This item was submitted to [Loughborough's Research Repository](#) by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

Are spatial-numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests no

PLEASE CITE THE PUBLISHED VERSION

<https://doi.org/10.1111/mbe.12093>

PUBLISHER

WILEY

VERSION

AM (Accepted Manuscript)

PUBLISHER STATEMENT

This is the peer reviewed version of the following article: Cipora, K., Patro, K. and Nuerk, H-C., 2015. Are spatial-numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests no. *Mind, Brain, and Education*, 9(4), pp. 190 - 206, which has been published in final form at <https://doi.org/10.1111/mbe.12093>. This article may be used for non-commercial purposes in accordance with Wiley Terms and Conditions for Use of Self-Archived Versions.

LICENCE

CC BY-NC-ND 4.0

REPOSITORY RECORD

Cipora, Krzysztof, K Patro, and Hans-Christoph Nuerk. 2015. "Are Spatial-numerical Associations a Cornerstone for Arithmetic Learning? the Lack of Genuine Correlations Suggests No". Loughborough University. <https://hdl.handle.net/2134/12179061.v1>.

Are spatial-numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests No

Krzysztof Cipora^{1,2*#}, Katarzyna Patro^{3#} & Hans-Christoph Nuerk^{3,4,5}

¹ Institute of Psychology, Jagiellonian University, Kraków, Poland

² Copernicus Center for Interdisciplinary Studies, Poland

³ Knowledge Media Research Center, Tübingen, Germany

⁴ Department of Psychology, University of Tübingen, Tübingen, Germany

⁵ LEAD Graduate School, University of Tübingen, Tübingen, Germany

These authors contributed equally to this work.

Cipora, K., Patro, K., & Nuerk, H.-C. (2015). Are spatial - numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests no. *Mind, Brain, and Education*, 9(4), 190-206. <https://doi.org/10.1111/mbe.12093>

Running head: Spatial-numerical associations & arithmetic skill

* Corresponding Author

Krzysztof Cipora

Institute of Psychology, Jagiellonian University, Kraków, Poland

Ul. Ingardena 6, 30-060 Kraków, Poland

Phone: +48 507 946 858

E-mail: krzysztof.cipora@gmail.com

Are spatial-numerical associations a cornerstone for arithmetic learning? The lack of genuine correlations suggests No

ABSTRACT

The mental number line metaphor describes that numbers are associated with space. These spatial-numerical associations (SNA) are subserved by parietal structures (mainly intraparietal sulcus IPS and posterior superior parietal lobule PSPL). Generally, it is assumed that this association is a basic cornerstone for arithmetic skills. In this review, we present a taxonomy of SNAs and outline which of them are related to arithmetic skills. Recent research suggests that not all SNAs are related to arithmetic skills; for instance, the Spatial Numerical Association of Response Codes (SNARC) is not or at least less related to arithmetic skills than SNAs assessed in the number line estimation task. In general, we conclude that the relationship between SNAs and arithmetic skills are rather weak or caused by mediating variables.

Nevertheless, interventions based on relations between space and numbers can be beneficial for arithmetic skills because space is a powerful tool to understand arithmetic concepts.

Spatial grounding of numerical representations

In the past few decades many signatures of elementary numerical processing have been described, shedding some light on how numbers might be represented in the human brain. Among the most thoroughly discussed properties of numerical processing is the fact that numerical magnitudes and other mathematical concepts (e.g., parity, Nuerk, Iversen & Willmes, 2004) can be grounded in physical dimensions such as space. For instance, numbers can be represented along a left-to-right continuum (forming a number line), they can *increase* or *decrease*, or they can be associated to physical volumes, distances, sizes and surfaces. Spatial grounding is often viewed as a kind of strategy that our brain employs to deal with abstract concepts with no explicit perceivable referents (e.g., time, word semantics or emotions).

All phenomena, revealed on a behavioural or neural level, which reflect mutual relations between spatial and numerical processing are jointly called Spatial-Numerical Associations (SNA; e.g., Fischer & Shaki, 2014; Patro, Nuerk, Cress & Haman, 2014). As we will further discuss, this term is very broad. Phenomena that can be classified into this category are largely varied; this is because numbers may interact with space in many different ways. Moreover, as we will show throughout this paper, observed correlates of SNA also largely depend on a particular SNA type.

Neural underpinnings of space-number relations

Already the first neurocognitive models of numerical cognition suggested that number magnitudes are subserved by brain areas around the intraparietal sulcus (IPS), which also subserve spatial (attentional) processing. In the extended version of the Triple-Code Model (Dehaene & Cohen, 1995), Dehaene, Piazza, Pinel and Cohen (2003) suggested that the parietal lobe supports numerical processing in a tripartite manner. Apart from domain-specific

activation within the horizontal segment of intraparietal sulcus (hIPS), and language-specific pattern in the angular gyrus (AG), processing numerical information involves the region of the posterior superior parietal lobe (PSPL) – the region that is mainly responsible for attentional functions and visuo-spatial behaviour such as grasping, eye movement or pointing. According to the authors' model, this region might play a key role in linking spatial functions to number processing. Another influential neurocognitive model in numerical cognition – ATOM theory (A Theory of Magnitude; Walsh, 2003) – assumes that numbers and space are jointly processed in the brain because they are both supported by the generalized magnitude system, presumably localized in the inferior parietal and prefrontal cortex. This theory accentuates, in particular, that numbers and space can be processed based on the same metric, as both of them are magnitudes (extensions) that can be quantified, compared or estimated. Also, from the perspective of cultural evolution and its potential effects on neural reorganization (cf. neural recycling hypothesis, Dehaene & Cohen, 2007), the spatial-numerical coupling seem to be justified, given that spatial and numerical processing are based on a very similar metric – distances can be considered as physical or numerical; spatial ordering rules can also be applied to numerical information, etc. Therefore, cultural acquisition of numerical knowledge could have easily co-opted evolutionarily old spatial-attentional circuits and adjusted them to new functions. What is important, the advocates of the neural recycling predict that once “cortical territories dedicated to evolutionarily older functions are invaded by novel cultural objects, their prior organization is never entirely erased” (Dehaene & Cohen, 2007 p. 384). Thus, whenever number magnitudes are processed by parietal structures, the previous, evolutionary purpose of these structures - spatial-attentional processing - may facilitate or interfere with processing of numbers (cf. Hubbard, Piazza, Pinel & Dehaene, 2005).

Are the above theoretical claims on a common spatial-numerical system in the brain

supported by the empirical data? In recent years, such evidence has indeed been collected. Though this evidence is not numerous, it suggests that numbers and space might interact at the neural level in many different ways, and that this interaction can be tested and observed in different populations. This seems to be the case even for non-human primates such as rhesus monkeys, whose intraparietal cortex contains functionally overlapping neurons for processing numerosities and length (Tudusciuc and Nieder, 2007). Such a neural overlap was found to be strongly related to monkeys' behaviour: when using a self-organizing map classifier, it could be seen that quantity-selective neurons were informative for both numerical and spatial judgement¹. In human adults, the parietal cortex also seems to play a role in connecting spatial representation to numerical processing and even to mental calculations. Knops, Thirion, Hubbard, Michel & Dehaene (2009b) identified a specific region in the human posterior superior parietal lobule (PSPL) that controlled two seemingly separate forms of behaviour: left- or rightwards saccades on the one hand, and solving subtraction or addition problems on the other hand.

What is important is that these neurocognitive data nicely corroborate the idea of Dehaene et al. (2003) about the special role of this parietal region in linking numbers to space. Other neurocognitive data suggest that intraparietal activation can even be susceptible to behavioural numerical training in school-aged children, providing the first cue for the potential importance of SNAs for math learning. In a study by Kucian and colleagues (2011), dyscalculic and control children were trained with a number line estimation (NLE) task (they had to mark a target number on a line flanked with other numbers, e.g., 0 and 10; NLE will be further discussed in following sections) and apart from an increase in performance in mental calculation tasks, they exhibited decreasing activation in frontal and (intra-)parietal areas after training (which might suggest better automatization of the calculation processes, Ischebeck, Zamarian, Egger, Schocke & Delazer, 2007; for related evidence from NLE see Bartlett, Man

& Booth, 2015).

Taken together, as the evidence shows, number and space processing interact with each other at least on a neural level, and this interaction is in line with three influential theoretical frameworks in numerical cognition: The Triple-Code Model, the ATOM and the neural recycling hypothesis. But is the anatomical closeness of spatial-numerical circuits of any use for our cognitive functioning? In particular - is it important for our arithmetic skills? Given that many signatures and connectivity patterns in the human brain have evolved to enhance information processing, one could indeed suppose such a possibility. If this is the case, it could have important consequences both for theories on numerical development and for educational practice in this domain.

In this short review, we present and discuss evidence that the relation between number-space processing and arithmetic skills might indeed exist, but this relation is not always evident and easy to interpret.

Variability in spatial-numerical associations

Spatial-numerical associations are not unitary and their relations to math skills are not either. In fact, both numerical and spatial processing are rather heterogeneous concepts which do not easily fall into one single definition. This variety, in turn causes the problem in defining “spatial-numerical associations”. As has been shown in the studies so far (which will be reviewed in subsequent paragraphs), numbers can be bound to spatial directions, but can also interact with spatial non-directional extensions (surfaces, lengths); on the other hand, spatial processing can be observed for cardinal numbers, as well as for ordinalities, intervals and operations. Before we go into the details of spatial-numerical representations, we will first explain the numerical concepts and in a second step the spatial concepts underlying the different associations.

Distinguishing mathematical concepts underlying spatial-numerical associations

Ordinality and cardinality are basic mathematical concepts. Ordinality refers to the fact that elements can be (verbally, temporally or spatially) ordered according to some criterion (for numbers, mostly magnitude). Cardinality refers to the retrieval of the set size (e.g., the number of dots in an array). In mathematics, cardinality can also be defined as an equivalence class under equinumerosity that includes all sets of the same size. Because set size can be determined verbally (“six”) or symbolically (“6”), we define the magnitude of both numbers as cardinality.

