Online Appendix

Bayesian Model Averaging for Dynamic Panels with an Application to a Trade Gravity Model

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Appendix A: Robustness using non-Gaussian errors

In this Appendix we relax the normality assumption for the idiosyncratic random error, v_{it} , and perform another Monte Carlo experiment to evaluate the model averaging and model selection properties. Results are discussed in Section 4 of the paper.

Table A1. Posterior probability of the true model
LIBMA summary statistics for various N , α , and σ_v^2

<i>Sample</i>	α	0.95			0.50		
	σ_v^2	0.05	0.10	0.20	0.05	0.10	0.20
<i>N=200</i>							
Mean		0.224	0.109	0.044	0.162	0.084	0.031
Variance		0.021	0.012	0.004	0.025	0.015	0.005
Q1		0.094	0.025	0.008	0.019	0.002	0.000
Median		0.222	0.071	0.023	0.115	0.023	0.003
Q3		0.342	0.161	0.053	0.283	0.118	0.023
<i>N=500</i>							
Mean		0.459	0.415	0.253	0.437	0.376	0.243
Variance		0.023	0.025	0.028	0.032	0.036	0.035
Q1		0.371	0.315	0.110	0.322	0.233	0.069
Median		0.497	0.447	0.231	0.487	0.412	0.211
Q3		0.580	0.543	0.386	0.582	0.534	0.397
<i>N=2000</i>							
Mean		0.653	0.632	0.643	0.640	0.643	0.651
Variance		0.021	0.025	0.025	0.026	0.025	0.024
Q1		0.607	0.573	0.585	0.592	0.595	0.598
Median		0.703	0.684	0.703	0.700	0.699	0.708
Q3		0.754	0.747	0.751	0.753	0.754	0.757

Notes:

1. The table presents means, variances, Q1, median (Q2), and Q3 of the posterior probability of the data generating process (true model) across the Monte Carlo replications. Higher values indicate that the true model is favored. We consider sample sizes $N = 200, 500, 2000$, two values for the coefficient of the lagged dependent variable, $\alpha = 0.95, 0.50$, and three values for the variance of the idiosyncratic random error, $\sigma_v^2 = 0.05, 0.10, 0.20$.
2. The error terms are constructed using discrete distributions (see Section 4.1.).

Table A2. Posterior probability ratio of true model versus best among the rest
LIBMA summary statistics for various N , α , and σ_v^2

<i>Sample</i>	α	0.95			0.50		
	σ_v^2	0.05	0.10	0.20	0.05	0.10	0.20
<i>N=200</i>							
Mean		1.587	0.828	0.350	1.205	0.662	0.254
Variance		1.767	1.043	0.348	1.813	1.168	0.420
Q1		0.463	0.168	0.053	0.099	0.011	0.001
Median		1.235	0.423	0.173	0.639	0.141	0.016
Q3		2.499	1.081	0.357	1.975	0.812	0.162
<i>N=500</i>							
Mean		3.384	3.050	1.783	3.258	2.721	1.715
Variance		4.835	4.355	3.257	5.489	4.810	3.590
Q1		1.532	1.318	0.437	1.205	0.867	0.258
Median		3.065	2.705	1.118	2.862	2.216	1.044
Q3		5.035	4.584	2.630	5.069	4.278	2.487
<i>N=2000</i>							
Mean		6.980	6.410	6.992	6.828	6.824	7.081
Variance		18.234	17.732	19.169	19.496	18.475	19.192
Q1		3.360	2.956	3.035	3.138	3.257	3.221
Median		6.676	5.738	6.824	6.335	6.464	6.762
Q3		10.195	9.520	10.229	9.999	10.021	10.578

Notes:

1. The table presents means, variances, Q1, median (Q2), and Q3 of the of the ratio of the posterior model probability of the true model to the highest posterior probability of all the other models (excluding the true model) across the Monte Carlo replications. Higher values indicate that the true model is favored. We consider sample sizes $N = 200, 500, 2000$, two values for the coefficient of the lagged dependent variable, $\alpha = 0.95, 0.50$, and three values for the variance of the idiosyncratic random error, $\sigma_v^2 = 0.05, 0.10, 0.20$.
2. The error terms are constructed using discrete distributions (see Section 4.1.).

Table A3. Probability of retrieving the true model
LIBMA summary statistics for various N , α , and σ_v^2

<i>Sample</i>	α	0.95			0.50		
	σ_v^2	0.05	0.10	0.20	0.05	0.10	0.20
<hr/>							
<i>N=200</i>							
% Correct		56	27	8	42	22	7
<i>N=500</i>							
% Correct		85	80	53	79	72	51
<i>N=2000</i>							
% Correct		93	92	93	92	92	93

Notes:

1. The table presents the percentage of the Monte Carlo replications that the true model has the highest posterior probability. Higher percentages suggest that the methodology recovers the true model more often. We consider sample sizes, $N = 200, 500, 2000$, two values for the coefficient of the lagged dependent variable, $\alpha = 0.95, 0.50$, and three values for the variance of the idiosyncratic random error, $\sigma_v^2 = 0.05, 0.10, 0.20$.

