Online Supplement for

Designing Supply Contracts: Buy-Now, Reserve and Wait-And-See

Jen-Yi Chen, Maqbool Dada; Qiaohai (Joice) Hu

Simplified Models With Recourse: AR, OR and R

In the AOR model, depending on c_0 , c , and the demand characteristics as summarized in Lemma A2 and Corollary 3, M sets prices p_o , p_e , and p_a so that R uses the advance purchase, option alternatives or both. Thus, for the three more restrictive models, AR, OR and R, a natural question to ask is how the qualifications of Lemma A2 and Corollary 3 can be relaxed. We summarize the results for two simplified settings that differ from the AOR model only in the commitment stage: M offers advance and recourse purchase alternatives (AR), or options and recourse alternatives (OR). Since these two models are special cases of the AOR model, the results summarized below are immediate from Lemma A2.

LEMMA A3. *If $f(\cdot)$ is PF_2 and $-5/11 \leq \beta \leq 1$, Π_M in AR model (OR model) is unimodal in K_a (K_o).*

Proof of Lemma A3 For the AR model, the result follows from the proof of Lemma A2(iii) since both p_e and $\epsilon_3(p_e)$ are irrelevant in the AR model, $g(K_a) = 0$ and the first domain of h_2 vanishes; whereas in the second domain, given K_a , h_2 is monotonically decreasing if $-5/11 \leq \beta \leq 1$. Overall h_2 can change sign at most once $(+, -)$. Then by Schoenberg's Theorem, $\frac{\partial \Pi_M}{\partial K_a}$ can change sign at most once $(+, -)$ if $f(\cdot)$ is PF_2 , so Π_M is unimodal in K_a . For the OR model, the result follows from the proof of Lemma A2(ii) since in the first domain, given K , h_2 is monotonically decreasing if $-5/11 \leq \beta \leq 1$. Overall h_2 and thus $\frac{\partial \Pi_M}{\partial K}$ changes sign at most once, so Π_M is unimodal in K if $f(\cdot)$ is PF_2 .

In the AO model, since p_r is irrelevant, the existence and uniqueness of the equilibrium require no constraint on β that determines the demand elasticity. In contrast, in models with recourse purchasing, M sets p_r after the market size is known, so we have to consider demand elasticity. As a result, the demand elasticity trickles up to the pricing decisions in the commitment stage: p_a , p_o , and p_e . Consequently, in addition to the restriction on $f(\cdot)$, Lemma A3 restricts $y(p)$ to satisfy (5) and $-5/11 < \beta \leq 1$ to ensure the uniqueness of the solutions but in a much more relaxed form than those in Lemma A2 which applies when all three alternatives are jointly considered. The β condition admits all linear and price proportional demand functions, but excludes iso-elastic functions with elasticity lower than $11/5$.

Lastly, we examine a much simplified model: the R model in which all decisions are made after uncertainty has been resolved. As a consequence, only conditions that qualify the demand specification are needed, which have been summarized in Section 2.1, to characterize the equilibrium. The results follow by setting $K_a^* = K_o^* = 0$ in the recourse stage equilibrium of Theorem 1.

REMARK . In the R model, $p_r^* = \frac{c+\alpha}{1+\beta}$ and $\Pi_M^* = (1+\beta)\Pi_R^* = \mathbb{E}[\epsilon] \left(\frac{\alpha-\beta c}{1+\beta} \right) y \left(\frac{c+(2+\beta)\alpha}{(1+\beta)^2} \right)$.

Proof of Remark It follows immediately from Theorem 1.

One would expect that the recourse price increases as market realization ϵ increases. However, as stated above, counter-intuitively, the recourse price in the R model is invariant to ϵ , and thus it can essentially be pre-determined in the commitment stage without altering the equilibrium outcome. The R model is also equivalent to the O model with $p_r^* = p_e^* = \frac{c+\alpha}{1+\beta}$ and $p_o^* = 0$ (Theorem 7). We also note that in the R model, each player takes a constant share of the channel profit. Although M is the Stackelberg leader, it may expect a larger profit share than R if and only if $\beta > 0$ in the demand specification.

Simplified Models Without Recourse: A and O

For the two special non-recourse models: the advance purchasing only model (A) and the option only model (O). Most results follow immediately from the analysis of the AO model. Theorem 7 exactly identifies the equilibrium for the O model since the analysis merely entails setting K_a to 0. For the A model, however, considerably sharper results can be obtained by specializing the demand function.

THEOREM A1. If (i) $y(p)$ is iso-elastic, or (ii) $y(p)$ is price-proportional and $f(\cdot)$ is IGFR, or (iii) $y(p)$ is linear and $f(\cdot)$ is IFR, or (iv) $y(p)$ satisfies (5) and $f(\cdot)$ is PF_2 , then the A model has a unique equilibrium K_a^* that satisfies

$$c_0 = \int_{\epsilon_1}^{\infty} [(1+\beta)^2 y^{-1}(K_a/\epsilon) - \alpha(2+\beta)] dF(\epsilon). \quad (\text{A.16})$$

Proof of Theorem A1 For the A model, $\Pi_M(K_a) = (p_a - c_0)K_a$ where p_a is defined in (15). Then,

$$\frac{d\Pi_M}{dK_a} = -c_0 + \int_{\epsilon_1}^{\infty} [(1+\beta)^2 \hat{p} - \alpha(2+\beta)] dF(\epsilon), \quad (\text{A.17})$$

and $\frac{d^2\Pi_M}{dK_a^2} = \int_{\epsilon_1}^{\infty} \left[\frac{(1+\beta)^2}{\epsilon y'(\hat{p})} \right] dF(\epsilon) + \frac{\alpha}{y(p(0))} f(\epsilon_1)$.