For adults, it seems obvious that the end point of ordinal counting of set size is identical with the cardinality (the principle behind it is called “cardinal counting principle”; Gelman & Gallistel, 1978). However, for children this is less obvious (Le Corre & Carey, 2007; Wynn, 1992). Indeed, the dyscalculia test Tedi-Math (Kaufmann et al., 2009, for the German version) tests just this relation. Many kindergarten children at the age of about 4,5 years are able to correctly count items (ordinal task), but they are not able to say how many items there are altogether (cardinal task for set size) and how many there are when counting from the other direction (independence of cardinality from counting directionality). Just because adults can easily make inferences from ordinality to cardinality in our counting system does not mean that ordinality and cardinality are represented in the same way in general (for differences see Turconi, Jemel, Rossion & Seron, 2004) and in their relation to space. Therefore, these two concepts are distinguished in our taxonomy, although for natural numbers there is a close relation between them.

Interval-based numerical relations differ from ordinal numerical relations. In ordinal relations, differences between ordinal numbers are not meaningful, because they can mean something very different. The ranks in a football league tell us little about the true difference

in the related measure (points, goal differences) which are used to build such ranks. Point difference between the first and the third may be much larger than between the 7th and the 9th of the league, although the difference in ranks is the same. This trivial example illustrates the important difference between ordinality and interval representations for SNA. In interval representations, we demand that means and differences between numbers are meaningful, while for ordinal sequences this must not be the case. In psychological research, this is why we demand interval scaling whenever we want to work with means and parametric statistical tests. For SNAs, a typical interval-scaled task is the number line estimation task (Siegler & Opfer, 2003), where numbers should be located on a number line not only in the correct order but also with the correct interval-spaced distances between them.

Finally, another important concept for our SNA taxonomy are operations. Not only number magnitude itself is associated with space but also arithmetic functions like addition or subtraction. This mathematical concept goes one step further. While previous concepts were related to the links between numbers and their magnitude, this mathematical concept describes a function, i.e., some sort of mapping of an input to an output via some rule. For our SNAs, two or more numbers are mapped towards a third one by two different rules, namely, addition or subtraction. Of course, there is an infinite number of other possible functions, but addition and subtraction are those which have been mainly researched with regard to SNAs.

In sum, different numerical concepts underlie our taxonomy of numbers and space. In order to gain a more thorough insight into the problem of the relation between SNA and arithmetic, we will now categorize and systematize different number-space associations within a taxonomy, and then we will try to compare those effects in terms of their putative relations to arithmetic skills.

A preliminary proposal of SNA taxonomy has been presented by Patro et al. (2014) who systematized and classified empirical data collected from studies with preschool children

and infants. We have constructed our taxonomy based on this original proposal with some minor changes described below.

The central distinction of our taxonomy is based on *non-directional* (henceforth called extensions) vs. *directional* associations between numerical and physical space. This distinction is directly borrowed from the taxonomy by Patro et al. (2014). Although the current literature on spatial processing distinguishes between several other types of representation such as intrinsic vs. extrinsic spatial relations (Uttal et al., 2013) or body-related vs. external spatial representations (Tversky, 2005), we have decided to keep the directional vs. extension distinction for the following reasons. First of all, spatial extensions and directions have a different ontological status; whereas the first concept (extensions) describes certain spatial *qualities* of an object (e.g., x is wide and high), the second one is *topological* in nature, as it only refers to an object's location within certain reference frames (e.g., x stands to the left of y). Even though both kinds of spatial information have to be coordinated in order to navigate in the environment or to perform certain motor tasks (e.g., grasping an object requires adjusting a grip to its size as well as directing the hand towards an object), they seem to be guided by partially dissociated neurocognitive mechanisms (e.g., Jeannerod, Decety & Michel, 1994). Second, the boundary between directional and extensional space representations seems to be naturally drawn by the studies on spatial-numerical associations. Researchers usually employ different paradigms to examine each of these two forms of associations and formulate different predictions regarding their developmental course and susceptibility to cultural learning. As we will further see, they also examine slightly different aspects of number representations in reference to each type of spatial processing.

Taking into account this basic division in space representation, we looked in the next step at other potential distinctions that could be made within each of these two spatial

categories, to make final sub-categories as homogenous as possible. In the case of non-directional SNAs, Patro et al. (2014) proposed to distinguish between effects based on spatial and numerical cardinal sizes on the one hand, and those based on spatial and numerical intervals (i.e. distinguished parts of the physical or numerical whole) on the other hand which reflected widely available children's data. This distinction also fits in well with the data obtained from adults (and makes theoretical sense since intervals are not computed identically to the whole; rather, they are computed in relation to the whole). Therefore, we decided to keep it also for our taxonomy.

Within directional SNAs, Patro et al. (2014) distinguished between cardinal and ordinal numerical processing. This time, the proposed crossovers between spatial and numerical categories seem to be well adjusted only to children's data, but are not exhaustive enough to embrace all forms of SNA found now in adults (like spatial biases for arithmetic operations). For that reason, we modified this part of the taxonomy. We still wanted to distinguish between different forms of numerical representations like ordinal vs. cardinal because their processing is known to be partially dissociated (Turconi et al., 2004). At the same time, however, another relevant distinction comes to the fore: explicit vs. implicit activation of directional representation. In some tasks (e.g., parity judgment of centrally presented numbers), spatial (directional) responses are not explicitly required. In other tasks (e.g. counting spatially aligned objects), they are explicitly enforced. Because the salience of spatial information might affect the strength and properties of its link to numbers, one should also take into account this distinction when discussing differences between SNA effects in the whole population. Note that so far the evidence on explicit directional SNA has come almost entirely from tasks based on numerical ordering. In this sense, this category actually fits in well with the ordinal-directional SNA proposed by Patro et al. (2014). In our model we postulate the existence of explicit directional representations of cardinals and arithmetic

operations as well. All these three categories have been demonstrated in the case of implicit directional spatial numerical representations.

Altogether, we have created four major categories of SNAs (two of them consisting of three subcategories), which approximately fit in with the categories proposed by Patro and Haman (2012), however, they now allow for including and systematizing a larger scope of effects and making further predictions for other possible effects. Among SNAs based on non-directional space representation we distinguished between (i) associations of spatial and numerical cardinal sizes, where number cardinality (i.e., set size) is mapped onto spatial extension (e.g., object size), (ii) associations of spatial and numerical intervals, in which differences/relationships between numbers are mapped onto physical distances or line length. Among SNAs based on directional space representation we distinguished between (i) associations of implicit directions and number cardinality representations. Not being forced by task demands, number cardinality is mapped onto directions in space (usually on a left-to-right dimension). This category is further divided into three sub-categories (one for cardinal numbers, one for ordinal numbers and one for arithmetic operations), (ii) associations of explicit directions and numbers. These mappings can be observed when participants are explicitly asked to map numbers onto spatial locations. Although so far they have been demonstrated only for ordinalities, we postulate the existence of such mappings for cardinalities and operations as well. The overview of the taxonomy is presented in Figure 12.

<Figure 1 around here>

Main types of SNAs and their relation to arithmetic skills

SNAs based on spatial extension representations

SNA-Extension: Spatial and numerical cardinalities

Overview. The first SNA type refers to the representation of numerical cardinalities and spatial extensions (length, size, etc). The basic hallmark of this category is that

representational metrics underlying numerical and spatial extensions can be considered as homologous - both extensions can be quantified and referred to as magnitudes (cf. Walsh, 2003). For instance, whereas numbers can increase in their numerical size (1, 2, 3, ...), analogous increase can be observed for objects expanding in physical space ($x \times x \times \dots$).

Paradigms. Evidence for this SNA category comes from two types of experiments. The first type aims to demonstrate that numbers and space are represented - especially by young children, infants and neonates - as two equivalent and interchangeable dimensions. It has been shown that children spontaneously extract certain rules referring to one dimension (length) and transfer it to another dimension (numbers). For instance, they generalize increase or decrease in numerical magnitudes to increase or decrease in length of lines (de Hevia & Spelke, 2010; de Hevia, Izard, Coubart, Spelke & Streri, 2014a; see also Lourenco & Longo, 2010 for a similar transfer of colours and patterns between spatial and numerical stimuli). This remarkable cross-dimensional transfer can already be observed in human neonates (de Hevia et al., 2014a), suggesting that our brain is naturally adjusted to capture and process this kind of spatial-numerical correlations. Further neural support for this SNA category comes from Tudusciuc's and Nieder's (2007) aforementioned study about functionally overlapping neurons for processing numerosities and length in the parietal cortex (IPS) of rhesus monkeys.

The second type of experimental evidence demonstrates a kind of interference between numerical and spatial magnitude processing. Among this type of evidence one should list the *size congruity effect* (e.g., Henik & Tzelgov, 1982) which shows that our performance in a (numerical or physical) magnitude comparison task is dependent on the correspondence or conflict between the physical size of a printed number and its numerical value (e.g., 2_6 are easier to compare than 2_6). Numerical magnitudes also affect the judgment of the physical midpoint of a line: while bisecting a horizontal line with both endpoints marked by numerical

flankers, children and adults overestimate physical distance from the line's true midpoint to a larger number and, thereby, shift their responses towards this number (de Hevia & Spelke, 2009). This relation can also occur in the other direction: spatial distance between numbers can accordingly affect numerical judgment. To give an example, numbers that are widely spaced are also judged to be numerically more distant (Lonnemann, Krinzinger, Knops & Willmes, 2008) and, if participants perform some computations over such physically distant numbers, they tend to overestimate the numerical outcome. Furthermore, interference between spatial arrangement and processing numerical information goes beyond processing single numbers. It was demonstrated that congruency between operation order (e.g., $2*2 + 2 = ?$) leads to more efficient calculation performance compared to incongruent spatial arrangement (e.g., $2 * 2+2 = ?$; Landy & Goldstone, 2010). All these interference effects demonstrate that even in adult participants, numerical and spatial information processing largely overlaps.

