance, which is stated in a contract) and is usually allowed to deviate by up to 2% (Sia 2007). Landfill height, cover soil friction and cohesion, as well as their uncertainties, are not significant parameters, and hence are kept at their base case values for development of the design chart. Cover soil thickness, soil density and their uncertainty have some influence, but are also adopted at base case values to limit the number of variables in the design chart. Nevertheless, different design charts could be created for different cover soil thicknesses and soil densities.

4.3. Selection of target probability of failure

During the development of a reliability-based design chart, the curves are generated such that the minimum interface

Table 5. Base case values of coefficient of variation V for reliability-based veneer cover design sensitivity study

Parameter	$V[H]$ (%)	$V[b]$ (%)	$V[\gamma_a]$ (%)	$V[\phi]$ (%)	$V[c]$ (%)	$V[h]$ (%)	$V[\delta]$ (%)	$V[a]$ (%)
Base value	5	5	5	10	10	5	5	20

Figure 9. Tornado charts to illustrate sensitivity of (a) input parameters and (b) their uncertainty (i.e. expressed in terms of standard deviation) to change in reliability index

shear strength or maximum slope inclination can be estimated, which satisfy both the target probability of failure, P_{ft} and the required factor of safety. The values of P_{ft} are determined based on judgement using an existing scale of P_{ft} that has been established in the literature and discussed by Dixon *et al.* (2006). Assuming that a landfill cover soil performed below average, based on USACE (1999), owing to construction equipment and loading, was exposed to moderate risk according to Gilbert (2001), and requires only minor repairs upon unsatisfactory performance conforming with Cole (1980), P_{ft} is estimated to be 1×10^{-2} , which is located between the annual probability of failure of mine slopes and foundations in Figure 11.

The assigned P_{ft} for the reliability-based design chart of veneer cover soil stability is higher than the recommended value of 5×10^{-3} for 'typical' consequences ($\leq$ US\$1 000 000) for barrier function failure (Koerner 2002), between 'below' and 'above average' performance level given in USACE (1999), and classed as 'medium' degree of system redundancy if failure occurs (D'Hollander 2002). Sabatini *et al.* (2002) used a conservative P_{ft} value of 1×10^{-4} for waste containment system stability analyses. The use of a value for P_{ft} that is higher than given in the literature could be compensated for by the use of relatively high degrees of uncertainty associated with the construction tolerances (e.g. $V[H]$, $V[b]$ and $V[h]$) of 5% applied in the design chart, as expert elicitation has indicated that it is typically less than 5% in practice (Sia 2007).

Figure 10. Tornado charts to illustrate sensitivity of: (a) uncertainty in input parameters to uncertainty in FS; (b) contribution of input parameters uncertainty to uncertainty in β (i.e. uncertainty is expressed in terms of standard deviation)

Figure 11. Empirical rates of failure for civil engineering facilities (after Baecher 1987)

5. RELIABILITY-BASED DESIGN CHART

Analyses to produce a reliability-based design chart were carried out in a spreadsheet with an Excel add-in, @RISK version 4.5, using Monte Carlo simulation. The coefficient of variation, V , for base case values is stated in Table 5. The geometries of the veneer cover (i.e. H , h and b) are assigned uniform distributions, which express equal chances of error from a lack of construction quality

control and achievable construction tolerances. The cover soil properties are assigned normal distributions with variations in accordance to Duncan (2000), and the interface shear strength parameters are assumed to be log-normally distributed, as both parameters are usually positive in value for dry conditions. Alternatively, both parameters can be postulated as normally distributed, with distribution tails truncated at minimum and maximum plausible values (Sia and Dixon 2007). Both types of assumed distribution would yield similar outcomes (e.g. reliability index or probability of failure), because the variability of the interface shear strength parameters is relatively small.