- (i) For the iso-elastic y , since $\alpha = 0$, $\frac{d^2\Pi_M}{dK_a^2} = \int_{\epsilon_1}^{\infty} \left[\frac{(1+\beta)^2}{\epsilon y'(\hat{p})} \right] dF(\epsilon) < 0$ and therefore Π_M is concave.
- (ii) For the price-elasticity y , $\frac{d^2\Pi_M}{dK_a^2} = \frac{c}{ab} f\left(\frac{c}{a} K_a\right) \left[-\frac{a}{K_a f\left(\frac{c}{a} K_a\right) / \bar{F}\left(\frac{c}{a} K_a\right)} + 1 \right]$ and therefore Π_M is concave-convex and thus also unimodal if $f(\cdot)$ is IGFR.
- (iii) For the linear y , $\frac{d^2\Pi_M}{dK_a^2} = \frac{2}{b} \bar{F}(2K_a/a) \left[\int_{\frac{2K_a}{a}}^{\infty} -\frac{2}{\epsilon} \frac{dF(\epsilon)}{\bar{F}(2K_a/a)} + \frac{f(2K_a/a)}{\bar{F}(2K_a/a)} \right]$ where the term in square brackets is increasing in K_a since $\frac{d}{dK_a} \left[\int_{\frac{2K_a}{a}}^{\infty} -\frac{2}{\epsilon} \frac{dF(\epsilon)}{\bar{F}(2K_a/a)} + \frac{f(2K_a/a)}{\bar{F}(2K_a/a)} \right] = \frac{2}{K_a} \frac{f(2K_a/a)}{\bar{F}(2K_a/a)} \left[1 - \int_{\frac{2K_a}{a}}^{\infty} \frac{2K_a/a}{\epsilon} \frac{dF(\epsilon)}{\bar{F}(2K_a/a)} \right] + \frac{d}{dK_a} \left[\frac{f(2K_a/a)}{\bar{F}(2K_a/a)} \right] > \frac{d}{dK_a} \left[\frac{f(2K_a/a)}{\bar{F}(2K_a/a)} \right] \geq 0$ if $f(\cdot)$ is IFR. Therefore, Π_M is concave-convex and thus also unimodal if $f(\cdot)$ is IFR.
- (iv) For y satisfying (5), let $\xi = K_a - \epsilon$, then $\frac{d\Pi_M}{dK_a} = \int_{-\infty}^{K_a} h_4 f(K_a - \xi) d\xi$ where

$$h_4(\xi) = \begin{cases} -c_0 + (1+\beta)^2 y^{-1}(K_a/(K_a - \xi)) - \alpha(2+\beta) & \text{for } \xi \in (-\infty, K_a - \epsilon_1) \\ -c_0 & \text{for } \xi \in [K_a - \epsilon_1, K_a] \end{cases}$$

Given K_a , h_4 is piecewise decreasing and changes sign at most once (+, -). Then, by Schoenberg's

theorem, if $f(\cdot)$ is PF_2 , $d\Pi_M/dK_a$ also changes sign at most once and thus Π_M is unimodal in K_a .

Should any unimodality conditions be satisfied, $\left. \frac{d\Pi_M}{dK_a} \right|_{K_a=0} = -c_0 + \alpha/\beta > 0$ and thus $K_a^* > 0$ always.

Theorem A1 lays out four combinations of conditions on $y(p)$ and $f(\cdot)$, each sufficient for the uniqueness of the equilibrium in the A model. As the condition on $y(p)$ becomes more relaxed, the condition on $f(\cdot)$ becomes more restrictive. Note that the A model is essentially the wholesale price contract with price postponement in Granot and Yin (2008). However, they restrict the demand function to be linear or iso-elastic, and require increasing $\epsilon f(\epsilon)$ to ensure the uniqueness of the solution. Theorem A1 contributes to this line of work in the literature by providing a unified and more general treatment of both the demand function and the random multiplier. Furthermore, the advance purchase quantity in the A and AO models can be ranked:

COROLLARY A1. *If $y(p)$ satisfies (5) and $f(\cdot)$ is PF_2 , then K_a^* is greater in the A model than in the AO model.*

Proof of Corollary A1 $\frac{\partial \Pi_M^A}{\partial K_a} - \frac{\partial \Pi_M^{AO}}{\partial K_a} = \int_{\epsilon'_2}^{\infty} [(1+\beta)^2 \hat{p} - \alpha(2+\beta) - c] dF(\epsilon) > \int_{\epsilon'_2}^{\infty} (1 + \beta) \left(p_e^* - \frac{c+\alpha}{1+\beta} \right) dF(\epsilon) = 0$ where $\partial \Pi_M^A / \partial K_a$ and $\partial \Pi_M^{AO} / \partial K_a$ are as defined in (A.17) and (A.15), respectively.