Relation to arithmetic skills. Taking into account the very early onset of this SNA category, it is natural to ask whether the ability to map numerical and spatial extensions could support the acquisition of basic mathematical skills already in very young children. Several mathematical concepts are constructed based on this form of spatial-numerical mapping. This is especially evident in the theory of measurement, in which numbers are assigned to the circumference or surface of geometrical shapes. Also, mapping numbers onto spatial extensions seems to be a common and natural tool in educational practice; for instance, teachers often explain or illustrate more sophisticated numerical concepts in terms of spatial relations (e.g., a function might *increase* or *decrease*). Given that, it seems reasonable to expect that this very basic ability to create number-space connections could provide some ground for formal mathematical learning.

Unfortunately, only few studies so far have addressed this question. What is more, the nature of the relation between SNA and math skills emerging from these studies appears to be more complex and problematic than supposed. Whereas some studies reveal that there is no relation between children's math performance and the size congruity effect (Bugden & Ansari, 2011; Rodic et al., 2014), other studies report only a partial relation for other number-space interference effects. In particular, Lonnemann and colleagues (2008) described a sort of interaction between the judgment of numerical and spatial distances in 8-9-year-olds. They found that boys who exhibited stronger SNA performed better in addition and subtraction tasks. For girls, however, a marginal negative correlation was observed. The authors argued that spatial-numerical mapping might not always be supportive for arithmetical reasoning but it might be in some cases even detrimental. The nature of this relation might depend on a specific thinking style preferable by a child: whereas boys might prefer solving calculations by relying on visuospatial thinking, girls might rely more on a verbal thinking style. Indirect evidence may be provided by a study of Kaufmann & Nuerk (2006) with children with ADHD who often encounter math problems. They also exhibit particular difficulties in the size-congruity effect as compared to control children even when they do not have observable math learning difficulties. This may imply that although a positively selected sample of children with ADHD (without observable math problems) have been tested, they nevertheless have problems in this basic SNA.

The fact that joint spatial-numerical processing might not always be beneficial for math competence is also widely demonstrated in studies on the approximate number system (ANS), which are based on comparison of non-symbolic numerosities. Usually, in such studies, it is necessary to control for non-numerical parameters of objects arrays (elements' size, total surface - which are usually termed as "magnitudes", but can also be viewed as "spatial" in nature, cf. Lourenco & Longo, 2010) to make sure that participants indeed

process numerical information instead of relying solely on perceptual cues. Recent studies consistently show that performance in trials in which numerical and visual (spatial) cues are negatively correlated (e.g., larger numerosities occupy smaller space) might be the most predictive for children's arithmetic skills (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; Sasanguie, Defever, Maertens & Reynvoet, 2014). Those subjects who easily disregard spatial-visual cues and focus on numerical information usually perform better in mathematical tasks. Performance in trials in which spatial and numerical cues are congruent does not correlate with arithmetic skill. Therefore, this effect seems to be mediated by inhibitory control skills (which are of major importance for math learning in general). Resolving such conflicts by extracting one of the display aspects and inhibiting other aspects may rely on more domain-general control functions playing a vital role in any problem solving task, being relevant for children's math achievement in general, including operations on symbolic material (Clark, Pritchard & Woodward, 2010; Gilmore et al., 2013; Bull, Espy, Wiebe, Sheffield, & Nelson, 2011). The evidence from these kind of studies suggests that the ability to disregard spatial confounds in numerical tasks might be as important as the ability to map both dimensions onto each other. We admit that overcoming such conflicts might be important for older children and adults, or for tasks that require processing numbers on a more abstract (e.g., verbal) level. Henceforth, it may require more domain-general capacities, not directly related to elementary numerical processing.

However, this conclusion might be age-specific. We still do not know whether early developing abilities to create mapping between space and numbers, or to make analogies between both dimensions (and not learning how to ignore naturally-occurring number-space correlations) could serve as a kind of scaffold for the acquisition and the understanding of basic mathematical concepts (e.g., measurement, fractions) in earlier age.

SNA-Extension: Spatial and numerical intervals

Overview. The second SNA type also refers to spatial extension. However, rather than relating associations between spatial and numerical extensions in an unspecified way, it refers specifically to intervals and the principle of equidistance. It is founded on the ability to represent equidistant relations between consecutive numbers (the distance between 1 and 2 is the same as 8 and 9) in terms of spatial intervals (line segments on a number line). This ability increases with age. Whatever underlying model one prefers, the conceptual description of performance change is clear: at the beginning, small children are not able to map equidistant numerical intervals to equidistant spatial intervals across the whole number range, whereas later they learn to do this. The models differ in their explanations as to why children fail to reach equidistance in the beginning and master this endeavour later. For instance, one of the models predicts that early in the development the mapping follows the logarithmic pattern (i.e. distances between small numbers are larger than respective distances between large numbers) and, subsequently, it gradually changes to the linear pattern (Siegler & Opfer, 2003, but see Ebersbach, Luwel, Frick, Onghena & Verschaffel, 2008; Moeller, Pixner, Kaufmann & Nuerk, 2009; Barth, Slussel, Cohen & Paladino, 2011, for alternative views).

Paradigms. In the most popular paradigm called number line estimation (NLE), participants have to mark a position of numerical targets (Arabic digits or dot arrays) on the line whose ends are labelled with other numbers (e.g., 0 and 100) (Barth, et al., 2011; Moeller et al., 2009; Opfer & Siegler, 2007; Siegler & Booth, 2004). The positioning of marking is assumed to reflect the shape of numerical representation. The fitting of this shape can be linear (i.e. equidistant positions of numbers irrespective of their magnitude) or not. The typical linear shape may be caused by a linear representation (Siegler & Opfer, 2003), by proportional strategies (Barth et al., 2011), by good place-value integration of multi-digit

numbers (Moeller et al., 2009) or by better integration of familiar and unfamiliar number ranges (Ebersbach et al., 2008).

It should be noted at this point that NLE might not necessarily assess SNAs but it might rather assess the strategic use of proportional judgments (e.g., the participant must assess which part of the 0-100 interval would correspond to 37). Then, the performance would largely depend on how many reference points are used by the participant. At least in older children, participants start making virtual reference points, first the start and the endpoint, then the middle of the line, then eventually by dividing virtual halves onto halves and so on. Apart from increasing number of reference points, the overall accuracy increases as well (Slusser, Santiago & Barth, 2013, see also Barth et al. 2011). The successful application of such proportional processes might lead to a seemingly linear estimation without linear representation.

Whether the typical non-linear shape is logarithmic (i.e. larger distances between small numbers and more compressed large numbers; Siegler & Opfer, 2003) or multi-linear with a break at 10 (Ebersbach et al., 2008; Helmreich et al., 2011) is also a matter of debate. In any case, whenever a linear shape reflects the correct solution (see Huber, Moeller & Nuerk, 2014, for an exception), linear performance is usually associated with better math performance³.

NLE in which both endpoints of a line are marked with numbers was later named *bounded NLE*, to contrast it from *unbounded NLE*. In the latter, the line only has the start point, and instead of a numerical label on the end point the subject is presented with the length of a single unit (Cohen & Blanc-Goldhammer, 2011; Link, Nuerk & Moeller, 2014). We will see that this distinction is important for the relation to arithmetic skills.

Relationship to arithmetic skills. The relation of this SNA type to arithmetic and mathematical skills is probably the most thoroughly studied. The linear representation

(indexed as percentage of variance in a child's estimation explained by numerical magnitude) correlates strongly with standardized math achievement test scores ($r \approx .50$; Booth & Siegler, 2006). The relationship between NLE performance to arithmetic skills is more pronounced in the case of addition and subtraction compared to multiplication (Link et al. 2014). It can be caused by the fact that multiplication is solely based on memory retrieval, not calculation (Dehaene et al., 2003). Also in children revealing math difficulties NLE is particularly impaired (Geary, Hoard, Nugent & Bailey, 2012).

NLE can be trained; these interventions lead to considerable improvements in math performance, and even to observable changes on the neural level (Kucian et al., 2011). First of all, Opfer and Siegler (2007) demonstrated that providing feedback during NLE leads to a rapid and notable improvement in the performance of this task. The improvement was explained in terms of shaping proper linear representation of number magnitudes. On the other hand, adult subjects can be trained with non-linear mappings in NLE (e.g., sigmoid), when adequate feedback is provided. What is interesting is that such training leads to correct estimation, but does not change the underlying number representation (Huber et al., 2014).

Beneficial for math performance are also trainings based on playing linear board games (which enforce equidistant differences between number units; Siegler & Ramani, 2009). Linear mapping in the NLE was also trained within tasks employing whole body movements (i.e. walking along the line, Link, Moeller, Huber, Fischer & Nuerk, 2013) which led not only to more improvements in spatial-numerical mappings but also in transfer tasks like addition. It appears to be even more efficient than control training - its effects partially generalize over untrained skills (e.g., number comparison or addition performance) and are more beneficial to children who perform poorly in the pre-test.

To measure interval-scaled SNAs, Cohen and Blanc-Goldhammer (2011) introduced the unbounded NLE. Different processes seem to play a role in this unbounded version of the

NLE in adults. In a first study with children, Link et al. (2014) found significant correlations between task performance and addition, subtraction and magnitude comparison for the bounded NLE, but not for the unbounded NLE. This suggests that not the associations between linear spatial and numerical intervals are important for the relation to arithmetic skills (as otherwise correlations should have been observed for the bounded and the unbounded versions), but rather the proportional strategies unique to the bounded version of the task.