2. The error terms are constructed using discrete distributions (see Section 4.1.).

Table A4. Model recovery: medians and variances of posterior inclusion probability for each variable
True model vs. BMA posterior inclusion probability for various N , α , and σ_v^2

Sample	α σ_v^2	0.95						0.50					
		0.05			0.20			0.10			0.20		
		Model	Median	Variance	Median	Variance	Median	Variance	Median	Variance	Median	Variance	
<i>N=200</i>													
yt-1	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00103	1.00000	0.00000	1.00000	0.00001
x1	1	0.98533	0.03332	0.84439	0.07089	0.54259	0.08866	0.94050	0.05592	0.69674	0.08352	0.46005	0.08605
x2	0	0.18906	0.01873	0.19277	0.02222	0.18649	0.02084	0.18470	0.01901	0.19136	0.02213	0.18425	0.02000
x3	0	0.18712	0.01748	0.18462	0.02122	0.17957	0.01710	0.18935	0.02013	0.18396	0.01668	0.18991	0.02314
x4	1	0.98708	0.02926	0.83962	0.07696	0.49877	0.08407	0.94699	0.05413	0.63426	0.08962	0.37623	0.07540
x5	0	0.18621	0.01949	0.18930	0.02272	0.18612	0.02089	0.18920	0.01543	0.19009	0.01948	0.18938	0.02186
x6	1	0.96697	0.04240	0.78891	0.08121	0.47078	0.08273	0.91206	0.05738	0.64015	0.08609	0.33444	0.07223
w1	0	0.19013	0.04195	0.21127	0.05684	0.26817	0.05852	0.13175	0.03713	0.10253	0.04647	0.06670	0.04599
w2	1	0.98807	0.05995	0.96496	0.07656	0.95082	0.06951	0.64884	0.15207	0.27934	0.15240	0.09806	0.12459
<i>N=500</i>													
yt-1	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000
x1	1	1.00000	0.00000	0.99975	0.00718	0.96429	0.04964	1.00000	0.00017	0.99984	0.01012	0.98394	0.04787
x2	0	0.12575	0.02023	0.12433	0.01388	0.12599	0.01826	0.12894	0.02064	0.12785	0.02530	0.12986	0.02306
x3	0	0.12148	0.01611	0.12167	0.01720	0.12262	0.01699	0.12757	0.02872	0.13145	0.02755	0.12779	0.02544
x4	1	1.00000	0.00003	0.99964	0.00677	0.95426	0.05240	1.00000	0.00041	0.99959	0.01251	0.96482	0.06551
x5	0	0.12695	0.01818	0.12891	0.01844	0.12531	0.01848	0.12775	0.02117	0.13564	0.02351	0.12837	0.02320
x6	1	1.00000	0.00025	0.99865	0.01006	0.93790	0.06465	1.00000	0.00151	0.99912	0.01641	0.90998	0.08230
w1	0	0.11931	0.01722	0.12270	0.02526	0.12990	0.02686	0.10587	0.02217	0.10632	0.02910	0.10812	0.03094
w2	1	1.00000	0.00000	1.00000	0.00059	0.99995	0.00698	0.99997	0.01091	0.99912	0.04088	0.98931	0.08415
<i>N=2000</i>													
yt-1	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000
x1	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000
x2	0	0.06637	0.00923	0.06548	0.00837	0.06676	0.01228	0.06620	0.01005	0.06804	0.01255	0.06577	0.01074
x3	0	0.06416	0.01034	0.06834	0.01250	0.06516	0.00777	0.06966	0.01613	0.06630	0.00990	0.06572	0.00927
x4	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00002
x5	0	0.06750	0.00914	0.07325	0.01962	0.06616	0.01124	0.06694	0.01088	0.06875	0.01403	0.06504	0.01124
x6	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00001	1.00000	0.00000	1.00000	0.00000	1.00000	0.00005
w1	0	0.06465	0.01128	0.06403	0.00890	0.06718	0.01530	0.06199	0.01127	0.06351	0.01194	0.06393	0.01324
w2	1	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000	1.00000	0.00000

Notes:

1. The table presents median and variances of the posterior inclusion probabilities for all the variables considered across the Monte Carlo replications. Higher (lower) values of the inclusion probability for a given variable suggest that the data favors (does not favor) the inclusion of that variable.
2. We consider sample sizes $N = 200, 500, 2000$, two values for the coefficient of the lagged dependent variable, $\alpha = 0.95, 0.50$, and three values for the variance of the idiosyncratic random error, $\sigma_v^2 = 0.05, 0.10, 0.20$.
3. The true model is shown in column 2. The universe of 9 potential explanatory variables includes the lagged dependent variable (y_{t-1}), 6 exogenous ($x1 - x6$), and 2 endogenous ($w1$ and $w2$) variables. A value of 1 (0) indicates that the variable is included (excluded).
3. The error terms are constructed using discrete distributions (see Section 4.1.).

