Supporting Information for

Circular Dichroism Metamirrors with Near-Perfect Extinction

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In this supplemental material, we derive the reflection matrix for circular dichroism metamirrors and predict the requirements of structural symmetry for different cases. The detail of structural optimization based on the transfer matrix method is then presented. Subsequently, we discuss the influence of interaction between two metamaterial interfaces and explain why the performance of the designed metamirror is insensitive to the incident angle.

Section A: Reflection matrix for perfect circular dichroism metamirrors

The reflection matrix $\mathbf{R}$ connects the incident and reflected fields, which are generally complex values, in the following format:

$$\begin{pmatrix} E_r^x \\ E_r^y \end{pmatrix} = \begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} \begin{pmatrix} E_i^x \\ E_i^y \end{pmatrix} = \mathbf{R} \begin{pmatrix} E_i^x \\ E_i^y \end{pmatrix}. \quad (\text{S1})$$

By changing the Cartesian base to circular base, the reflection matrix for circular polarization

states is attained ¹

$$\mathbf{R}_{circ} = \begin{pmatrix} r_{++} & r_{+-} \\ r_{-+} & r_{--} \end{pmatrix} = \Lambda^{-1} \mathbf{R} \Lambda = \frac{1}{2} \begin{pmatrix} r_{xx} + r_{yy} + i(r_{xy} - r_{yx}) & r_{xx} - r_{yy} - i(r_{xy} + r_{yx}) \\ r_{xx} - r_{yy} + i(r_{xy} + r_{yx}) & r_{xx} + r_{yy} - i(r_{xy} - r_{yx}) \end{pmatrix}, \quad (\text{S2})$$

where $\Lambda = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}$ is the change of the base matrix. Here, ‘+’ and ‘-’ are defined as the clockwise and counterclockwise circularly polarized waves when viewed along +z direction, respectively. Consequently, each reflection coefficient has different physical meaning compared with transmission cases, because the wavevectors are in opposite directions for transmitted and reflected light. For example, r_{++} and r_{-+} are the cross- and co-polarized reflection coefficient for RCP incident light, respectively. The incident waves are assumed to propagate along +z direction. Therefore, in the main text, the reflection coefficients are defined as $r_{LR} = r_{++}$, $r_{RR} = r_{-+}$, $r_{LL} = r_{+-}$, $r_{RL} = r_{--}$.

For metamirrors absorbing RCP incidences only, we can consider following three cases.

Case (1): $r_{++} = r_{-+} = r_{--} = 0$, $|r_{+-}| = 1$.

In this case, the incident RCP waves are totally absorbed and the LCP waves can be reflected without reversing the handedness. The only solution for this case is given by

$$\begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} = \frac{e^{i\alpha}}{2} \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix}, \quad (\text{S3})$$

Where α is an arbitrary phase shift. This is exactly the case discussed in the main text.

Case (2): $r_{++} = r_{-+} = r_{+-} = 0$, $|r_{--}| = 1$.

In this case, incident LCP waves will be converted to RCP states in reflection. A unique solution is then given by

$$\begin{pmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{pmatrix} = \frac{e^{i\alpha}}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}, \quad (\text{S4})$$

Case (3): $r_{++} = r_{-+} = 0$, $|r_{+-}| \neq 0$ and $|r_{--}| \neq 0$

In this case, $r_{xx} \neq \pm r_{yy}$ and $r_{xy} \neq \pm r_{yx}$. Only a part of the incident LCP waves can be converted to RCP modes in reflection. No unique solution existed in this case.

Section B: Consideration of the structural symmetry

Two kinds of symmetries are considered in this paper. One is mirror symmetry with respect to an incident plane, and the other is rotational symmetry with respect to the z -axis. As mentioned in the main text, the conditions for rotational and mirror symmetries are

$$\sin(\varphi) \begin{pmatrix} r_{xy} + r_{yx} & r_{yy} - r_{xx} \\ r_{yy} - r_{xx} & -r_{xy} - r_{yx} \end{pmatrix} = \mathbf{0} \quad (\text{S5})$$

and

$$\begin{cases} r_{xy} - r_{yx} = 0 \\ \sin(2\varphi)(r_{xx} - r_{yy}) + 2\cos(2\varphi)r_{xy} = 0 \end{cases} \quad (\text{S6})$$

respectively.

Following the procedures discussed in the main text, we can draw the conclusions on the structural symmetry for the preceding three cases in section A.

Case (1): $r_{++} = r_{-+} = r_{+-} = 0$, $|r_{+-}| = 1$. Neither n -fold rotational ($n > 2$) symmetry nor mirror symmetry is allowed.