Corollary A1 states the intuitive result that when options are not available, M tends to induce R to buy more in advance by offering a lower advance purchase price. As to the O model, it performs equally well as the AO model if and only if $c_0 \geq c$ according to Theorem 7.

Integrated Channel and Coordinating Contracts

In this section, we consider the integrated case in which M and R are vertically integrated as a centralized channel, denoted by INT . We then investigate how to design coordinating contracts to achieve this first-best solution using advance purchase, option, and recourse contracts.

In the integrated case, the channel first decides K_a to produce in advance at cost c_0 , and then after uncertainty ϵ is resolved, determines the sales quantity q , or equivalently, how much more to produce at cost c if any. Therefore, given the advance produced quantity K_a , the channel's recourse stage profit function can be expressed as follows:

$$\pi_{INT}(p|K_a) = qp - c(q - K_a)^+ \quad (\text{A.17})$$

where $q = \epsilon y(p)$. Note that the channel would incur marginal cost c for each unit of recourse production if $q > K_a$. Which of the solutions, $q > K_a$, $q = K_a$, or $q < K_a$, is optimal depends on the channel's marginal revenue for the realized ϵ . In the commitment stage, the channel selects K_a to produce at unit cost c_0 to maximize its expected profit.

$$\Pi_{INT}(K_a) = -c_0 K_a + \mathbb{E}[\pi_{INT}]. \quad (\text{A.18})$$

The analysis proceeds as in the decentralized cases using backward induction. We first summarize the recourse stage equilibrium below.

THEOREM A2. *The recourse stage subgame has a unique subgame perfect Nash equilibrium, as summarized in Table 4:*

where ϵ_1 and ϵ_2 are as defined in Table 1.

Proof: Same as the proof for Theorem 1 of the AOR model with $p_e = p_r = c$.

Table 4 Recourse Stage Equilibrium for Integrated Model

ϵ	q^*	p^*	π_{INT}^*
$[0, \epsilon_1)$	$\epsilon y(p^*)$	$p(0)$	$q^* p^*$
$[\epsilon_1, \epsilon_2)$	K_a	$y^{-1}(K_a/\epsilon)$	$q^* p^*$
$[\epsilon_2, \infty)$	$\epsilon y(p^*)$	$p(c)$	$q^*(p^* - c) + K_a c$

Theorem A2 identifies the three cases in which the marginal revenue at $q = K_a$ is less than 0, between 0 and c , and greater than c , respectively. In the first case, demand is weak so the channel holds back inventory to ensure non-negative marginal revenue; in the second case, the marginal gain from selling more than K_a is not enough to recoup the marginal cost so the channel depletes and sells exactly K_a ; in the third case, strong demand encourages the channel to produce and sell more until marginal revenue reaches marginal cost. The two prices $p(0)$ and $p(c)$ are chosen such that the marginal revenue matches marginal cost 0 and c , respectively. As in the AO model, we only require increasing price elasticity to uniquely characterize the recourse stage equilibrium in the integrated case.

Having characterized the recourse stage equilibrium, we now examine the channel's commitment stage problem. Theorem A2 together with (A.18) yields the channel's commitment stage expected profit function. We can then summarize the equilibrium through its FOC.

THEOREM A3. *For the integrated channel (INT model), the optimal advance production $K_a^* = 0$ for $c_0 \geq c$; otherwise, $K_a^* > 0$ and uniquely satisfies*

$$c_0 = \int_{\epsilon_1}^{\epsilon_2} \left[\hat{p} + \frac{y(\hat{p})}{y'(\hat{p})} \right] dF(\epsilon) + \int_{\epsilon_2}^{\infty} c dF(\epsilon). \quad (\text{A.19})$$

where $\hat{p} = y^{-1}(K_a/\epsilon)$.

Proof: The concavity in K_a follows exactly in the same way as in (A.6) and (A.8) with c_0 , 0, and c replacing p_a , p_o , and p_e , respectively.

Theorem A3 shows that the channel would produce in advance if and only if advance production is cheaper. Therefore, when recourse production is cheaper, it would always defer production to the recourse stage after uncertainty is resolved.

Having characterized the optimal solution for the integrated channel, next we examine how M can provide the right incentives to align R 's interest so to attain this first-best outcome. We note that such a coordinating contract must present R with the same marginal cost function as the integrated channel faces in both stages. With that in mind, we consider the following nonlinear pricing scheme for the advance purchase, option, and recourse contracts:

$$(\text{Advance Purchase}) \quad C_1(K_a) = A_1 \cdot \mathbb{I}\{K_a > 0\} + c_0 K_a,$$

$$(\text{Option}) \quad C_2(K_o) = A_2 \cdot \mathbb{I}\{K_o > 0\} + p_o K_o, \text{ with } p_e = c,$$

$$(\text{Recourse}) \quad C_3(q) = A_3 \cdot \mathbb{I}\{q > K\} + c(q - K)^+.$$

where $A_i \geq 0$ for $i = 1, 2, 3$, and $\mathbb{I}\{\cdot\}$ is the indicator function, taking the value 1 if the argument is true and 0 otherwise. Note that, for the option contract, additional variable cost may be incurred upon execution.