Table A5. Model recovery: medians and variances of estimated parameter values
True model vs. BMA coefficients' estimated values for various N , α , and σ_v^2

Sample	α σ_v^2	0.95						0.50						
		0.05			0.10			0.05			0.10			
		Median	Variance	True Value	Median	Variance	True Value	Median	Variance	True Value	Median	Variance	True Value	
N=200														
yt-1	0.95	0.95265	0.00003	0.95088	0.00001	0.95004	0.00001	0.50	0.54517	0.02128	0.58207	0.01934	0.64121	0.01742
x1	0.05	0.04877	0.00029	0.04260	0.00059	0.02708	0.00092	0.05	0.04642	0.00044	0.03736	0.00076	0.02658	0.00135
x2	0.00	0.00000	0.00004	0.00019	0.00009	-0.00012	0.00016	0.00	0.00034	0.00006	0.00031	0.00017	0.00056	0.00029
x3	0.00	0.00003	0.00004	0.00024	0.00008	-0.00006	0.00012	0.00	-0.00038	0.00007	-0.00021	0.00011	0.00031	0.00038
x4	-0.05	-0.05008	0.00028	-0.04235	0.00060	-0.02513	0.00083	-0.05	-0.04809	0.00045	-0.03288	0.00076	-0.02041	0.00094
x5	0.00	-0.00019	0.00004	0.00065	0.00009	0.00011	0.00017	0.00	0.00004	0.00005	0.00051	0.00014	-0.00025	0.00039
x6	0.05	0.04781	0.00035	0.04159	0.00066	0.02405	0.00089	0.05	0.04483	0.00048	0.03542	0.00087	0.01822	0.00102
w1	0.00	0.00241	0.00053	0.00393	0.00124	0.00591	0.00173	0.00	0.00245	0.00034	0.00232	0.00082	0.00164	0.00104
w2	0.13	0.14234	0.00168	0.13564	0.00206	0.13346	0.00206	0.13	0.10195	0.00457	0.04429	0.00481	0.01747	0.00432
N=500														
yt-1	0.95	0.95153	0.00001	0.95077	0.00000	0.95017	0.00000	0.50	0.47206	0.00211	0.46203	0.00390	0.46398	0.00707
x1	0.05	0.05043	0.00006	0.05011	0.00014	0.04933	0.00042	0.05	0.04855	0.00005	0.04995	0.00015	0.05036	0.00039
x2	0.00	-0.00007	0.00001	0.00006	0.00002	-0.00016	0.00005	0.00	0.00001	0.00001	0.00001	0.00003	0.00024	0.00005
x3	0.00	0.00003	0.00001	0.00024	0.00002	-0.00005	0.00004	0.00	-0.00020	0.00001	-0.00007	0.00003	-0.00018	0.00006
x4	-0.05	-0.05098	0.00006	-0.04964	0.00015	-0.04733	0.00043	-0.05	-0.05015	0.00006	-0.04761	0.00014	-0.04602	0.00042
x5	0.00	-0.00004	0.00001	0.00037	0.00002	-0.00009	0.00005	0.00	0.00001	0.00001	0.00015	0.00003	-0.00005	0.00005
x6	0.05	0.05027	0.00007	0.04868	0.00017	0.04719	0.00048	0.05	0.04829	0.00007	0.04852	0.00016	0.04171	0.00047
w1	0.00	0.00051	0.00003	0.00045	0.00010	0.00050	0.00026	0.00	0.00092	0.00005	0.00109	0.00016	0.00093	0.00028
w2	0.13	0.13947	0.00010	0.13760	0.00016	0.13619	0.00033	0.13	0.14756	0.00048	0.14869	0.00126	0.14721	0.00234
N=2000														
yt-1	0.95	0.95070	0.00000	0.95034	0.00000	0.95015	0.00000	0.50	0.49044	0.00025	0.48638	0.00026	0.48469	0.00023
x1	0.05	0.05046	0.00001	0.05039	0.00003	0.05011	0.00005	0.05	0.04907	0.00001	0.05096	0.00002	0.05250	0.00005
x2	0.00	-0.00003	0.00000	-0.00005	0.00000	-0.00013	0.00001	0.00	0.00000	0.00000	-0.00003	0.00000	0.00001	0.00001
x3	0.00	0.00003	0.00000	0.00014	0.00000	0.00003	0.00000	0.00	-0.00011	0.00000	-0.00003	0.00000	-0.00004	0.00000
x4	-0.05	-0.05108	0.00001	-0.04958	0.00003	-0.04965	0.00005	-0.05	-0.05115	0.00001	-0.04866	0.00002	-0.04908	0.00005
x5	0.00	-0.00007	0.00000	0.00025	0.00001	-0.00007	0.00001	0.00	0.00001	0.00000	0.00009	0.00000	-0.00001	0.00000
x6	0.05	0.05037	0.00002	0.04906	0.00003	0.04932	0.00005	0.05	0.04907	0.00002	0.04903	0.00003	0.04560	0.00005
w1	0.00	0.00011	0.00001	0.00005	0.00001	0.00023	0.00002	0.00	0.00020	0.00001	0.00026	0.00001	0.00019	0.00002
w2	0.13	0.13350	0.00003	0.13164	0.00003	0.13103	0.00003	0.13	0.13613	0.00006	0.13498	0.00007	0.13677	0.00007