In summary, performance in typical bounded number line estimation correlates with early arithmetic skills. Nevertheless, it is questionable whether the results of this task reflect the numerical representation type or rather familiarity with numbers, place value system and understanding proportions (i.e. abilities that are not basic and require relatively complex numerical processing and conceptual knowledge). In that case, the observed correlation seems to be more trivial, since one correlates math abilities with math abilities themselves. However, in our view, these controversies do not undermine the benefit of NLE trainings. It seems that training conducted in a relatively simple form leads to increases in understanding several relatively complex mathematical concepts and abilities. Therefore, NLE trainings allow several relatively complex processes of mathematical reasoning to be trained in a relatively easy and attractive way, even within a short-term intervention session (cf. Link et al. 2013).

SNA based on directional spatial representation

SNA-Directions: Implicit spatial coding for numerical cardinalities

Overview. This SNA type refers to numerical representations that are mapped onto directions in space. It captures another important aspect of number line representation – its orientation. The mapping is implicit in nature – it appears automatically and is not required by task demands or experimental instructions. The direction of this mapping usually follows the

reading direction of participants (Dehaene, Bossini & Giraux, 1993; Shaki, Fischer & Petrusic, 2009), but some forms of directional SNA can also be observed in preliterate children (de Hevia, Girelli, Addabbo & Cassia, 2014b; Hoffmann, Hornung, Martin, Schiltz, 2013; Patro & Haman, 2012). In general, participants immersed in Western cultures associate small magnitude numbers with the left hand side and large magnitude numbers with the right hand side. Another kind of spatial mapping is present in vertical dimension, in which small numbers are associated with lower, whereas large numbers with upper positions (Schwarz & Keus, 2004; Shaki & Fischer, 2012).

Paradigms. The most thoroughly studied phenomenon within this SNA type is the SNARC effect (Dehaene et al., 1993). In a typical experimental setup, participants judge parity or magnitude of centrally displayed numbers. They are usually faster with their left hand (left button) when a number is relatively small and with the right hand when it is relatively large.

SNARC was replicated several times with varied experimental paradigms, stimuli used and response format (Nuerk, Wood & Willmes, 2005a; Wood, Willmes, Nuerk & Fischer, 2008 for meta-analysis; Fischer & Shaki, 2014 for a current review). It was found for pointing (leftward/rightward movement faster after small/large target number detection; Fischer, 2003a), bipedal (Schwartz & Müller, 2006) and oculomotor responses (Fischer, Warlop, Hill & Fias, 2004). Apart from various forms of symbolic material (Arabic numbers, number words, spoken number words, multi-digit and negative numbers or fractions, Dehaene et al., 1993; Fischer, 2003b; Fischer & Rottmann, 2005, Bonato, Fabbri, Umiltà & Zorzi, 2007), it was also investigated for non-symbolic arrays (Nuerk et al., 2005a; Patro & Haman, 2012).

Importantly, the spatial mapping is present even if the response criterion does not directly relate to the meaning of the number (*Is the number written upright or in italics?*, Fias,

Lauwereyns & Lammertyn, 2001) or does not require task-relevance of numerical attributes (bisecting a line composed of numbers, Calabria & Rossetti, 2005, passive viewing of numerical primes followed by lateral target detection, Fischer, Castel, Dodd & Pratt, 2003). Another line of research focuses on demonstrating how body movement can be engaged in directional number processing. In these paradigms, the participants are asked to provide random numbers from a specified number range (Random Number Generation - RNG task) or provide estimates of some numerical values and execute some motor actions. It was shown that when people move their head (Loetscher, Schwarz, Schubiger, Brugger, 2008), they walk (Shaki & Fischer, 2014) or lean in certain directions (Eerland, Guadalupe & Zwaan, 2011) – the direction of such movement strongly affects RNG or numerical estimation. Importantly, these associations are bidirectional – RNG influences motion direction (i.e. leftward/rightward turns are more likely after a small/large magnitude random number was generated) as well as motion direction influences RNG (i.e. generating small/large random numbers is more likely after leftward/rightward turns; Shaki & Fischer, 2014).

Relationship to arithmetic skills. Traditionally, investigation of SNARC-like effects had a form of demonstrating main effects and showing general phenomena. These early studies already demonstrated that there are considerable interindividual differences in this form of SNA. For instance, it has been shown that SNARC is not a ubiquitous pattern: it is present only in about 70% of participants (Wood et al., 2008). Moreover, even in those participants who reveal the SNARC effect a large variability can be observed (in strength and direction of this association). These interindividual differences leave the field open for a differential approach in studying SNA's. Indeed, as it was recently demonstrated by Viarouge, Hubbard and McCandliss (2014), the SNARC effect can be reliably measured and, therefore, its magnitude can be used as a differential measure.

The level of math/arithmetic skill is one of the possible candidates explaining variability in the SNARC effect (for a discussion on differentiation between arithmetic skill and math competence see Cipora & Nuerk, 2013). Less proficient participants were thought to reveal a stronger SNARC effect. However, in adult studies the results are rather inconsistent. Some of the experiments revealed a tendency towards a stronger SNARC effect in less proficient participants (Dehaene et al., 1993 exp 1; Fischer & Rottman, 2005). Nevertheless, the difference was far from being significant. A null effect was also found by Bonato et al. (2007) and Bull, Cleland & Mitchell (2013). Direct support for a null correlation of the SNARC effect with arithmetic skill was provided by Cipora & Nuerk (2013). Their null finding was very likely not due to a power problem as confirmed by the results of the Bayesian analysis.

So, while there might be little correlation of the size of the SNARC effect in the normal range, there might be some relation when extreme groups are included in the analysis. Hoffman, Mussolin, Martin & Schiltz (2014a) have recently found that participants with math difficulties revealed a stronger SNARC effect compared to controls and math professionals who revealed the weakest SNARC effect. Extreme group effects seem to hold not only for the lower spectrum (participants with math difficulties) but also for the upper spectrum (professional mathematicians). They seem to exhibit a significantly smaller SNARC effect than controls (Cipora et al., 2015).

Such findings are partially corroborated by SNARC studies in children with and without difficulties. Bachot, Gevers, Fias & Roeyers (2005) found that children (age range 7 to 12) with visuospatial disorder (VSD) and developmental dyscalculia (these groups were not differentiated in the study) did not reveal the SNARC effect, contrary to a control group. On the other hand, a recent study by Crollen and Noël (2015) revealed that 4th graders with poor visuospatial abilities did not differ from their peers with typical visuospatial abilities in

the SNARC effect. Therefore, these authors suggest that the findings of Bachot et al. (2005) can be attributed more to developmental dyscalculia than to visuospatial disorders.

Studies conducted with normally developing children also show inconsistent results. Hoffman et al. (2013) found that the SNARC effect observed in the magnitude classification was present in children who did not use simple counting in order to make magnitude judgments. The SNARC effect observed in magnitude classification was also more pronounced in children whose number knowledge was better (who could write more Arabic numerals in a correct sequence). Contrarily, the SNARC effect observed in a colour judgment task (assumed to rely on approximate number representation) did not correlate with any observed measure of numerical knowledge. Schneider, Grabner & Paetsch (2009) found in a large-scale study virtually no relationship between the SNARC effect and school math achievement.

To sum up, in adults the differences in the SNARC effect depending on arithmetic skill level are rather small. If they exist, they do so only or mainly for extreme groups (i.e., high and low math performers) but not or only very little in the normal range. Studies on children are very scarce and have produced inconsistent results so far. Some studies observed a relation of the SNARC effect and arithmetic skills in children with and without disorders, while others did not. Altogether, a significant relation of the SNARC effect to arithmetic skills is absent or small or only present in extreme groups. Unfortunately, other SNA within this category have not yet been investigated regarding their relationship to arithmetic skill level, therefore our conclusions here are only limited to the SNARC effect. Nevertheless, there is no theoretical reason to expect high correlations of other effects with the level of arithmetic skills.

On the other hand, some data supporting the causal relationship between this SNA type and arithmetic skills come from at least one training study. When directional SNA was

trained in pre-schoolers, so that the response to small/large magnitude numbers was associated with leftward/rightward whole body movement, significant improvements in math performance were observed (Fischer, Moeller, Bientzle, Cress & Nuerk, 2011). However, in this training study a number line was visually presented as well – therefore, it is unclear whether the training effect is due to explicit directional response or implicit interval (or proportional) learning.

SNA-Directions: Implicit spatial coding for numerical ordinalities

Overview. Within this SNA type the ordinal aspect of numerical information is implicitly mapped onto directional spatial locations.

Paradigms. To the best of our knowledge only one study has demonstrated this SNA type. Van Dijck and Fias (2011) showed that the SNARC effect can be observed for an object's position in a sequence within a given experimental set. When participants are asked to memorize random sequences of numbers, they reveal a SNARC effect based on the position in a sequence, but not on numerical magnitude. The same was observed when participants were asked to memorize arbitrary sequences of fruit and vegetable names and then made binary decisions on them. Noteworthy, this experimental paradigm – referring to the key role of verbal working memory in temporary storage and elaboration of numerical information (Fias, van Dijck & Gevers, 2011; but see Ginsburg et al., 2014; van Dijck et al., 2014) - strongly undermines typical explanations referring to stable spatial mapping of number representations.

Relationship to arithmetic skills: Unfortunately, the relationship of SNA to arithmetic skills has not been investigated yet. We can already suppose, however, that when arbitrary sequences are built temporarily, they may demand stronger engagement of cognitive control

processes as well as working memory – two crucial functions supporting our arithmetical reasoning.

Interrelations between directional implicit coding of ordinalities and cardinalities

One must keep in mind that both ordinal and cardinal numerical processing offer an explanation for the SNARC effect. Ordinal information can be activated even if the task primarily focuses on magnitude (cardinality) processing, and vice versa. This might happen because both ordinality and cardinality constitute the semantics of natural numbers. Nevertheless, for some tasks one sort of information might appear to be more salient than others (e.g., when a subject has to memorize a random numerical order). Therefore, we decided to keep the distinction between both kinds of directional SNAs and formulate the questions on their potential relation to arithmetic skills separately.

