Online Supplement for

Designing Fee Tables for Retail Delivery Services by Third-party Logistics Providers

by

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Validity of inequalities

Proof of Proposition 1 (cumulative range width forcing constraints)

Constraint (19) is valid: If the [FTD] solution does not select a range k that spans intervals j_1 through j and ends at interval j , then $\sum_{j' \leq j_1} X_{j',j}^k = 0$, and constraint (19) follows from the non-negativity of the range selection variables. If $\sum_{j' \leq j_1} X_{j',j}^k = 1$, then the width of range k is at least $(d_j - d_{j_1-1})$. In this case, the [FTD] must choose a range $k+1$ beginning with interval $j+1$ (to satisfy range contiguity constraints (3)) and have a width of at least $(d_j - d_{j_1-1})$ (from the increasing range width constraints (13)); consequently, the solution must select an ending interval j_3 for range $k+1$ such that $(d_{j_3} - d_j) \geq (d_j - d_{j_1-1})$, implying (19).

The proof of the validity of constraints (20) follows a similar logic. ■

Proof of Proposition 3 (block-based variable lower bound constraints)

Constraint (22) is valid: Suppose the [FTD] solution selects some ending interval $j' \geq j$ for the distance range that includes interval j (from (2) and (3), we know that interval j must belong to exactly one of the K distance ranges). Similarly, suppose weight interval $i' \geq i$ is the ending interval for a range that spans weight interval i . In this case, the fee lower bound constraints (9) and the range-to-fee linkage constraints (4) and (7) ensure that P_{ijt} must be at least as much as the highest lower bound ($l_{ij't}$) in the block containing cell $\langle i, j \rangle$, and constraint (22) follows. A similar argument establishes constraint (23). ■

Proof of Proposition 4 (block-based variable upper bound constraints)

Constraint (25) is valid: Suppose the [FTD] solution selects some starting interval $j' \leq j$ for the distance range that includes interval j . Similarly, suppose weight interval $i' \leq i$ is the starting interval for a range that spans weight interval i . In this case, the fee upper bound constraints (9) and the range-to-fee linkage constraints (4) and (7) ensure that P_{ijt} is no more than the lowest upper bound $u_{ij't}$ of the block containing cell $\langle i, j \rangle$ and constraint (25) follows. A similar argument establishes constraint (26). ■

Proof of Proposition 5 (fee difference forcing constraints)

Constraint (27) is valid: Consider an [FTD] solution that includes both distance interval j and $j + 1$ in the same distance range. In this case, $\sum_{k, j_1 \leq j} X_{j_1, j}^k = 0$ and constraints (27) follow from the range-to-fee

linkage constraints in [FTD]. If not and if the solution chooses weight range Z_{i_1, i_2}^h to span weight interval i (i.e., $i_1 \leq i \leq i_2$), then the RHS evaluates to $(u_{i_1, j+1, t} - l_{i_2, j, t})$ which is valid because cells $(i_1, j+1)$, (i_2, j) , (i, j) , and $(i, j+1)$ belong to the same fee block. Similarly, starting from the weight ranges and then integrating the distance range selections, we can show the validity of constraint (28). ■

Proof of Proposition 6 (block selection forcing constraints)

Constraint (29) is valid: If the intervals j_1 and j_2 belong to *different* distance ranges, the RHS of constraint (29) is zero and the constraint follows from the non-negativity of the weight range selection variables. If j_1 and j_2 belong to the same range, then the RHS of the constraint is one. Consider all of the eligible weight ranges that span weight interval i . The weight range selection constraints (5) and (6) ensure that the sum of all the weight ranges that span any interval i is one. If the solution selects $i_1 \leq i \leq i_2$ with $u_{i_1, j_1, t} < l_{i_2, j_2, t}$ for any t , then the fee value for the block containing cell $\langle i, j \rangle$ cannot simultaneously satisfy its lower and upper bounds. Therefore, the LHS of constraint (29) must be one in this case. We can establish the validity of constraint (30) following a similar argument. ■

Variable elimination rules**Proof of Proposition 2 (variable elimination rules)**

First range. The first range must start with the first interval, therefore $j_1 = 1$. Next, the width of the first range must be the least of the range widths and as a result must be less than or equal to the average range width ($\frac{d_N}{K}$) of the subsequent $K - 1$ ranges that span a distance of $(d_N - d_{j_2})$. Finally, because each of the remaining $K - 1$ ranges must occupy at least one interval, $j_2 + (K - 1)$ must be less than or equal to N .

Middle range. Because $k - 1$ ranges must have been completed with each occupying at least one interval, we know that $j_1 \geq k$. Analogously, the $K - k$ ranges that follow range k must each cover at least one interval. Therefore, $j_2 + (K - k)$ must be less than or equal to N and part (b) follows. To see part (c), consider a distance range selection X_{j_1, j_2}^k that implies a width of $d_{j_2} - d_{j_1-1}$ for range k . In this case, the previous ranges, each with a width no greater than that of range k , can have an average width of at most $\frac{d_{j_1-1}}{k-1}$, which range k 's width must exceed and (c) follows. Next, each following range must have a width

of at least $d_{j_2} - d_{j_1-1}$. Therefore, the average width $\frac{d_N - d_{j_2}}{K - k}$ of the $K - k$ ranges that follow k must be greater than or equal to range k 's width and (d) follows.

Last range. The last range must include the last interval N , therefore $j_2 = N$. Next, the previous $K - 1$ ranges must include at least one interval each; consequently, $j_1 \geq K$. Finally, the width $(d_N - d_{j_1})$ of the last range must be at least as much as the average width $\frac{d_{j_1-1}}{K-1}$ of the previous $K - 1$ ranges and (c) follows.