Notes:

Notes:

1. The table presents medians and variances of the estimated parameters across the Monte Carlo replications. Higher (lower) values of the inclusion probability for a given variable suggest that the data favors (does not favor) the inclusion of that variable. We consider sample sizes $N = 200, 500, 2000$, two values for the coefficient of the lagged dependent variable, $\alpha = 0.95, 0.50$, and three values for the variance of the idiosyncratic random error, $\sigma_v^2 = 0.05, 0.10, 0.20$.
2. The true model is shown in column 2. The universe of 9 potential explanatory variables includes the lagged dependent variable (y_{t-1}), 6 exogenous ($x1 - x6$), and 2 endogenous ($w1$ and $w2$) variables. A value of 1 (0) indicates that the variable is included (excluded).
3. The error terms are constructed using discrete distributions (see Section 4.1.).

Appendix B: LIBMA predictive properties

In this Appendix, we examine the predictive properties of our methodology against more parsimonious models. Results are discussed in Section 4 of the paper.

Table B1. RMSE Comparison between LIBMA and various parsimonious models
1-step to 3-step ahead forecasts for $a = 0.50$ and various N and σ_v^2

σ_v^2 Sample	0.05				0.10				0.20						
	LIBMA	Model A	Model B	Model C	Model D	LIBMA	Model A	Model B	Model C	Model D	LIBMA	Model A	Model B	Model C	Model D
1-step															
N=200	0.07184	0.85169	0.76642	0.76336	0.75921	0.18569	1.23220	1.03130	1.02830	1.02200	0.39203	1.97010	1.52940	1.52140	1.51820
N=500	0.03916	0.85051	0.76632	0.76329	0.75909	0.07832	1.23450	1.03390	1.03080	1.02550	0.17640	1.97470	1.53750	1.52990	1.52720
N=2000	0.04576	0.84951	0.76456	0.76185	0.75760	0.09055	1.23560	1.03480	1.03230	1.02670	0.17558	1.97680	1.54010	1.53310	1.53040
2-step															
N=200	0.10466	0.95197	0.70114	0.69303	0.68960	0.26067	1.37380	0.88462	0.87435	0.87300	0.55064	2.21220	1.23640	1.22310	1.22020
N=500	0.07655	0.95058	0.70125	0.69315	0.69001	0.12472	1.37980	0.89052	0.88023	0.87964	0.23884	2.23120	1.24700	1.23370	1.23180
N=2000	0.08307	0.94853	0.69892	0.69104	0.68789	0.13926	1.38060	0.89119	0.88140	0.88027	0.23758	2.22740	1.24740	1.23490	1.23250
3-step															
N=200	0.13662	0.96964	0.70666	0.69916	0.69581	0.31494	1.40370	0.89491	0.88533	0.88253	0.64086	2.27450	1.26090	1.24840	1.24450
N=500	0.10908	0.96732	0.70629	0.69889	0.69562	0.15974	1.40240	0.89659	0.88752	0.88505	0.27710	2.28150	1.26750	1.25550	1.25180
N=2000	0.11632	0.96585	0.70447	0.69724	0.69413	0.17683	1.40490	0.89864	0.88976	0.88728	0.27631	2.28010	1.26800	1.25650	1.25290

Notes:

1. The table compares estimated LIBMA RMSE with four Models A, B, C, and D. We consider three sample sizes, $N = 200, 500, 2000$, $a = 0.50$, and three values for the variance of the idiosyncratic random error, $\sigma_v^2 = 0.05, 0.10, 0.20$.
2. Model A includes y_{t-1} ; Model B includes y_{t-1} and $w1$; Model C includes y_{t-1} , $w1$, and $w2$; Model D includes the true model that is, y_{t-1} , $x1$, $x4$, $x6$, and $w2$.