Case (2): $r_{++} = r_{-+} = r_{+-} = 0$, $|r_{--}| = 1$. Rotational symmetry is allowed but mirror symmetry is not permitted.

Case (3): $r_{++} = r_{-+} = 0$, $|r_{+-}| \neq 0$ and $|r_{--}| \neq 0$. Neither n -fold rotational ($n > 2$) symmetry nor mirror symmetry is allowed.

Section C: Structure optimization based on transfer matrix approach

For a single metamaterial interface between two media n and m , the 4×4 transfer matrix

$\mathbf{M}_{mn}$ relates the forward and backward propagating fields on each side of the interface ²

$$\begin{pmatrix} E_{mx}^{(f)}, E_{my}^{(f)}, E_{mx}^{(b)}, E_{my}^{(b)} \end{pmatrix}^T = \mathbf{M}_{mn} \begin{pmatrix} E_{nx}^{(f)}, E_{ny}^{(f)}, E_{nx}^{(b)}, E_{ny}^{(b)} \end{pmatrix}^T. \quad (\text{S7})$$

The transfer matrix can be expressed in terms of the reflection and transmission coefficients

$$\mathbf{M}_{mn} = \begin{pmatrix} 1 & 0 & -r_{mx;mx} & -r_{mx;my} \\ 0 & 1 & -r_{my;mx} & -r_{my;my} \\ 0 & 0 & t_{nx;mx} & t_{nx;my} \\ 0 & 0 & t_{ny;mx} & t_{ny;my} \end{pmatrix}^{-1} \begin{pmatrix} t_{mx;n\sigma} & t_{mx;ny} & 0 & 0 \\ t_{my;n\sigma} & t_{my;ny} & 0 & 0 \\ -r_{nx;n\sigma} & -r_{nx;ny} & 1 & 0 \\ -r_{ny;n\sigma} & -r_{ny;ny} & 0 & 1 \end{pmatrix}, \quad (\text{S8})$$

where $r_{m\sigma';n\sigma}$ and $t_{m\sigma';n\sigma}$ are the reflection and transmission coefficients. The subscripts indicate that light is incident from medium n with polarization $\sigma (= x, y)$ and propagates to medium m with polarization σ' . Here the subscripts (f) and (b) denote the forward and backward propagation lights. For a structure composed of several metamaterial interfaces surrounded by homogeneous dielectrics, the overall transfer matrix is given by $\mathbf{M} = \dots \mathbf{M}_{43} \mathbf{P}_3 \mathbf{M}_{32} \mathbf{P}_2 \mathbf{M}_{21}$, where $\mathbf{P}_a = \text{diag}(e^{ik_0 n_a d_a}, e^{ik_0 n_a d_a}, e^{-ik_0 n_a d_a}, e^{-ik_0 n_a d_a})$ is the propagation matrix within a homogeneous medium a (thickness d_a , refractive index n_a), and k_0 is the wave number in free space.

Figure S1. Comparison between the simulated absorption spectra and the theoretical results from the transfer matrix method (TMM).

The calculation accuracy from transfer matrix method (TMM) is presented in Fig. S1. As the separation distance between the two layers of metamaterials is gradually reduced to deep subwavelength scales, the calculated absorption starts to deviate from the numerical simulation

result (CST Microwave Studio). This is because of the emergence of the interaction between the top and bottom metallic structures. Nevertheless, the theoretical prediction still shows the correct trend of the absorption for RCP states.

Figure S2. The dependence of the reflection coefficients of the bottom layer metamaterial on the incident angle.

To better understand the physical mechanism of the angular insensitivity discussed in the main text, the reflection coefficients for the bottom metamaterial interface are plotted in Fig. S2. It is noticeable that the resonance frequency for the bottom rod does not change even when the incident angle is up to 60 degrees. Consequently, the resonance frequency of the total structure is relatively insensitive to oblique illuminations.

Figure S3. The dependence of the absorption of the metamirror on the twisted angle β . (a) RCP incidence. (b) LCP incidence.

Since $\pi/4$ is not the only twisted angle to break the rotational and mirror symmetries, we further investigate the performance of the metamirror for other twisted angles β (from 15° to 75°). The simulation results plotted in Fig. S3 clearly indicate that the best performance occurs at $\beta = 45^\circ$, where we obtain the maximum absorption for RCP light and minimum absorption for LCP light at the operating wavelength. This is consistent with the predicted phenomenon of interference in the main text, which is explained by the Pancharatnam-Berry phase. Within about a range of 30 degrees centered at 45° (that is, $30^\circ < \beta < 60^\circ$), we can achieve a reasonably good circular dichroism contrast.

References

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