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**Commentary on Discussion of ‘On the theory of standing waves in
tyres at high vehicle speeds’ by V.V. Krylov and O. Gilbert,
Journal of Sound and Vibration 329 (2010) 4398-4408**

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1. Introduction

In the discussion [1], Graham questions the suitability of the model of a one-dimensional tensioned beam on elastic foundation subjected to a moving load that was used in Ref. [2] to develop a new physically explicit theory of the well-known phenomenon of standing waves in tyres at high vehicle speeds (see paper [2] for a more detailed description of this phenomenon). His main reservation is that such a model does not take into account the effects of rotation, namely the effects of the Coriolis and centripetal accelerations, which he considers to be crucially important for the existence of standing waves in tyres. It will be argued that the effects of rotation are not important for the existence of standing waves in rotating tyres, and thus the tensioned beam model of tyre standing waves proposed in Ref. [2] is appropriate.

2. Discussion

Note that the issue of the effects of rotation on tyre standing waves is not new, and Graham's arguments (see Ref. [1]) are largely based on the results and conclusions of the earlier theoretical works by Endo et al. [3] and Huang and Soedel [4]. Both these papers argue that the effects of rotation are crucially important for the existence of the phenomenon of

standing waves in rotating rings (that can be used for modelling vehicle tyres - VVK). In particular, Huang and Soedel [4], who refer to the paper [3], state (see [4], page 274) that "it was shown for the stationary load case that the rotating ring solution yields no resonant speeds", which means that there is no resonant excitation of a rotating ring by a stationary force at any speed of the ring's rotation. More specifically, Huang and Soedel identify two main rotation-related mechanisms influencing the vibration response of a rotating ring to a stationary force. These are centrifugal (centripetal) tension and Coriolis acceleration. The authors show (see [4], page 273) that centrifugal tension considered separately does not lead to the disappearance of standing waves in a rotating ring, whereas "the Coriolis acceleration imposes more significant effects on the rotating ring than does the centrifugal force" and prevents the existence of standing waves in a rotating ring at any speed of rotation.

Regarding the application of this result to vehicle tyres, one comes to the inevitable conclusion that there should be no tyre standing waves on rotating tyres at any vehicle speed. Note however, that this obvious conclusion is not drawn by the authors of the paper [4] themselves, apparently because it contradicts numerous experiments and practical observations on rotating tyres, either on rotating drums or on roads, that demonstrate that standing waves on rotating tyres at high vehicle speeds do exist (see e.g. [5, 6]). It is interesting that even the word 'tyre' is not mentioned in Ref. [4] at all, let alone 'standing waves' in tyres. The impression is that the authors of paper [4] published in 1987 are not confident enough about their results and therefore prefer to present them as purely academic and not associated with any specific applications.

Note in this connection that the results of Ref. [4] are also in contradiction with the earlier paper of one of the authors, Soedel [7], dealing with a thin shell model of a tyre that does not take into account the effects of rotation. According to the paper [7], in this case the resonant excitation of the ring does take place at load speeds exceeding a certain critical speed equal to a minimum wave velocity. Note that in this paper, that was published 12 years earlier than paper [4], this resonant excitation is directly related to the phenomenon of tyre standing waves. Moreover, the author even calculates the value of the vehicle critical speed for a belted radial tyre (MichelinX135 13) and states that the calculated value of 137.6 km/h falls "inside the typical range of standard tire critical speeds" (see Ref. [7], page 243). It would have been natural if Huang and Soedel [4] have discussed the above-mentioned striking discrepancy between the predictions of the papers [7] and [4]. However, they do not even mention the paper [7] in the reference list of paper [4], which certainly does not help members of the tyre research community to appreciate the problem. Regretfully, the same ambiguity about this

issue remains in the latest 3rd edition of the book of Soedel "Vibrations of Shells and Plates" [8] published in 2004 (see pages 270 and 428 for discussions of Refs. [7] and [4] respectively).

Not mentioned in [4] is also the important theoretical paper of Padovan [9] on standing waves in rotating tyres published in 1976, even though Huang and Soedel are well familiar with his work and cite two of his earlier papers of a more general nature published in 1971 and 1973. The predictions of the paper of Padovan [9] are opposite to those of the paper by Huang and Soedel [4]. Namely, he shows that the effects of rotation do not change the predictions significantly, and tyre standing waves are being generated at high vehicle speeds. This agrees with numerous experiments and practical observations on rotating tyres (see e.g. [5, 6]). Obviously, a comparative theoretical analysis would be needed to understand the reasons for the discrepancy between the predictions made in papers [4] and [9], which is beyond the scope of this commentary.