We first look at the recourse stage incentive problem. After uncertainty unfolds, under the pricing scheme constructed above, R faces almost the same cost function as does the integrated channel, (A.17), except that R incurs additional cost A_3 when exhausting the options and buying more in the recourse stage. Therefore,

$$\pi_R(p|K_a, K) = qp - c(q - K_a)^+ - A_3 \cdot \mathbb{I}\{q > K\}$$

To motivate R to choose the coordinated quantity, R 's incentive compatibility (IC) and individual rationality (IR) constraints must be satisfied for any $K \geq K_a \geq 0$ and any $\epsilon \geq 0$:

$$(\text{IC}) \quad \pi_R(p^*|K_a, K) \geq \pi_R(p|K_a, K) \text{ for any } p \geq c$$

$$(\text{IR}) \quad \pi_R(p^*|K_a, K) \geq 0$$

where p^* is as characterized in Theorem A2. We note that there are infinitely many A_3 satisfying the IC and IR constraints. For instance, A_3 could be 0, $[\epsilon y(p(c)) - K](p(c) - c)$, or any number in between.

Having characterized the coordinating recourse contract, we next look at the commitment stage incentive problem with the following expected profit function for R .

$$\begin{aligned} \Pi_R(K_a, K) = & -c_0 K_a - A_1 - p_o(K - K_a) - A_2 + \int_0^{\epsilon_1} \epsilon y(p(0))p(0)dF(\epsilon) + \int_{\epsilon_1}^{\epsilon_2} K_a y^{-1}(K_a/\epsilon)dF(\epsilon) \\ & + \int_{\epsilon_2}^{\epsilon_4} [\epsilon y(p(c))(p(c) - c) + cK_a]dF(\epsilon) + \int_{\epsilon_4}^{\infty} [\epsilon y(p(c))(p(c) - c) + cK_a - A_3]dF(\epsilon). \end{aligned}$$

Setting its first-order partial derivative with respect to K_a to zero yields the following FOC.

$$c_0 = \int_{\epsilon_1}^{\epsilon_2} \left[\hat{p} + \frac{y(\hat{p})}{y'(\hat{p})} \right] dF(\epsilon) + \int_{\epsilon_2}^{\infty} c dF(\epsilon) - \left(\frac{dA_1}{dK_a} + \int_{\epsilon_4}^{\infty} \frac{\partial A_3}{\partial K_a} dF(\epsilon) - p_o \right).$$

To align R 's marginal profit function (8) with the channel's (A.19), the last term of the RHS in (8) must vanish. Therefore,

$$A_1^* = p_o K_a - \int_{\epsilon_4}^{\infty} A_3 dF(\epsilon) + \mathcal{C}.$$

Note that the constant term, $\mathcal{C}$, must satisfy R 's commitment stage participation constraint, i.e., $\Pi_R(K_a^*, K) \geq 0$ where K_a^* uniquely satisfies (A.19). Therefore, in equilibrium, $\mathcal{C}$ should be chosen by M to extract all R 's profit so that $\Pi_R(K_a^*, K) = 0$. We note that R 's choice of $K_o = K - K_a$ and thus the option price function $C_2(K_o)$ are irrelevant in designing the coordinating contracts as it only affects the flow of cash transferred between the two parties, but not the expected profits between the two parties. Lastly, since p_o , A_2 and A_3 are not unique, there exist infinitely many coordinating contracts that differ only in the level of certainty in the cash flow for each party; that is, the more revenue there is from advance purchases and options, the less uncertain is the profit of the manufacturer.