The process to create a reliability-based design that satisfies FS of 1.5 and P_{ft} of 1×10^{-2} is shown in Figure 12. Initially, δ to achieve FS of 1.5 is calculated using Equations 11 and 12 by assuming a value of α , e.g. 0 kPa. The factor R is also assumed, e.g. 0.2. The slope inclination can then be calculated, as ϕ is taken at the base case value of 30° . Similarly, other design parameters and their uncertainty are also assigned base case values. In each simulation FS is calculated for different values of the design parameters, which are randomly sampled based on their probability distribution using the Monte Carlo method. Subsequently, the distribution of FS shown in Figure 13 with its mean (μ_{FS}) and standard deviation (σ_{FS}) is obtained after the simulation achieves convergence (i.e. changes in statistical moments of FS are less than 0.5% for subsequent simulations). The reliability index β and corresponding failure probability P_f are calculated by assuming that FS is normally distributed. Steps 2 to 5 in Figure 12 are repeated for different values of R , ranging from 0.2 to 3.0. An example intermediary chart as shown in Figure 14 can then be plotted, and R values corresponding to P_f of 1×10^{-2} are obtained from the chart. The

Figure 13. FS distribution after simulation for α of 5 kPa; $V[\alpha] = 10\%$, $V[\delta] = 7\%$ and $R = 1.2$

Figure 14. One of the intermediary charts used to estimate factor R to satisfy $P_f = 1 \times 10^{-2}$. Note: $V[\alpha] = 10\%$, $V[\delta] = 7\%$

Figure 12. Flowchart illustrating one cycle of simulation to create reliability-based design chart that satisfies $P_{ft} = 1 \times 10^{-2}$ and $FS = 1.5$

resulting maximum slope angle ($b_{\max}$) can be computed from the R value (i.e. $\phi = 30^\circ$), and τ_{IN} and the minimum requirement for interface friction angle ($\delta_{\min}$) to satisfy FS of 1.5 can also be calculated from Equations 11 and 12, respectively. The entire process outlined in Figure 12 is repeated for different combinations of α values of 0, 2, 5 and 10 kPa, $V[\alpha]$ of 10%, 20% and 40% or $V[\delta]$ ranging from 5% to 40%, while maintaining other insignificant input parameters at their base case values. The reliability-based design chart for veneer cover soil stability illustrated in Figure 15 can finally be plotted. It expresses the relationship between $\delta_{\min}$ and $b_{\max}$ to achieve FS of 1.5 and P_f of 1×10^{-2} for the specific slope geometry and cover soil properties.

5.1. Application of reliability-based design chart

The reliability-based design chart shown in Figure 15 can be used to find the minimum interface shear strength parameters to achieve FS of 1.5, in addition to satisfying the failure probability of 1×10^{-2} for the associated uncertainty in the design parameters. Additionally, the chart can be used to find the maximum slope angle for veneer cover, given that the strength parameters and their uncertainties are known (e.g. from laboratory testing).

The four dotted lines extending almost diagonally from left to right in Figure 15 represent interface adhesions of 0, 2 kPa, 5 kPa and 10 kPa. These lines are drawn so that the corresponding interface friction angle at a certain slope angle will yield a factor of safety against sliding of 1.5. Other types of line, namely dark-coloured continuous lines, dashed lines and light-coloured continuous lines, express the uncertainty in the interface adhesion in terms of coefficient of variation. The dark-coloured continuous lines indicate the coefficient of variation for an interface adhesion of 10% with different magnitudes of uncertainty in interface friction angle. Similarly, the dashed lines record uncertainty for an interface adhesion of 20%, and the light-coloured continuous lines designate the spread for an interface adhesion of 40%. These lines connect the

four dotted adhesion lines together, and any uncertainty that is beyond these lines in Figure 15 should not be interpolated. The percentages stated alongside the different types of line in Figure 15 indicate the uncertainty in the interface friction angle in terms of coefficient of variation.