SNA-Directions: Implicit spatial coding for arithmetic operations

Overview. This SNA refers to spatial directional mapping of *arithmetic operations*. In analogy to SNARC-like effects for numbers, arithmetic operations seem to be associated with numerical gain or loss. Addition and multiplication are expected to trigger rightward biases, and subtraction and division should trigger leftward biases. Such operational momentum effects have been shown for actual symbolic and non-symbolic calculations, but recently also with the presentation of the operation sign only.

Paradigms. The operational momentum (OM) describes the tendency to overestimate the results of addition problems and to underestimate the results of subtraction problems, with the size of over/underestimation increasing with the increasing size of the outcome (McCrink, Dehaene & Dehaene-Lambertz, 2007; McCrink & Wynn, 2009). This systematic tendency, found for both symbolic and non-symbolic stimuli (Knops, Viarouge & Dehaene, 2009a) and also present for non-symbolic multiplication and division (Katz & Knops, 2014), was

interpreted in terms of hypothetical mental movement along a spatially oriented number line while computing numerical outcome. OM biases were also demonstrated when participants had to perform movements along a number line (Pinhas & Fischer, 2008), and are accompanied by specific eye movements (Klein, Huber, Nuerk & Moeller, 2014, Hartmann, Mast & Fischer, 2015) with spatial displacement of manual responses (Knops et al., 2009a; Pinhas & Fischer, 2008), with lateral attentional shifts (Masson & Pesenti, 2014) as well as oriented movement with an arm (Wiemers, Bekkering & Lindemann, 2014) or whole body movement (Lugli et al., 2013), associated with adding up or subtracting two numbers. However, at least some part of the effect could be due to confounds, i.e., that the size of the calculation result is usually larger for addition than for subtraction (see Klein et al., 2014, for evidence).

However, the whole variance of the OM effect probably cannot be explained by confounds with results magnitude, because even operation signs (- and +) themselves were found to be spatially mapped, with a minus (-) sign linked to the left hand side and a plus (+) sign to the right hand side (OSSA: Operation Sign Spatial Association, Pinhas, Shaki & Fischer, 2014). Since numbers are absent in Pinhas et al. (2014), confounds with number magnitude cannot explain the results.

Relationship to arithmetic skills. Studies on SNA's for arithmetic operations are relatively new and mostly concerned with the establishment of the main effects. Nevertheless, the interindividual variability in these phenomena calls for differential studies. For instance, Knops, Zitzmann, McCrink (2013) demonstrated that efficiency of attention reorienting is one of the determinants of operational momentum in children. Hopefully, future studies will address the relationship of spatial mapping of arithmetic operations to arithmetic skills properly.

SNA-Directions: Explicit spatial coding for numerical ordinalities

Overview. Besides spatial-directional biases that emerge implicitly irrespective of task instructions, there are also biases that can be observed when a subject is explicitly required to provide a spatial response associated with a number's position. These are mostly spatial biases in numerical ordering, usually examined in children and less so in adults. To map numbers onto certain spatial locations (e.g., by counting spatially distributed tokens), a participant has to explicitly decide between different possible directions of this mapping, when he/she does not want to introduce directional shifts and jumps within the counting procedure. This is an important difference to the SNARC-like effects which are implicit and for which null effects can easily be observed. If a participant has to count, he/she has to choose directionality for each counting step (which is usually consistent across all counting steps).

Paradigms. The most popular way of assessing this SNA is to observe the direction of counting. Counting can take the form of enumerating objects that are distributed horizontally, vertically or in any other way. Several independent studies have consistently demonstrated that not only adults but also preschool children count in line with cultural habits dominant in their societies (e.g., left-to-right, top-to-bottom in Western countries; Göbel, 2015; Opfer, Thompson & Furlong, 2010; Shaki, Fischer & Göbel, 2012). Some other variations of a counting task have also been proposed, for instance, adding or removing tokens to or from a row (the dominant pattern in Western societies is to do so on the right endpoint of a row, Opfer & Thompson, 2006). However, counting direction is heavily influenced by the body. Patro, Nuerk & Cress (2015) showed that the counting hand influences the counting direction in preschoolers, especially in peripersonal space, in which it is usually assessed.

One specific instance of a counting activity is counting with fingers. The major difference between object and finger counting is that in the first case number words are

mapped onto extracorporeal physical space, and in the latter case onto specific body parts – fingers. In that sense, SNA based on finger counting biases can be viewed as more body-related. Another substantial difference between both tasks is that the direction of finger counting does not simply follow cultural habits. Instead, it seems to reflect the hand preference of participants – both adults and children asked to count spontaneously usually start from the thumb of their right hand and proceed to the left (Wasner, Moeller, Fischer & Nuerk, 2014, 2015). This preference is, however, flexible and can be changed towards the cultural pattern when fingers are presented to participants on a computer screen (Lindemann, Alipour & Fischer, 2011; Fischer, 2008). In any case, it is important to point out that studies examining actual counting behaviour and studies examining reported counting behaviour come to opposite results. While about 2/3 of German participants started counting spontaneously with the right hand, 2/3 of German participants actually reported starting counting with the left hand, when they had to respond in a questionnaire, where fingers were horizontally aligned (Wasner et al., 2014).

The third line of evidence on spatial-numerical ordering comes from the searching paradigm (Opfer & Furlong, 2011). In a typical searching task, children are asked to identify the location of an object that is hidden in one of several compartments numbered from left to right or from right to left. It has been shown that already 4-year-olds perform better in such a task when compartments are numbered from left to right.

Relation to arithmetic skills. The ability to count is conditioned by having at least basic numerical knowledge (Gelman & Gallistel, 1978). For instance, to count correctly, children have to understand the 1-to-1 correspondence between numbers and their spatial positions. They should also know the proper order of number words in a counting list. Clearly, without basic understanding of those principles it is difficult to count at all, and even more to count consistently in one direction. The counting direction of children can also be

related to a certain set of false beliefs regarding counting procedures. For instance, many preschoolers wrongly believe that the only correct way of counting is to start from the left side (Briars & Siegler, 1984; Kamawar et al., 2010); this false belief might, paradoxically, also reinforce the culturally-expected pattern of spatial-numerical mapping.

However, the main question that has not been addressed in the studies so far is to what extent spatial preferences in numerical ordering could affect our arithmetic learning. Although there is no direct empirical evidence on which we could base our remarks, we would like to formulate here certain predictions on that matter which could be hopefully tested in future studies.

We propose that consistent spatial biases in numerical ordering might provide a basis for understanding certain properties of the system of natural numbers. Combined visual, auditory and proprioceptive experience of a fixed spatial structure of a counting list (in which number words are always presented in the same order and occupy the same relative positions in a sequence) could obviously be supportive for capturing the idea of the successor function, the discrete nature of numerical symbols (because each number refers only to one distinctive location), the cardinal principle of counting (because cardinality of the whole set of objects can be inferred from the last number word – in both temporal and spatial sense), etc. What is more, spatial structure of counting reflects in a way the structure of a number line, and in this sense the number line can be visually and haptically accessible to children as soon as they are able to count. By counting in a spatially-consistent way children might already learn equidistant relations between neighbouring numbers which appear to be crucial for solving number line estimation tasks and many arithmetical tasks (see discussion on SNA – extensions 2-Intervals). Support for this view was obtained by Opfer and Thompson (2006) who observed a relation between the direction in which children added up and subtracted tokens and the accuracy of assigning numbers to spatial positions on a number line (which is

known to be a strong predictor of arithmetical skills). Given that frequency of culture-consistent counting systematically increases with the age of participants (reaching the level of almost 100% in adulthood, Shaki et al., 2012), one might suppose that the culture-consistent counting pattern is also a kind of strategy which is beneficial, at least to some extent, for the development of our numerical ordering skills. However, to date no data exist.

SNA-Directions: Explicit spatial coding for numerical cardinalities

To the best of our knowledge, no operationalization of this construct exists, but one is easily conceivable. One could simply present participants with numbers of different magnitudes, be they symbolic or non-symbolic and explicitly ask them to choose the most appropriate place for them in one or more horizontally aligned slots. Our expectation would be that in left-to-right reading cultures, larger cardinalities are also located more to the right in such an explicit task. However, as for all directional spatial-numerical effects, we do not believe that there will be a strong relation to arithmetic skills.

SNA-Directions: Explicit spatial coding for operations

To the best of our knowledge, no operationalization of this construct exists as well. One could present participants with vertically aligned addition and subtraction operations and ask them to place a card with the correct solution to the right or the left below that operation. In line with implicit operational momentum effects, we would expect that subtraction results are more on the left and additions more on the right even when overall magnitude is matched (cf. Klein et al., 2014) However, as with all directional spatial-numerical effects, we do not believe that there will be a large relation to arithmetic skills.

On the need to distinguish SNA constructs and not just operationalizations and effects

In our taxonomy we decided to include constructs underlying observed phenomena instead of classifying effects due to experimental tasks employed to investigate them. For instance, ordinality is a mathematical and – in our view – a psychological construct. Counting is just one of many possible operationalizations of assessing ordinality and its spatial directionality. Many other operationalizations are conceivable. We could explicitly ask participants to order dot sets or symbolic numbers of different magnitudes and see which directional ordering principle they prefer. Or we could use an implicit SNARC task where we assess whether the ordinal position in a sequence is associated with a certain spatial location, i.e., ordinality with spatial directionality.

Speaking in terms of personality psychology paradigms, one can view this taxonomy within the framework of a multi-trait multi-method approach (Campbell and Fiske, 1959). We suggest that the personality traits, (i.e. constructs like cardinality, ordinality for numbers; directionality, extension for space) should be the relevant structural principles in the taxonomy, and not the method employed (e.g., SNARC RT differences in a choice reaction task).