Equilibrium for Uniform and Weibull Distributions

Table 5 **Equilibrium for Uniform Distribution and Linear Demand**

c	0	5	10	11	12	13	14	15	20	25	30	35	40	45	50
AOR															
Π_M^*	339.44	274.95	217.24	207.24	198.76	191.54	185.25	179.42	156.65	143.56	138.29	138.30	140.01	140.11	140.11
Π_R^*	183.17	148.36	117.23	111.98	107.77	104.36	102.09	100.34	95.10	94.16	95.00	95.06	91.79	90.55	90.55
Π_{Ch}^*	522.61	423.31	334.47	319.22	306.53	295.90	287.33	279.76	251.76	237.72	233.29	233.36	231.81	230.66	230.66
p_o^*	13.20	11.88	10.56	10.30	10.03	9.77	∞	∞	∞	∞	∞	∞	∞	∞	∞
p_e^*	12.50	16.25	20.00	20.75	21.50	22.25	∞	∞	∞	∞	∞	∞	∞	∞	∞
p_a^*	∞	∞	∞	30.30	30.07	29.87	29.54	29.17	27.50	26.20	25.36	25.05	25.34	25.47	25.47
K^*	5.54	4.98	4.43	4.32	4.21	4.10	4.28	4.59	6.11	7.47	8.51	9.11	9.13	9.06	9.06
K_o^*	5.54	4.98	4.43	2.87	1.51	0.33	0	0	0	0	0	0	0	0	0
K_a^*	0	0	0	1.45	2.70	3.77	4.28	4.59	6.11	7.47	8.51	9.11	9.13	9.06	9.06
AR															
Π_M^*	313.08	257.63	212.96	205.37	198.22	191.51	185.25	179.42	156.65	143.56	138.29	138.30	140.01	140.11	140.11
Π_R^*	156.76	130.65	111.84	108.97	106.39	104.10	102.09	100.34	95.10	94.16	95.00	95.06	91.79	90.55	90.55
Π_{Ch}^*	469.84	388.28	324.80	314.34	304.61	295.61	287.33	279.76	251.76	237.72	233.29	233.36	231.81	230.66	230.66
p_o^*	35.33	33.21	31.12	30.71	30.32	29.93	29.54	29.17	27.50	26.20	25.36	25.05	25.34	25.47	25.47
K_a^*	0.52	1.66	3.06	3.36	3.67	3.97	4.28	4.59	6.11	7.47	8.51	9.11	9.13	9.06	9.06
OR															
Π_M^*	339.44	274.95	217.24	206.52	196.06	185.88	175.97	166.33	122.20	84.86	54.31	30.55	13.58	3.39	0
Π_R^*	183.17	148.36	117.23	111.44	105.80	100.30	94.95	89.75	65.94	45.79	29.31	16.48	7.33	1.83	0
Π_{Ch}^*	522.61	423.31	334.47	317.96	301.86	286.18	270.92	256.08	188.14	130.65	83.62	47.03	20.90	5.23	0
p_o^*	13.20	11.88	10.56	10.30	10.03	9.77	9.51	9.24	7.92	6.60	5.28	3.96	2.64	1.32	0
p_e^*	12.50	16.25	20.00	20.75	21.50	22.25	23.00	23.75	27.50	31.25	35.00	38.75	42.50	46.25	50.00
K_o^*	5.54	4.98	4.43	4.32	4.21	4.10	3.99	3.88	3.32	2.77	2.22	1.66	1.11	0.55	0
R															
Π_M^*	312.50	253.13	200.00	190.13	180.50	171.13	162.00	153.13	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R^*	156.25	126.56	100.00	95.06	90.25	85.56	81.00	76.56	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}^*	468.75	379.69	300.00	285.19	270.75	256.69	243.00	229.69	168.75	117.19	75.00	42.19	18.75	4.69	0
p_r^*	25.00	27.50	30.00	30.50	31.00	31.50	32.00	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
AO															
Π_M^*	312.50	253.13	200.00	190.69	182.64	175.68	169.67	164.49	148.01	141.59	140.14	140.11	140.11	140.11	140.11
Π_R^*	156.25	126.56	100.00	95.67	92.46	90.14	88.51	87.41	86.62	88.70	90.38	90.55	90.55	90.55	90.55
Π_{Ch}^*	468.75	379.69	300.00	286.36	275.10	265.82	258.18	251.91	234.63	230.29	230.51	230.66	230.66	230.66	230.66
p_o^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e^*	25.00	27.50	30.00	30.50	31.00	31.50	32.00	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
p_a^*	∞	∞	∞	29.43	28.93	28.48	28.08	27.73	26.45	25.77	25.49	25.47	25.47	25.47	25.47
K^*	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_o^*	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_a^*	0	0	0	1.13	2.14	3.03	3.83	4.55	7.10	8.46	9.01	9.06	9.06	9.06	9.06
A															
Π_M^*	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11	140.11
Π_R^*	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55	90.55
Π_{Ch}^*	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66	230.66
p_a^*	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47	25.47
K_a^*	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06	9.06
O															
Π_M^*	312.50	253.13	200.00	190.13	180.50	171.13	162.00	153.13	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R^*	156.25	126.56	100.00	95.06	90.25	85.56	81.00	76.56	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}^*	468.75	379.69	300.00	285.19	270.75	256.69	243.00	229.69	168.75	117.19	75.00	42.19	18.75	4.69	0
p_o^*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e^*	25.00	27.50	30.00	30.50	31.00	31.50	32.00	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
K_o^*	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
INT															
Π_{Ch}^*	625.00	506.25	400.00	382.26	368.29	357.19	348.35	341.30	322.88	318.55	318.33	318.33	318.33	318.33	318.33
K_a^*	0	0	0	4.02	7.29	9.96	12.18	14.02	19.58	21.64	21.93	21.93	21.93	21.93	21.93