For discussion, consider the coefficient of variation for an interface adhesion of 10%, which is represented by the dark-coloured continuous lines. If common practice such as ignoring the interface adhesion is adopted, the maximum allowable uncertainty in the interface friction angle based on Figure 15 is 13.5%. Any variability of interface friction angle greater than 13.5% will yield a higher probability of failure (i.e. greater than 1×10^{-2}). Similarly, if the interface adhesions are 2 kPa, 5 kPa and 10 kPa with 10% associated uncertainty, the maximum allowable variations for interface friction angle are 24%, 40% and >40%, respectively. Table 6 states the maximum allowable coefficients of variation for interface friction angle, given that the coefficients of variation for interface adhesion are 10%, 20% and 40%. Interface properties that have variation less than stated in Table 6 will satisfy a probability of failure not greater than 1×10^{-2} . The term 'NS' indicates that the interface friction angle should be

Table 6. Maximum allowable $V[\delta]$ (%) for interface adhesion of 0, 2 kPa, 5 kPa and 10 kPa and $V[\alpha]$ of either 10%, 20% or 40% to achieve P_f not greater than 1×10^{-2}

α (kPa)	$V[\alpha]$ (%)		
	10	20	40
0	13.5	13.5	13.5
2	24.0	18.8	12.3
5	40.0	17.0	7.0
10	>40.0	13.5	NS ^a

^aNS = not satisfied.

Figure 15. Reliability-based design chart for target FS of 1.5 and P_f of 1×10^{-2} (after Sia and Dixon 2006)

determined with certainty. Therefore, if interface adhesion is equal to or greater than 10 kPa but the estimation is subjected to high uncertainty up to 40% or greater, the interface friction value must be ascertained to achieve a probability of failure not greater than 1×10^{-2} .

Given that the interface materials have been selected, the steps in utilising the reliability-based design charts are as follows.

1. Using Figure 15, determine the maximum slope angle for the landfill to achieve FS against sliding of 1.5 with the interface shear strength parameters obtained from a testing programme.
2. Pinpoint the intersection of the interface shear strength parameters and the maximum slope angle between two similar types of line, which indicate similar variation for interface adhesion. For example, the intersection point may be located between two dark-coloured continuous lines (i.e. $V[\alpha] = 10\%$), or between two dashed lines (i.e. $V[\alpha] = 20\%$), or between two light-coloured continuous lines (i.e. $V[\alpha] = 40\%$).
3. Observe the numbers specifying the variation for interface friction angles for those two lines, and interpolate the variation from the intersection point.
4. The variation of interface shear strength parameters obtained or expected from laboratory tests should not be greater than that extracted in steps 2 and 3, so that the probability of failure of less than 1×10^{-2} is satisfied.

One limitation of the reliability-based design chart is that it fails to answer the question of how increase in FS is required to compensate for higher uncertainty in the interface shear parameters.

Example 3: Using reliability-based design chart

Adopting the same configuration of cover soil as used in Example 1, the lining materials constituting the weakest interface should be selected based on the combination of statistical moments asserted in Table 7 to achieve FS of 1.5 and P_{ft} of 1×10^{-2} . To obtain the values stated in Table 7, the steps described above are employed as follows.

1. Assuming that the interface adhesion and friction angle obtained from laboratory testing are 2 kPa and

Table 7. Combination of statistical moments for weakest interface given that slope angle b is 26.6°

α (kPa)	δ (degrees)	$V[\delta]$ %		
		$V[\alpha] = 10\%$	$V[\alpha] = 20\%$	$V[\alpha] = 40\%$
0	36.2	10.0	10.0	10.0
2	31.2	13.3	13.0	11.5
5	22.8	19.5	15.8	NS
10	6.3	NS ^a	NS	NS

^aNS = not satisfied.