Surprisingly, Graham in his discussion [1] says (see the footnote on page 6030) that, in his view, the results of the paper of Huang and Soedel [4] "carry the greater authority" than the results of the paper by Padovan [9]. This is in spite of the above-mentioned facts that the former contradicts the existing experiments on rotating tyres, whereas the latter does not. In this author's opinion, it is the experiments that bring ultimate judgement to any theoretical paper. However sophisticated a theoretical paper may be and however high the authority of the authors are, if the paper contradicts the experiments it should be considered as wrong.

It is interesting to note that in the paper of Lin and Soedel [10] published in 1988 and not cited by Graham [1], the authors, who now mention explicitly the problem of standing waves in tyres, state that tyre standing waves and critical vehicle speeds do exist, which is quite opposite to the earlier predictions of the paper [4] co-authored by Soedel. The authors attribute this radical change to the fact that in Ref. [10] the strain nonlinearity has been taken into account properly. Although Ref. [4] is present in the reference list of the paper [10], there is no discussion of this paper in Ref. [10] even though its main conclusions are in direct contradiction with the paper [10].

3. Conclusions

In the light of the above, this author believes that the point of view expressed by Graham [1] regarding the alleged unsuitability of the one-dimensional model of a tensioned beam under a moving load proposed in Ref. [2] to describe the phenomenon of standing waves in

rotating tyres is incorrect because it is based on controversial predictions of the theoretical papers [3, 4] that contradict the existing experiments on rotating tyres. In this author's opinion, the effects of rotation are not important for the existence of tyre standing waves, although they may influence the values of the predicted critical vehicle speed and the wave forms. This implies that the model of a one-dimensional tensioned beam on elastic foundation subjected to a moving load [2] is suitable for the description of the phenomenon of standing waves in rotating tyres. Note in this connection that the use of an infinite beam in the model of Ref. [2], instead of a finite ring, is in agreement with the results of the paper [11], which is also noted in the discussion [1]. Another confirmation of the suitability of the model of Ref. [2] is its good agreement with the experiments and with the existing numerical calculations. In particular, the values of critical vehicle speed and their dependence on tyre parameters and air inflation pressure predicted in [2] agree well with the results of the numerical calculations [12] that take account of the tyre rotation.

References

- [1] W.R. Graham, Discussion of 'On the theory of standing waves in tyres at high vehicle speeds' by V.V. Krylov and O. Gilbert, *Journal of Sound and Vibration* 329 (2010) 4398-4408. *Journal of Sound and Vibration* 332 (2013) 6029-6031.
- [2] V.V. Krylov, O. Gilbert, On the theory of standing waves in tyres at high vehicle speeds. *Journal of Sound and Vibration* 329 (2010) 4398-4408.
- [3] M. Endo, K. Hatamura, M. Sakata, O. Taniguchi, Flexural vibration of a thin rotating ring. *Journal of Sound and Vibration* 92 (1984) 261–272.
- [4] S.C. Huang, W. Soedel, Effects of Coriolis acceleration on the free and forced in-plane vibrations of rotating rings on elastic foundation. *Journal of Sound and Vibration* 115 (1987) 253–274.
- [5] R.A. Brockman, J.H. Champion, J.P. Medzorian, Finite element analysis of tire critical speeds. *Computers and Structures* 43 (1992) 581-593.
- [6] A. Chatterjee, J.P. Cusumano, J.D. Zolock, On contact-induced standing waves in rotating tires: experiment and theory. *Journal of Sound and Vibration* 227 (1999) 1049-1081.
- [7] W. Soedel, On the dynamic response of rolling tyres according to thin shell approximations. *Journal of Sound and Vibration* 41 (1975) 233–246.

- [8] W. Soedel, *Vibrations of Shells and Plates*, 3rd Edition, Marcel Dekker, New York, 2004.
- [9] J. Padovan, On viscoelasticity and standing waves in tires. *Tire Science and Technology* 4 (1976) 233-246.
- [10] J.L. Lin, W. Soedel, On the critical speeds of rotating thick or thin rings. *Mechanics of Structures and Machines: An International Journal* 16 (1988) 439-483.
- [11] A.V. Metrikine, M.V. Tochilin, Steady-state vibrations of an elastic ring under a moving load. *Journal of Sound and Vibration* 232 (2000) 511–524.
- [12] J.R. Cho, K.W. Kim, H.S. Jeong, Numerical investigation of tire standing wave using 3-D patterned tire model. *Journal of Sound and Vibration* 305 (2007) 795–807.