Table 6 Equilibrium for Exponential Distribution

c	0	5	10	10.1	15	15.3	15.4	20	25	30	35	40	45	50
AOR														
Π_M	332.12	269.02	212.56	211.50	170.15	168.16	167.50	141.59	121.79	109.50	103.16	100.99	100.88	100.92
Π_R	179.02	145.01	114.57	114.00	93.21	92.28	92.01	83.62	78.52	76.09	74.96	74.18	73.40	73.20
Π_{Ch}	511.14	414.02	327.13	325.50	263.36	260.44	259.51	225.21	200.31	185.58	178.12	175.17	174.28	174.13
p_o	11.20	10.08	8.96	8.94	7.51	7.42	∞	∞	∞	∞	∞	∞	∞	∞
p_e	13.37	17.03	20.70	20.77	24.75	25.00	∞	∞	∞	∞	∞	∞	∞	∞
p_a	∞	∞	∞	29.64	28.53	28.48	28.45	26.79	25.37	24.37	23.77	23.53	23.54	23.57
K	4.70	4.23	3.76	3.75	3.30	3.27	3.26	4.35	5.45	6.35	7.00	7.35	7.44	7.44
K_o	4.70	4.23	3.76	3.68	0.22	0.05	0	0	0	0	0	0	0	0
K_a	0	0	0	0.08	3.07	3.22	3.26	4.35	5.45	6.35	7.00	7.35	7.44	7.44
AR														
Π_M	312.84	255.86	208.16	207.31	170.13	168.15	167.50	141.59	121.79	109.50	103.16	100.99	100.88	100.92
Π_R	156.55	129.16	107.89	107.53	92.94	92.24	92.01	83.62	78.52	76.09	74.96	74.18	73.40	73.20
Π_{Ch}	469.39	385.02	316.05	314.84	263.07	260.39	259.51	225.21	200.31	185.58	178.12	175.17	174.28	174.13
p_a	35.28	33.02	30.73	30.69	28.61	28.49	28.45	26.79	25.37	24.37	23.77	23.53	23.54	23.57
K_a	0.31	1.05	2.03	2.05	3.17	3.24	3.26	4.35	5.45	6.35	7.00	7.35	7.44	7.44
OR														
Π_M	332.12	269.02	212.56	211.49	162.74	159.96	159.04	119.56	83.03	53.14	29.89	13.28	3.32	0
Π_R	179.02	145.01	114.57	114.00	87.72	86.22	85.73	64.45	44.76	28.64	16.11	7.16	1.79	0
Π_{Ch}	511.14	414.02	327.13	325.50	250.46	246.18	244.77	184.01	127.79	81.78	46.00	20.45	5.11	0
p_o	11.20	10.08	8.96	8.94	7.84	7.77	7.75	6.72	5.60	4.48	3.36	2.24	1.12	∞
p_e	13.37	17.03	20.70	20.77	24.36	24.58	24.65	28.02	31.68	35.35	39.01	42.67	46.34	∞
K_o	4.70	4.23	3.76	3.75	3.29	3.26	3.25	2.82	2.35	1.88	1.41	0.94	0.47	0
R														
Π_M	312.50	253.13	200.00	199.00	153.13	150.51	149.65	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R	156.25	126.56	100.00	99.50	76.56	75.26	74.82	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}	468.75	379.69	300.00	298.50	229.69	225.77	224.47	168.75	117.19	75.00	42.19	18.75	4.69	0
AO														
Π_M	312.50	253.13	200.00	199.00	159.29	157.36	156.73	132.99	116.65	107.29	102.73	101.15	100.92	100.92
Π_R	156.25	126.56	100.00	99.50	82.75	82.09	81.88	75.27	72.57	72.12	72.55	73.04	73.21	73.21
Π_{Ch}	468.75	379.69	300.00	298.51	242.04	239.45	238.61	208.26	189.22	179.42	175.29	174.20	174.13	174.14
p_o	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e	25.00	27.50	30.00	30.05	32.50	32.65	32.70	35.00	37.50	40.00	42.50	45.00	47.50	50.00
p_a	25.00	27.50	30.00	29.94	27.55	27.43	27.40	25.93	24.86	24.17	23.78	23.60	23.57	23.57
K	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_o	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_a	0	0	0	0.06	2.58	2.71	2.75	4.44	5.76	6.63	7.15	7.39	7.44	7.44
A														
Π_M	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92	100.92
Π_R	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21	73.21
Π_{Ch}	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14	174.14
p_a	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57	23.57
K_a	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44	7.44
O														
Π_M	312.50	253.13	200.00	199.00	153.13	150.51	149.65	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R	156.25	126.56	100.00	99.50	76.56	75.26	74.82	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}	468.75	379.69	300.00	298.50	229.69	225.77	224.47	168.75	117.19	75.00	42.19	18.75	4.69	0
p_o	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e	25.00	27.50	30.00	30.05	32.50	32.65	32.70	35.00	37.50	40.00	42.50	45.00	47.50	50.00
K_o	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
INT														
Π_{Ch}	625.00	506.25	400.00	398.01	326.16	322.99	321.96	285.36	262.72	251.09	246.13	244.73	244.60	244.60
K_a	0	0	0	0.22	8.54	8.93	9.05	13.67	16.83	18.70	19.66	20.02	20.07	20.07