This basic assumption can be tested in multi-trait, multi-method approaches (see Campbell & Fiske, 1959 for details). In general, the correlations within the same construct over different methods should be higher than correlations within the same method over different constructs. So, SNARC effects for ordinal and cardinal concepts should not be strongly correlated, while different ways of measuring ordinal knowledge should be more highly correlated.

Just because we are measuring the same effects on average with the same method, this does not mean that we are assessing the same underlying constructs. For instance, even SNARC effects for different notations in the same task are mostly not significantly correlated with one another (Wood, Nuerk, & Willmes, 2006). This suggests that even within the

construct field cardinality in implicit directional space, different sub-constructs depending on symbolic or non-symbolic magnitude may need to be assumed in the future. However, this goes beyond the scope of this paper, especially since there are no data in which association with space is more closely related to arithmetic skills and learning.

Conclusions and recommendations

In most articles about spatial-numerical associations (aka the mental number line), these associations are regarded as one of the basic cornerstones of numerical and arithmetic skills. Our review surprisingly shows that substantial and consistent relations between SNA and math skills were only observed in some instances of SNA and under specific conditions. First of all, relationships are present for congruency of spatial and numerical intervals in children. Nevertheless, this effect probably originates from associated proportional arithmetic strategies and not because a spatial interval is matched to a numerical interval. Otherwise, Link et al. (2014) should have observed relations to arithmetic skills not only for the bounded but also for the unbounded tasks. In adult studies, only a small relationship between the SNARC effect and math skills was observed. However, the effect usually only appears when extreme groups are tested. Therefore, its practical importance is largely limited. For all other SNAs, the studies are either lacking or they show no or a very small relationship to arithmetic skills. These conclusions are also summarised in Table 1.

<Insert Table 1 around here>

Altogether, this overview shows that the results are somewhat disillusioning and do not support the common narrative of numerical cognition that SNAs are extremely important for math performance. Importantly, these small or null correlations are not due to null variance: Considerable interindividual variability can be found in all SNA categories.

Open questions, limitations, and recommendations

Apart from training studies, the observed relationship between elementary numerical processing (including SNAs) and arithmetic skill is correlational in nature. Therefore, in most cases there is no possibility to draw strong causal inferences. From a theoretical point of view, when correlations exist, at least three causal models are possible (see also Figure 2).

<Insert Figure 2 around here>

- (i) Extensive training in mathematics could influence and modify number representations so that they are more optimized and efficient.
- (ii) An individual's characteristics of elementary numerical processing underlie arithmetic skills. There are some experimental data showing that interventions aimed at shaping proper numerical representation (e.g., equidistant spatial intervals between numbers differing the same in magnitude) generalize to arithmetic skills. This was present in the case of number line trainings, especially those involving physical movements in space (Link et al., 2014). Several studies show that efficiency of SNA is a significant predictor of future arithmetic skills (see Siegler, 2009). Nevertheless, this field requires further exploration. What is even more important is that, up to now, training only in selected instances of SNA was shown to influence arithmetic skills. It also seems that the effects are observed rather early in development. There is still no evidence for such improvements in adults.
- (iii) There is a more general and individually varied trait that can account for both elementary numerical processing and arithmetic skills. A possible candidate

here would be fluid intelligence (including working memory efficiency) which may account for arithmetic skills. Moreover, fluid intelligence is also highly correlated with efficiency of cognitive control, including inhibition capacities. Cognitive control efficiency then influences SNA (as it was demonstrated for instance in the case of the SNARC effect; Hoffmann, Pigat & Schiltz, 2014b). In some studies, these influences were controlled as covariates. Usually, they significantly predict math skills, but elementary numerical processing explains some additional variance as well (Link et al., 2014). It is thus very important to carefully control for general cognitive capacities in future studies as well.

Finally, the small correlations between SNAs and arithmetic skills do not imply that spatial-numerical trainings, especially using variants of the NLE do not work. Many studies have shown that such trainings function well and training success transfers to other skills. However, the reason for this might not be that the spatial-numerical representations underlying arithmetic skills themselves improve but rather associated strategies and arithmetic skills are trained more powerfully with a spatial-numerical task. In essence, SNAs may be a good tool or mediator to effectively train arithmetic skills, but not necessarily the cornerstone representation on which complex arithmetic skills are built upon.

On the basis of this review, we wish to formulate several take-home-messages: First, the neural circuits responsible for spatial and numerical processing largely overlap; therefore, space and numbers are related in many different categories and paradigms. Second, however, different SNAs (especially those clusters concerning non-directional and directional SNAs) need to be distinguished. They are conceptually different and their correlations to arithmetic and educational skills differ depending on the category. Third, the (cor-)relation between basic SNAs and complex arithmetic skills, which is the goal of education, is generally rather poor. Fourth, even if such relations exist, good SNAs might not be the cause of good

arithmetic skills, but the relation might be the other way round or mediated by other general cognitive variables (especially inhibition capacities). Fifth, the generally low correlations between SNAs and arithmetic skills should not tell educators to refrain from spatial metaphors. They are successful and can be a powerful tool for the acquisition of arithmetic skills, even though good SNAs may not be the fundamental cause for such acquisitions.

REFERENCES

- Bachot, J., Gevers, W., Fias, W., & Roeyers, H. (2005). Number sense in children with visuospatial disabilities: Orientation of the mental number line. *Psychology Science*, 47(1), 172-183.
- Barth, H., Slusser, E., Cohen, D., & Paladino, A. (2011). A sense of proportion: Commentary on Opfer, Siegler, and Young. *Developmental Science*, 14(5), 1205-1206.
- Berteletti, I., Man, G., & Booth, J. R. (2015). How number line estimation skills relate to neural activations in single digit subtraction problems. *NeuroImage*, 107, 198-206.
- Bonato, M., Zorzi, M., & Umiltà, C. (2012). When time is space: evidence for a mental time line. *Neuroscience & Biobehavioral Reviews*, 36(10), 2257-2273.
- Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of numerical fractions: Real or integer?. *Journal of Experimental Psychology: Human Perception and Performance*, 33(6), 1410-1419.
- Booth, J. L., & Siegler, R. S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 42(1), 189-201.
- Briars, D., & Siegler, R. S. (1984). A featural analysis of preschoolers' counting knowledge. *Developmental Psychology*, 20(4), 607-618.
- Bugden, S., & Ansari, D. (2011). Individual differences in children's mathematical competence are related to the intentional but not automatic processing of Arabic numerals. *Cognition*, 118(1), 32-44.
- Bull, R., Espy, K. A., Wiebe, S. A., Sheffield, T. D., & Nelson, J. M. (2011). Using confirmatory factor analysis to understand executive control in preschool children: sources of variation in emergent mathematic achievement. *Developmental Science*, 14(4), 679-692.

- Bull, R., Cleland, A. A., & Mitchell, T. (2013). Sex differences in the spatial representation of number. *Journal of Experimental Psychology: General*, 142(1), 181-192.
- Calabria, M., & Rossetti, Y. (2005). Interference between number processing and line bisection: a methodology. *Neuropsychologia*, 43(5), 779-783.
- Campbell, D. T., & Fiske, D. W. (1959). Convergent and discriminant validation by the multitrait-multimethod matrix. *Psychological bulletin*, 56(2), 81-105.
- Cipora, K., & Nuerk, H.-C. (2013). Is the SNARC effect related to the level of mathematics? No systematic relationship observed despite more power, more repetitions, and more direct assessment of arithmetic skill. *The Quarterly Journal of Experimental Psychology*, 66(10), 1974-1991.
- Cipora, K., Hohol, M., Nuerk, H.-C., Willmes, K., Brożek, B., Kucharzyk, B., & Nęcka, E. (2015). Professional mathematicians differ from controls in their spatial-numerical associations. *Psychological Research* DOI: 10.1007/s00426-015-0677-6.
- Clark, C. A., Pritchard, V. E., & Woodward, L. J. (2010). Preschool executive functioning abilities predict early mathematics achievement. *Developmental psychology*, 46(5), 1176-1191.
- Cohen, D. J., & Blanc-Goldhammer, D. (2011). Numerical bias in bounded and unbounded number line tasks. *Psychonomic Bulletin & Review*, 18(2), 331-338.
- Crollen, V., & Noël, M. P. (2015). Spatial and numerical processing in children with high and low visuospatial abilities. *Journal of Experimental Child Psychology*, 132, 84-98.
- de Hevia, M. D., & Spelke, E. S. (2009). Spontaneous mapping of number and space in adults and young children. *Cognition*, 110(2), 198-207.
- de Hevia, M. D., & Spelke, E. S. (2010). Number-space mapping in human infants. *Psychological Science*, 21(5), 653-660.

- de Hevia, M. D., Izard, V., Coubart, A., Spelke, E. S., & Streri, A. (2014a). Representations of space, time, and number in neonates. *Proceedings of the National Academy of Sciences*, *111*(13), 4809-4813.
- de Hevia, M. D., Girelli, L., Addabbo, M., & Cassia, V. M. (2014b). Human infants' preference for left-to-right oriented increasing numerical sequences. *PloS one*, *9*(5), e96412.
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General*, *122*(3), 371-396.
- Dehaene, S., & Cohen, L. (1995). Towards an anatomical and functional model of number processing. *Mathematical Cognition*, *1*(1), 83-120.
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*(3-6), 487-506.
- Dehaene, S., & Cohen, L. (2007). Cultural recycling of cortical maps. *Neuron*, *56*(2), 384-398.
- Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5-to 9-year old children: Evidence for a segmented linear model. *Journal of Experimental Child Psychology*, *99*(1), 1-17.
- Ebersbach, M., Luwel, K., & Verschaffel, L. (2015). The relationship between children's familiarity with numbers and their performance in bounded and unbounded number line estimations. *Mathematical Thinking and Learning*, *17*(2-3), 136-154.
- Eerland, A., Guadalupe, T. M., & Zwaan, R. A. (2011). Leaning to the Left Makes the Eiffel Tower Seem Smaller Posture-Modulated Estimation. *Psychological science*, *22*(12), 1511-1514.