- 31.2°, the maximum slope angle for the cover soil to achieve FS against sliding of 1.5 using Figure 15 is 26.6° .
2. The intersection point of the interface shear strength parameters and the maximum slope angle are pinpointed between two similar types of line. For example, if the variation in interface adhesion $V[\alpha]$ is expected to be 10%, the maximum variation for interface friction angle $V[\delta]$ to satisfy P_f of 1×10^{-2} is located between two dark-coloured continuous lines. Similarly, if $V[\alpha]$ is expected to be 40%, $V[\delta]$ is located between two light-coloured continuous lines.
3. For interface adhesion of 2 kPa and $V[\alpha]$ of 10%, the intersection point is located along the adhesion line (i.e. dotted line) for 2 kPa and between two dark-coloured continuous lines with spreads in interface friction angle $V[\delta]$ of 12% and 13.5%, as shown in Figure 15. However, if $V[\alpha]$ is expected to be 40%, the intersection point is situated along the adhesion line for 2 kPa and between two light-coloured continuous lines with $V[\delta]$ of 11% and 12%.
4. Therefore, by interpolating along the adhesion line for 2 kPa and between two dark-coloured continuous lines for $V[\delta]$ of 12% and 13.5%, the maximum allowable $V[\delta]$ to satisfy P_f of 1×10^{-2} is 13.3%. Similarly, if $V[\alpha]$ is 40%, interpolating along the adhesion line of 2 kPa and between two light-coloured continuous lines yields a maximum allowable $V[\delta]$ of 11.5%. Finally, the interface materials should be selected such that the interface shear strengths are greater than, but the associated variations are less than, the values stated in Table 7 in order to satisfy both the FS and P_{ft} criteria.

5.2. Influence of P_{ft} on reliability-based design chart

Comparison of curves to achieve P_{ft} of 1×10^{-2} and 1×10^{-4} is illustrated in Figure 16. If a stricter criterion is imposed on P_{ft} (e.g. 1×10^{-4}) for similar design parameter values and their corresponding degree of uncertainty, it is observed that the slope inclination of the cover soil needs to be lowered significantly, which is undesirable for the landfill operator. There needs to be a

Figure 16. Comparison between P_{ft} of 1×10^{-2} and 1×10^{-4} on reliability-based design chart

consensus between operator, designer and legislator on the degree of acceptable failure or unacceptable performance before reliability-based design can be practised to optimise design.

6. CONCLUSIONS

A design chart is a useful graphical tool for assessing the behaviour of a stability model for preliminary design, and to obtain both a safe and an optimum design. Two types of design chart have been produced, based on the modified limit equilibrium two-wedge method for evaluating the stability of veneer cover soil. The deterministic design chart provides information regarding the minimum interface shear strength or maximum slope angle to achieve FS of 1.5; the reliability-based design chart is capable of providing similar information, and also the allowable uncertainties associated with the design parameters. However, the latter chart imposes a more stringent criterion in the selection of lining materials, and requires practitioners to conduct more interface shear tests. This produces higher confidence in the selection of lining materials for design. The reliability-based design chart presented in the paper is not currently able to consider different cover soil characteristics, types of loading or their corresponding uncertainties. Nevertheless, it is a step forward towards reliability-based design, and demonstrates the importance of understanding uncertainty in design. The current practice using target factors of safety is not able to reflect this uncertainty, and therefore its use will at times result in unsafe designs. The reliability approach could be standardised, and may become a requirement in the future for landfill engineering design. A key consideration is the relationship between the financial risk that results from an unsafe design and the additional costs incurred in obtaining input data for, and carrying out, a reliability-based design.

APPENDIX. DERIVATIVES OF FS WITH INPUT PARAMETERS

Interface adhesion:

$$\frac{\partial FS}{\partial \alpha} = \frac{1}{\gamma_d h \sin b} \quad (22)$$

Interface friction:

$$\frac{\partial FS}{\partial \delta} = \frac{\cos b(1 + \tan^2 \delta)}{\sin b} \quad (23)$$