Table 7 **Equilibrium for Weibull(2) Distribution**

c	0	5	10	10.1	11.2	11.3	15	20	25	30	35	40	45	50
AOR														
Π_M	341.16	276.34	218.34	217.33	208.52	207.84	185.79	165.28	154.09	149.89	149.81	150.81	151.04	151.04
Π_R	180.80	146.45	115.71	115.16	110.23	109.98	102.45	97.16	95.12	93.72	91.41	88.51	87.55	87.54
Π_{Ch}	521.96	422.79	334.05	332.49	318.75	317.82	288.24	262.44	249.22	243.61	241.21	239.33	238.58	238.58
p_o	16.99	15.29	13.59	13.55	13.06	∞	∞	∞	∞	∞	∞	∞	∞	∞
p_e	10.22	14.20	18.18	18.26	19.26	∞	∞	∞	∞	∞	∞	∞	∞	∞
p_a	∞	∞	∞	31.78	31.78	31.75	30.52	29.07	28.04	27.52	27.49	27.74	27.85	27.85
K	5.02	4.52	4.02	4.01	3.90	3.92	4.98	6.32	7.42	8.15	8.48	8.50	8.46	8.46
K_o	5.02	4.52	4.02	2.84	0.03	0	0	0	0	0	0	0	0	0
K_a	0	0	0	1.16	3.87	3.92	4.98	6.32	7.42	8.15	8.48	8.50	8.46	8.46
AR														
Π_M	313.51	259.64	217.05	216.31	208.52	207.84	185.79	165.28	154.09	149.89	149.81	150.81	151.04	151.04
Π_R	157.02	131.64	113.51	113.22	110.24	109.98	102.45	97.16	95.12	93.72	91.41	88.51	87.55	87.54
Π_{Ch}	470.54	391.28	330.56	329.54	318.76	317.82	288.24	262.44	249.22	243.61	241.21	239.33	238.58	238.58
p_a	35.60	33.92	32.19	32.16	31.78	31.75	30.52	29.07	28.04	27.52	27.49	27.74	27.85	27.85
K_a	0.82	2.14	3.54	3.57	3.89	3.92	4.98	6.32	7.42	8.15	8.48	8.50	8.46	8.46
OR														
Π_M	341.16	276.34	218.34	217.25	205.44	204.38	167.17	122.82	85.29	54.59	30.70	13.65	3.41	0
Π_R	180.80	146.45	115.71	115.14	108.87	108.31	88.59	65.09	45.20	28.93	16.27	7.23	1.81	0
Π_{Ch}	521.96	422.79	334.05	332.39	314.31	312.69	255.76	187.91	130.49	83.51	46.98	20.88	5.22	0
p_o	16.99	15.29	13.59	13.55	13.18	13.15	11.90	10.20	8.49	6.79	5.09	3.40	1.70	∞
p_e	10.22	14.20	18.18	18.26	19.13	19.21	22.15	26.13	30.11	34.09	38.07	42.04	46.02	∞
K_o	5.02	4.52	4.02	4.01	3.90	3.89	3.51	3.01	2.51	2.01	1.51	1.00	0.50	0
R														
Π_M	312.50	253.13	200.00	199.00	188.18	187.21	153.13	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R	156.25	126.56	100.00	99.50	94.09	93.61	76.56	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}	468.75	379.69	300.00	298.50	282.27	280.82	229.69	168.75	117.19	75.00	42.19	18.75	4.69	0
AO														
Π_M	312.50	253.13	200.00	199.06	190.58	189.91	171.72	159.01	153.41	151.45	151.06	151.04	151.04	151.04
Π_R	156.25	126.56	100.00	99.55	95.86	95.59	89.64	87.26	87.05	87.34	87.52	87.54	87.54	87.54
Π_{Ch}	468.75	379.69	300.00	298.61	286.44	285.51	261.37	246.26	240.46	238.80	238.58	238.58	238.58	238.58
p_o	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e	25.00	27.50	30.00	30.05	30.60	30.65	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
p_a	25.00	27.50	30.00	29.98	29.72	29.69	29.00	28.39	28.05	27.90	27.85	27.85	27.85	27.85
K	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_o	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_a	0	0	0	0.89	3.02	3.13	5.70	7.28	8.03	8.36	8.45	8.46	8.46	8.46
A														
Π_M	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04	151.04
Π_R	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54	87.54
Π_{Ch}	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58	238.58
p_a	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85	27.85
K_a	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46	8.46
O														
Π_M	312.50	253.13	200.00	199.00	188.18	187.21	153.13	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R	156.25	126.56	100.00	99.50	94.09	93.61	76.56	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}	468.75	379.69	300.00	298.50	282.27	280.82	229.69	168.75	117.19	75.00	42.19	18.75	4.69	0
p_o	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e	25.00	27.50	30.00	30.05	30.60	30.65	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
K_o	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
INT														
Π_{Ch}	625.00	506.25	400.00	398.17	382.98	381.85	354.42	338.77	333.16	331.66	331.46	331.45	331.45	331.45
K_a	0	0	0	2.51	8.37	8.68	15.09	18.48	19.84	20.31	20.40	20.40	20.40	20.40