- Fias, W., Lauwereyns, J., & Lammertyn, J. (2001). Irrelevant digits affect feature-based attention depending on the overlap of neural circuits. *Cognitive Brain Research*, 12(3), 415-423.
- Fias, W., van Dijck, J. P., & Gevers, W. (2011). How number is associated with space? The role of working memory. In S. Dehaene & E. Brannon (Eds.), *Space, time and number in the brain: Searching for the foundations of mathematical thought* (pp. 133–148). London, UK.
- Fischer, M. (2003a). Spatial representations in number processing--evidence from a pointing task. *Visual cognition*, 10(4), 493-508.
- Fischer, M. H. (2003b). Cognitive representation of negative numbers. *Psychological Science*, 14(3), 278-282.
- Fischer, M. H. (2008). Finger counting habits modulate spatial-numerical associations. *Cortex*, 44(4), 386-392.
- Fischer, M. H., Castel, A. D., Dodd, M. D., & Pratt, J. (2003). Perceiving numbers causes spatial shifts of attention. *Nature Neuroscience*, 6(6), 555-556.
- Fischer, M. H., & Rottmann, J. (2005). Do negative numbers have a place on the mental number line. *Psychology Science*, 47(1), 22-32.
- Fischer, M. H., & Shaki, S. (2014). Spatial associations in numerical cognition—From single digits to arithmetic. *The Quarterly Journal of Experimental Psychology*, 67(8), 1461-1483.
- Fischer, M. H., Warlop, N., Hill, R. L., & Fias, W. (2004). Oculomotor bias induced by number perception. *Experimental Psychology*, 51(2), 91-97.
- Fischer, U., Moeller, K., Bientzle, M., Cress, U., & Nuerk, H.-C. (2011). Sensori-motor spatial training of number magnitude representation. *Psychonomic Bulletin & Review*, 18(1), 177-183.

- Fuhs, M. W., & McNeil, N. M. (2013). ANS acuity and mathematics ability in preschoolers from low-income homes: contributions of inhibitory control. *Developmental Science*, 16(1), 136-148.
- Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. *Journal of Educational Psychology*, 104(1), 206-223.
- Gelman, R., Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.
- Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., ... & Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS One*, 8(6), e67374.
- Ginsburg, V., van Dijck, J. P., Previtali, P., Fias, W., & Gevers, W. (2014). The impact of verbal working memory on number–space associations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 40(4), 976-986.
- Göbel, S. M. (2015). Up or down? Reading direction influences vertical counting direction in the horizontal plane—a cross-cultural comparison. *Frontiers in psychology*, 6.
- Hartmann, M., Mast, F. W., & Fischer, M. H. (2015). Spatial Biases During Mental Arithmetic: Evidence from Eye Movements on a Blank Screen. *Name: Frontiers in Psychology*, 6, 12.
- Helmreich, I., Zuber, J., Pixner, S., Kaufmann, L., Nuerk, H.-C., & Moeller, K. (2011). Language effects on children's non-verbal number line estimations. *Journal of Cross-Cultural Psychology*, 42(4), 598-613.
- Henik, A., & Tzelgov, J. (1982). Is three greater than five: The relation between physical and semantic size in comparison tasks. *Memory & Cognition*, 10(4), 389-395.

- Hoffmann, D., Hornung, C., Martin, R., & Schiltz, C. (2013). Developing number–space associations: SNARC effects using a color discrimination task in 5-year-olds. *Journal of Experimental Child Psychology*, 116(4), 775-791.
- Hoffmann, D., Mussolin, C., Martin, R., & Schiltz, C. (2014a). The impact of mathematical proficiency on the number-space association. *PLoS one*, 9(1), e85048.
- Hoffmann, D., Pigat, D., & Schiltz, C. (2014b). The impact of inhibition capacities and age on number–space associations. *Cognitive processing*, 15(3), 329-342.
- Hubbard, E. M., Piazza, M., Pinel, P., & Dehaene, S. (2005). Interactions between number and space in parietal cortex. *Nature Reviews Neuroscience*, 6(6), 435-448.
- Huber, S., Moeller, K., & Nuerk, H.-C. (2014). Dissociating number line estimations from underlying numerical representations. *The Quarterly Journal of Experimental Psychology*, 67(5), 991-1003.
- Ischebeck, A., Zamarian, L., Egger, K., Schocke, M., Delazer, M. (2007). Imaging early practice effects in arithmetic. *Neuroimage* 36, 993-1003.
- Jeannerod, M., Decety, J., & Michel, F. (1994). Impairment of grasping movements following a bilateral posterior parietal lesion. *Neuropsychologia*, 32(4), 369-380.
- Kamawar, D., LeFevre, J. A., Bisanz, J., Fast, L., Skwarchuk, S. L., Smith-Chant, B., & Penner-Wilger, M. (2010). Knowledge of counting principles: How relevant is order irrelevance?. *Journal of Experimental Child Psychology*, 105(1), 138-145.
- Katz, C., & Knops, A. (2014). Operational momentum in multiplication and division?. *PloS one*, 9(8), e104777.
- Kaufmann, L., & Nuerk, H.-C. (2008). Basic number processing deficits in ADHD: a broad examination of elementary and complex number processing skills in 9-to 12-year-old children with ADHD-C. *Developmental Science*, 11(5), 692-699.

- Kaufmann, L., Nuerk, H.-C., Graf, M., Krinzinger, H., Delazer, M., & Willmes, K. (2009). *TEDI-MATH Test zur Erfassung numerisch-rechnerischer Fertigkeiten vom Kindergarten bis zur 3. Klasse*. Bern: Huber.
- Klein, E., Huber, S., Nuerk, H.-C., & Moeller, K. (2014). Operational momentum affects eye fixation behaviour. *The Quarterly Journal of Experimental Psychology*, 67(8), 1614-1625.
- Knops, A., Viarouge, A., & Dehaene, S. (2009a). Dynamic representations underlying symbolic and nonsymbolic calculation: Evidence from the operational momentum effect. *Attention, Perception, & Psychophysics*, 71(4), 803-821.
- Knops, A., Thirion, B., Hubbard, E. M., Michel, V., & Dehaene, S. (2009b). Recruitment of an area involved in eye movements during mental arithmetic. *Science*, 324(5934), 1583-1585.
- Knops, A., Zitzmann, S., & McCrink, K. (2013). Examining the presence and determinants of operational momentum in childhood. *Frontiers in psychology*, 4.
- Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., Gälli, M., Martin, E. & von Aster, M. (2011). Mental number line training in children with developmental dyscalculia. *Neuroimage*, 57(3), 782-795.
- Landy, D., & Goldstone, R. L. (2010). Proximity and precedence in arithmetic. *The Quarterly Journal of Experimental Psychology*, 63(10), 1953-1968.
- Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, 105(2), 395-438.
- Lindemann, O., Alipour, A., & Fischer, M. H. (2011). Finger counting habits in middle eastern and western individuals: an online survey. *Journal of Cross-Cultural Psychology*, 42(4), 566-578.

- Link, T., Moeller, K., Huber, S., Fischer, U., & Nuerk, H.-C. (2013). Walk the number line—An embodied training of numerical concepts. *Trends in Neuroscience and Education*, 2(2), 74-84.
- Link, T., Nuerk, H.-C., & Moeller, K. (2014). On the relation between the mental number line and arithmetic competencies. *The Quarterly Journal of Experimental Psychology*, 67(8), 1597-1613.
- Loetscher, T., Schwarz, U., Schubiger, M., & Brugger, P. (2008). Head turns bias the brain's internal random generator. *Current Biology*, 18(2), R60-R62.
- Lonnemann, J., Krinzinger, H., Knops, A., & Willmes, K. (2008). Spatial representations of numbers in children and their connection with calculation abilities. *Cortex*, 44(4), 420-428.
- Lourenco, S. F., & Longo, M. R. (2010). General magnitude representation in human infants. *Psychological Science* 21(6), 873-881.
- Lugli, L., Baroni, G., Anelli, F., Borghi, A. M., & Nicoletti, R. (2013). Counting is easier while experiencing a congruent motion. *PloS one*, 8(5), e64500.
- Masson, N., & Pesenti, M. (2014). Attentional bias induced by solving simple and complex addition and subtraction problems. *The Quarterly Journal of Experimental Psychology*, 67(8), 1514-1526.
- McCrink, K., Dehaene, S., & Dehaene-Lambertz, G. (2007). Moving along the number line: Operational momentum in nonsymbolic arithmetic. *Perception & Psychophysics*, 69(8), 1324-1333.
- McCrink, K., & Wynn, K. (2009). Operational momentum in large-number addition and subtraction by 9-month-olds. *Journal of Experimental Child Psychology*, 103(4), 400-408.

- Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H.-C. (2009). Children's early mental number line: Logarithmic or decomposed linear? *Journal of Experimental Child Psychology*, 103(4), 503-515.
- Moyer, R. S., & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature*, 215, 1519-1520.
- Nuerk, H.-C., Bauer, F., Krummenacher, J., Heller, D., & Willmes, K. (2005b). The power of the mental number line: how the magnitude of unattended numbers affects performance in an Eriksen task. *Psychology Science*, 47, 34-50.
- Nuerk, H.-C., Iversen, W., & Willmes, K. (2004). Notational modulation of the SNARC and the MARC (linguistic markedness of response codes) effect. *Quarterly Journal of Experimental Psychology Section A*, 57(5), 835-863.
- Nuerk, H.-C., Wood, G., & Willmes, K. (2005a). The universal SNARC effect. *Experimental Psychology (formerly Zeitschrift für Experimentelle Psychologie)*, 52(3), 187-194.
- Opfer, J. E., & Siegler, R. S. (2007). Representational change and children's numerical estimation. *Cognitive Psychology*, 55(3), 169-195.
- Opfer, J. E., & Thompson, C. A. (2006). Even early representations of numerical magnitude are spatially organized: Evidence for a directional magnitude bias in pre-reading preschoolers. In R. Sun and N. Miyaki (Eds.), *Proceedings of the 28th Annual Conference of the Cognitive Science Society* (pp. 639-644). Mahwah, NJ: Erlbaum.
- Opfer, J. E., Thompson, C. A., & Furlong, E. E. (2010). Early development of spatial-numeric associations: evidence from spatial and quantitative performance of preschoolers. *Developmental Science*, 13(5), 761-771.
- Opfer, J. E., & Furlong, E. E. (2011). How numbers bias preschoolers' spatial search. *Journal of Cross-Cultural Psychology*, 42(4), 682-695.