Unit weight of cover soil:

$$\begin{aligned} \frac{\partial FS}{\partial \gamma_d} = & \frac{\cos b \tan \delta}{\gamma_d \sin b} - \frac{\alpha + \gamma_d h \cos b \tan \delta}{\gamma_d^2 h \sin b} \\ & + \frac{h \tan \phi / 2 \gamma_d H \cos b \sin b}{\cos b - \sin b \tan \phi} \\ & - \frac{c + (\gamma_d h \tan \phi / 2 \cos b)}{\gamma_d^2 H \sin b (\cos b - \sin b \tan \phi)} \end{aligned} \quad (24)$$

Cover soil thickness:

$$\begin{aligned} \frac{\partial FS}{\partial h} = & \frac{\cos b \tan \delta}{h \sin b} - \frac{\alpha + \gamma_d h \cos b \tan \delta}{\gamma_d h^2 \sin b} \\ & + \frac{\tan \phi}{2 H \cos b \sin b (\cos b - \sin b \tan \phi)} \end{aligned} \quad (25)$$

Cover soil cohesion:

$$\frac{\partial FS}{\partial c} = \frac{1}{\gamma_d H \sin b (\cos b - \sin b \tan \phi)} \quad (26)$$

Cover soil friction:

$$\begin{aligned} \frac{\partial FS}{\partial \phi} = & \frac{1 + \tan^2 \phi}{2 h H \cos b \sin b (\cos b - \sin b \tan \phi)} \\ & + \frac{c + (\gamma_d h \tan \phi / 2 \cos b)}{\gamma_d H (\cos b - \sin b \tan \phi)^2} - (1 + \tan^2 \phi) \end{aligned} \quad (27)$$

Landfill height:

$$\frac{\partial FS}{\partial H} = \frac{-[c + (\gamma_d h \tan \phi / 2 \cos b)]}{\gamma_d H^2 \sin b (\cos b - \sin b \tan \phi)} \quad (28)$$

Veneer slope inclination:

$$\begin{aligned} \frac{\partial FS}{\partial b} = & -\tan \delta - \frac{\alpha + \gamma_d h \cos b \tan \delta}{\gamma_d h \sin^2 b} \cos b \\ & + \frac{h \tan \phi}{2 H \cos^2 b (\cos b - \sin b \tan \phi)} \\ & - \frac{[c + (\gamma_d h \tan \phi / 2 \cos b)] \cos b}{\gamma_d H \sin^2 b (\cos b - \sin b \tan \phi)} \\ & - \frac{[c + (\gamma_d h \tan \phi / 2 \cos b)] (-\sin b - \cos b \tan \phi)}{\gamma_d H \sin b (\cos b - \sin b \tan \phi)^2} \end{aligned} \quad (29)$$

NOTATIONS

Basic SI units are given in parentheses:

c	cohesion of the cover soil (Pa)
h	thickness of cover soil (m)
C_A	resistance due to interface adhesion along active wedge (N/m)
C_P	resistance due to cohesion of cover soil along passive wedge (N/m)
E_A, E_P	resultant forces acting between active and passive wedge (N/m)
F_A, F_P	resultant force acting at base of the active or passive wedge (N/m)
FS	factor of safety against sliding (dimensionless)
H	Height of landfill (m)
N_A, N_P	normal resultant force acting at base of active or passive wedge (N/m)

T_{gs}	tension force from anchored geosynthetics (N/m)
$V[\cdot]$	coefficient of variation; defined as ratio of standard deviation over mean; mean value of parameter stated in the brackets (dimensionless)
W_A, W_P	weight of active or passive wedge (N/m)
α	apparent adhesion of interface (Pa)
β	reliability index (dimensionless)
γ_d	dry unit weight of cover soil (N/m ³)
δ	interface friction angle (degrees)
μ_{FS}	mean value of FS (dimensionless)
σ_{FS}	standard deviation of FS (dimensionless)
τ_{IN}	interface shear strength; defined in Equation 12 (Pa)
τ_{soil}	shear strength of cover soil; defined in Equation 13 (Pa)
ϕ	friction angle of cover soil (degrees)
$\Phi(\cdot)$	standard normal distribution function (dimensionless)

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