Table 8 **Equilibrium for Weibull(3) Distribution**

c	0	5	10	10.1	10.2	10.3	10.4	15	20	25	30	35	40	45	50
AOR															
Π_M	345.56	279.90	221.16	220.27	219.49	218.78	218.11	192.50	175.72	168.75	168.38	170.50	171.52	171.54	171.54
Π_R	182.08	147.48	116.53	115.99	115.51	115.05	114.76	106.30	102.78	101.82	99.62	94.99	91.30	91.12	91.12
Π_{Ch}	527.64	427.38	337.69	336.26	335.00	333.83	332.86	298.80	278.50	270.57	268.01	265.49	262.83	262.66	262.66
p_o	19.48	17.53	15.59	15.53	15.48	15.43	∞	∞	∞	∞	∞	∞	∞	∞	∞
p_e	8.39	12.55	16.71	16.81	16.90	17.00	∞	∞	∞	∞	∞	∞	∞	∞	∞
p_a	∞	∞	∞	32.32	32.33	32.34	32.32	30.89	29.54	28.66	28.45	28.79	29.18	29.20	29.20
K	5.42	4.88	4.34	4.33	4.32	4.31	4.32	5.77	7.22	8.33	8.94	9.05	8.94	8.93	8.93
K_o	5.42	4.88	4.34	1.44	0.69	0.17	0	0	0	0	0	0	0	0	0
K_a	0	0	0	2.89	3.62	4.13	4.32	5.77	7.22	8.33	8.94	9.05	8.94	8.93	8.93
AR															
Π_M	313.79	261.23	220.82	220.13	219.45	218.78	218.11	192.50	175.72	168.75	168.38	170.50	171.52	171.54	171.54
Π_R	157.22	132.68	115.78	115.52	115.26	115.01	114.76	106.30	102.78	101.82	99.62	94.99	91.30	91.12	91.12
Π_{Ch}	471.01	393.90	336.60	335.65	334.72	333.79	332.86	298.80	278.50	270.57	268.01	265.49	262.83	262.66	262.66
p_a	35.62	34.04	32.45	32.41	32.38	32.35	32.32	30.89	29.54	28.66	28.45	28.79	29.18	29.20	29.20
K_a	1.03	2.60	4.19	4.22	4.26	4.29	4.32	5.77	7.22	8.33	8.94	9.05	8.94	8.93	8.93
OR															
Π_M	345.56	279.90	221.16	220.05	218.95	217.85	216.76	169.32	124.40	86.39	55.29	31.10	13.82	3.46	0
Π_R	182.08	147.48	116.53	115.95	115.37	114.79	114.21	89.22	65.54	45.52	29.13	16.39	7.28	1.82	0
Π_{Ch}	527.64	427.38	337.69	336.00	334.32	332.64	330.97	258.54	189.94	131.91	84.42	47.49	21.11	5.28	0
p_o	19.48	17.53	15.59	15.54	15.50	15.47	15.42	13.64	11.68	9.74	7.79	5.84	3.89	1.95	∞
p_e	8.39	12.55	16.71	16.80	16.88	16.96	17.05	20.87	25.04	29.20	33.36	37.52	41.68	45.84	∞
K_o	5.42	4.88	4.34	4.33	4.32	4.31	4.29	3.80	3.25	2.71	2.17	1.63	1.08	0.54	0
R															
Π_M	312.50	253.13	200.00	199.00	198.01	197.01	196.02	153.13	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R	156.25	126.56	100.00	99.50	99.00	98.51	98.01	76.56	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}	468.75	379.69	300.00	298.50	297.01	295.52	294.03	229.69	168.75	117.19	75.00	42.19	18.75	4.69	0
AO															
Π_M	312.50	253.13	200.00	199.16	198.41	197.71	197.05	180.23	173.78	171.87	171.56	171.54	171.54	171.54	171.54
Π_R	156.25	126.56	100.00	99.60	99.26	98.94	98.64	92.57	91.18	91.06	91.11	91.12	91.12	91.12	91.12
Π_{Ch}	468.75	379.69	300.00	298.77	297.67	296.65	295.69	272.81	264.95	262.93	262.66	262.66	262.66	262.66	262.66
p_o	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e	25.00	27.50	30.00	30.05	30.10	30.15	30.20	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
p_a	25.00	27.50	30.00	29.99	29.98	29.97	29.95	29.55	29.32	29.23	29.20	29.20	29.20	29.20	29.20
K	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_o	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
K_a	0	0	0	2.17	2.73	3.12	3.43	7.34	8.45	8.84	8.93	8.93	8.93	8.93	8.93
A															
Π_M	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54	171.54
Π_R	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12	91.12
Π_{Ch}	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66	262.66
p_a	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20	29.20
K_a	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93	8.93
O															
Π_M	312.50	253.13	200.00	199.00	198.01	197.01	196.02	153.13	112.50	78.13	50.00	28.13	12.50	3.13	0
Π_R	156.25	126.56	100.00	99.50	99.00	98.51	98.01	76.56	56.25	39.06	25.00	14.06	6.25	1.56	0
Π_{Ch}	468.75	379.69	300.00	298.50	297.01	295.52	294.03	229.69	168.75	117.19	75.00	42.19	18.75	4.69	0
p_o	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
p_e	25.00	27.50	30.00	30.05	30.10	30.15	30.20	32.50	35.00	37.50	40.00	42.50	45.00	47.50	50.00
K_o	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
INT															
Π_{Ch}	625.00	506.25	400.00	398.40	397.02	395.75	394.57	369.25	361.91	360.27	360.10	360.10	360.10	360.10	360.10
K_a	0	0	0	5.37	6.74	7.70	8.45	17.36	19.44	20.01	20.10	20.11	20.11	20.11	20.11