- Patro, K., & Haman, M. (2012). The spatial–numerical congruity effect in preschoolers. *Journal of Experimental Child Psychology*, 111(3), 534-542.
- Patro, K., Nuerk, H.-C., Cress, U., & Haman, M. (2014). How number-space relationships are assessed before formal schooling: A taxonomy proposal. *Frontiers in Psychology*, 5.
- Patro, K., Nuerk, H.-C., & Cress, U. (2015). Does your body count? Embodied influences on the preferred counting direction of preschoolers. *Journal of Cognitive Psychology*, 27(4), 413-425.
- Pinhas, M., & Fischer, M. H. (2008). Mental movements without magnitude? A study of spatial biases in symbolic arithmetic. *Cognition*, 109(3), 408-415.
- Pinhas, M., Shaki, S., & Fischer, M. H. (2014). Heed the signs: Operation signs have spatial associations. *The Quarterly Journal of Experimental Psychology*, 67(8), 1527-1540.
- Rodic, M., Zhou, X., Tikhomirova, T., Wei, W., Malykh, S., Ismatulina, V., ... & Kovas, Y. (2015). Cross-cultural investigation into cognitive underpinnings of individual differences in early arithmetic. *Developmental Science*, 18(1), 165-174.
- Sasanguie, D., Defever, E., Maertens, B., & Reynvoet, B. (2014). The approximate number system is not predictive for symbolic number processing in kindergarteners. *The Quarterly Journal of Experimental Psychology*, 67(2), 271-280.
- Schneider, M., Grabner, R. H., & Paetsch, J. (2009). Mental number line, number line estimation, and mathematical achievement: Their interrelations in grades 5 and 6. *Journal of Educational Psychology*, 101(2), 359-372.
- Schwarz, W., & Keus, I. M. (2004). Moving the eyes along the mental number line: Comparing SNARC effects with saccadic and manual responses. *Perception & Psychophysics*, 66(4), 651-664.
- Schwarz, W., & Müller, D. (2006). Spatial associations in number-related tasks. *Experimental Psychology* 53, 4-15.

- Shaki, S., Fischer, M. H., & Petrusic, W. M. (2009). Reading habits for both words and numbers contribute to the SNARC effect. *Psychonomic Bulletin & Review*, 16(2), 328-331.
- Shaki, S., & Fischer, M. H. (2012). Multiple spatial mappings in numerical cognition. *Journal of Experimental Psychology: Human Perception and Performance*, 38(3), 804-809.
- Shaki, S., Fischer, M. H., & Göbel, S. M. (2012). Direction counts: a comparative study of spatially directional counting biases in cultures with different reading directions. *Journal of Experimental Child Psychology*, 112(2), 275-281.
- Shaki, S., & Fischer, M. H. (2014). Random walks on the mental number line. *Experimental Brain Research*, 232(1), 43-49.
- Siegler, R. S. (2009). Improving the numerical understanding of children from low-income families. *Child Development Perspectives*, 3(2), 118-124.
- Siegler, R. S., & Booth, J. L. (2004). Development of numerical estimation in young children. *Child Development*, 75(2), 428-444.
- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation evidence for multiple representations of numerical quantity. *Psychological Science*, 14(3), 237-250.
- Siegler, R. S., & Ramani, G. B. (2009). Playing linear number board games—but not circular ones—improves low-income preschoolers' numerical understanding. *Journal of Educational Psychology*, 101(3), 545-560.
- Slusser, E. B., Santiago, R. T., & Barth, H. C. (2013). Developmental change in numerical estimation. *Journal of Experimental Psychology: General*, 142(1), 193-208.
- Tudusciuc, O., & Nieder, A. (2007). Neuronal population coding of continuous and discrete quantity in the primate posterior parietal cortex. *Proceedings of the National Academy of Sciences*, 104(36), 14513-14518.

- Turconi, E., Jemel, B., Rossion, B., & Seron, X. (2004). Electrophysiological evidence for differential processing of numerical quantity and order in humans. *Cognitive Brain Research*, 21(1), 22-38.
- Tversky, B. (2005). Functional Significance of Visuospatial Representations. In: P. Shah & A. Miyake (Ed). *The Cambridge handbook of visuospatial thinking* p. 1-34. New York: Cambridge University Press
- Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: a meta-analysis of training studies. *Psychological Bulletin*, 139(2), 352-402.
- van Dijck, J. P., Abrahamse, E. L., Acar, F., Ketels, B., & Fias, W. (2014). A working memory account of the interaction between numbers and spatial attention. *The Quarterly Journal of Experimental Psychology*, 67(8), 1500-1513.
- van Dijck, J. P., & Fias, W. (2011). A working memory account for spatial–numerical associations. *Cognition*, 119(1), 114-119.
- Viarouge, A., Hubbard, E. M., & McCandliss, B. D. (2014). The Cognitive Mechanisms of the SNARC Effect: An Individual Differences Approach. *PloS one*, 9(4), e95756.
- Walsh, V. (2003). A theory of magnitude: common cortical metrics of time, space and quantity. *Trends in Cognitive Sciences*, 7(11), 483-488.
- Wasner, M., Moeller, K., Fischer, M. H., & Nuerk, H.-C. (2014). Aspects of situated cognition in embodied numerosity: the case of finger counting. *Cognitive Processing*, 15(3), 317-328.
- Wasner, M., Moeller, K., Fischer, M. H., & Nuerk, H.-C. (2015). Related but not the same: Ordinality, cardinality and 1-to-1 correspondence in finger-based numerical representations. *Journal of Cognitive Psychology*, 27(4), 426-441.

- Wiemers, M., Bekkering, H., & Lindemann, O. (2014). Spatial interferences in mental arithmetic: Evidence from the motion–arithmetic compatibility effect. *The Quarterly Journal of Experimental Psychology*, 67(8), 1557-1570.
- Wood, G., Nuerk, H.-C., & Willmes, K. (2006). Crossed Hands and the SNARC Effect: A failure to replicate Dehaene, Bossini and Giraux (1993). *Cortex*, 42(8), 1069-1079.
- Wood, G., Willmes, K., Nuerk, H.-C., & Fischer, M. H. (2008). On the cognitive link between space and number: A meta-analysis of the SNARC effect. *Psychology Science Quarterly*, 50(4), 489-525.
- Wynn K. (1992). Children's acquisition of number words and the counting system. *Cognitive Psychology*, 24, 220–251.

Footnotes

¹ Tudusciuc and Nieder (2007) interpreted their data in terms of continuous and discrete judgements which is not incompatible with our interpretation, but could be complementary. However, for the purpose of this review, it is important that spatial decisions (line length) and numerical decisions (non-symbolic magnitude, dot sets) were subserved by the same population of neurons in the IPS.

² Note that in our taxonomy we did not include two thoroughly studied phenomena: numerical distance effect and size effect (Moyer & Landauer, 1967). These phenomena reflect analogue nature of numerical representations; nevertheless they are not directly linked to spatial mapping (see Bonato, Zorzi & Umiltà, 2012 for similar claims). It also seems that magnitude can be processed without space since numerical distance effects can be obtained in the absence of spatial-numerical effects (see Nuerk et al., 2005b). For these reasons these phenomena are outside the scope of this paper.

³ Noteworthy, there is some recent evidence that especially in younger children this type of SNA has a directional component. In pre-schoolers the estimation accuracy is higher when the number line starts on the left and extends to the right than in the opposite direction. A similar pattern can be found in second graders but the difference between conditions is smaller (Ebersbach et al., 2015). Nevertheless, the effect requires further investigation, and now we have decided to include this SNA as referring to spatial extensions without directionalities since other studies have not reported a directional component (e.g. Link et al., 2013).

Table 1. Summary of observed relationships between SNA and math skills in children and adult groups. In the case of other discussed SNAs no attempts were taken in order to investigate its relation to math skills nor were null effects observed.

Age group	SNAs related to math skill
Children	<p>Ability to inhibit influences of non-numerical spatial cues correlates with math skill, but the effect is probably mediated by inhibition capacities.</p> <p>Bounded number line estimation – children who perform better and whose estimation pattern is linear reveal better math skill. Effect not present in unbounded number line estimation.</p> <p>Spatial training (number line estimation, linking number magnitudes in spatial directions, embodied training of number line estimation involving whole body movements) generalizes over untrained calculation skills.</p> <p>SNARC effect – attenuated in children with Visuospatial Disorder (VSD). No differences observed within normal math skill range. Nevertheless the results are inconsistent with more recent findings.</p>
Adults	<p>SNARC effect – smaller in individuals with high math skill and stronger in those with poor math skill – relationship is present mostly when extreme skill groups are considered.</p>

Figure 1. Overview of proposed SNA taxonomy. SNAs are classified based on spatial representation type (extension versus directionality) and numerical representation type (cardinality, interval, cardinality, operations). Theoretically postulated SNAs are marked with a dashed line.

Figure 2. Possible patterns of causal relationships between SNA and arithmetic skills. (I) extensive training shapes SNAs; (II) SNA is a base for arithmetic skills; (III) other variables determine both SNAs and arithmetic skills. For a further discussion please refer to the main text.