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Analytical prediction of shear angle and frictional behaviour in vibration-assisted cutting

Wei Bai^a, Anish Roy^b, Lingxi Guo^c, Jianfeng Xu^{a,*}, Vadim V. Silberschmidt^b

^a State Key Lab of Digital Manufacturing Equipment and Technology, Huazhong University of Science and Technology, Wuhan, 430074, China

^b Wolfson School of Mechanical, Electrical and Manufacturing Engineering, Loughborough University, Leicestershire, LE11 3TU, UK

^c Shanghai Aerospace Control Technology Institute, Shanghai, 201109, China

A B S T R A C T

Vibration-assisted cutting (VAC), a promising technique, proved to enhance the machinability of difficult-to-cut materials. Its significant superiority with regard to conventional cutting (CC) is considered to be closely related to variation of a shear angle in the primary shear zone and specific frictional behaviour at tool-chip interface. This paper analyses kinematics of VAC, focusing on critical stages of tool-workpiece interaction. Based on the evolution of kinematic parameters, a transient shear angle and a tool-chip contact length are investigated in a cycle according to these stages. To predict the transient parameters, an analytical model of the cutting process is proposed based on non-equidistant shear-zone and tool-chip sliding-sticking zone theories. This model for VAC can not only predict the dominant parameters of the cutting process (e.g., cutting force, friction coefficient), but also the secondary ones (e.g., shear strain). Experimental validation of the developed model is performed with orthogonal VAC of titanium alloy, and the shear angles are measured with optical microscopy of chip samples. For various process parameters, the effective shear angle in VAC is larger than that in CC. However, the average shear angle in VAC is smaller than the shear angle in CC. The proposed model can not only effectively predict the shear angle and frictional behaviour in VAC, but also other process parameters in a vibration cycle, enriching the theory of the VAC process.

Keywords:

Vibration-assisted cutting
Shear angle
Tool-chip contact length
Non-equidistant shear zone
Tool-chip sliding-sticking zone
Analytical model

1. Introduction

Vibration-assisted cutting (VAC) is a promising process, with various benefits compared to conventional cutting (CC). In VAC, micro-scale high-frequency vibration is applied to improve the machinability of difficult-to-machine materials. A wide range of applications demonstrated that it has several advantages such as reduction in the average cutting force, improvement of the cutting stability and surface quality, increase in tool life, etc. [1]. The cutting mechanism of VAC, especially in a vibration cycle, is not clear as a result of the evolution of complex thermo-mechanical processes at high-frequency vibration. For a complex cutting process, two main mechanisms can be identified: first, formation of a chip along the primary shear zone with intense shearing; second, a sustained movement of the chip along the tool-chip interface accompanied by friction effects and formation of a secondary shear zone [2].

In a cutting process, the shear angle is considered as a fundamental parameter that defines plastic deformation and the process geometry

[3]. It is important to have an access to a local distribution of tribological parameters controlling the tool-chip contact [4]. For the plane-strain conditions of orthogonal cutting, once the shear angle and tool-chip friction are defined, all other process parameters could be predicted [3–5].

The shear and frictional behaviours in cutting were studied with three fundamental approaches: empirical, analytical and numerical. A slip-line field was constructed in the chip-formation zone obtained from experiments to predict a shear angle by Oxley [6]. Wright [7] proposed an upper-bound analysis to predict a shear angle using strain-hardening characteristics of a workpiece. Besides, the Lee and Shaffer's [8] and Merchant's [9] shear-angle laws are widely adopted in orthogonal cutting, based on the principles of the maximum shear stress and the minimum energy, respectively. For tool-chip contact mechanics, several approaches were developed to characterize complexity of a tool-chip interface. A broadly used scheme with sticking and sliding zones, located away from a tool tip along its rake face, was proposed to describe the frictional behaviour [10,11]. Also, a numerical model can be used to

* Corresponding author.

E-mail address: jfxu@hust.edu.cn (J. Xu).

Fig. 1. Schematic diagram of VAC process.

analysis the contact characteristics [12]. Özel [13] employed a thermo-mechanical updated Lagrangian finite-element formulation accounting for the effects of friction of a tool-chip interface friction to simulate the chip formation in orthogonal cutting.

For VAC processes, the reduction of cutting forces was widely reported. Researchers tried to explain the reason of this phenomenon through the increase in a shear angle and the decrease in friction. However, the research on the shear-angle and frictional behaviours in VAC, especially their evolution in a single cycle of vibration, are still rare. Hu and Shao [14] proposed a slip-line cutting model of ultrasonically assisted cutting, that demonstrated that the shear angle in VAC was larger than that in CC. Zhang et al. [15] investigated the instantaneous effective rake and shear angles for different magnitudes of undeformed chip thickness in VAC. They also proposed an analytical model, derived from the force analysis of elastic-plastic deformation steps of low-frequency VAC for an equivalent shear angle [16]. However, they only predicted the equivalent shear angle for VAM. Bai et al. [17] analysed the changes of transient shear angle during a vibration cycle by dividing the cutting process into three stages in elliptical vibration

cutting. In addition, some studies investigated the effect of vibration on frictional behaviour. Amini and Kazemiyoun [18] performed quick-stop cutting experiments to measure the chip topography in vibration-assisted turning, and established that ultrasonic vibration could increase the shear angle and decrease the tool-chip contact length. Jamshidi and Nategh [19] proposed a theory for the tool-chip contact length by considering the effect of vibration on the cutting speed. The tool-chip contact length was expressed as the product of a tool-chip contact length in CC and a velocity correlation function. The latter was obtained as exponential function by fitting the experimental data. A two-dimensional FE model was developed by Lotfi and Amini [20] to simulate a frictional behaviour at tool-chip interface in ultrasonically assisted machining. Similarly, the equivalent length of the sticking zone at tool-chip interface was investigated, not the transient variables. Although some works focused on an equivalent or average shear angle and tool-chip contact length in VAC, a further investigation is needed to reveal the evolution of the shear angle and tool-chip friction behaviour in a vibration cycle.

In this study, a model predicting frictional behaviour and evolution of shear angle in VAC is investigated. First, kinematics of VAC is analysed to define the critical instants of tool-workpiece interaction in Section 2. The transient shear angle and tool-chip contact length are defined in a vibration cycle for these instants in Sections 3 and 4, respectively. A model for prediction of process parameters in VAC is proposed in Section 5, that can predict the dominant and secondary parameters of the cutting process. Also, a series of experiments for validation are performed in Section 6. The paper ends with some concluding remarks in Section 7. It should be stated that our previous work [21] is referred to here to support a new independent model in this paper. The study [21] proposed an analytical model of chip formation for precise prediction of orthogonal process in CC, including the scheme for a primary shear zone and a tool-chip interface. This paper analyses the shear angle (in the primary shear zone) and frictional behaviour (at the tool-chip interface) in VAC; their evolution in a single vibration cycle is modelled but the employed relationships between internal parameters

Fig. 2. Schematic of VAC process: (a) evolution of tool position, velocity and relative cutting velocity in single vibration cycle; (b) tool path relative to workpiece; (c) relative positions of tool and workpiece at different times.

in the primary shear zone and tool-chip interface are obtained from our published model [21].

2. Kinematics of vibration-assisted cutting process

During a VAC process, a tool vibrates along the cutting direction at a frequency f (vibration period $T = 1/f$). High-frequency vibration of the tool causes the tool to separate periodically from the workpiece as shown in Fig. 1.

The cutting-tool position x in VAC can be presented as follows:

$$x = a \sin(\omega t) = a \sin(2\pi f t) \quad (1)$$

where a is the vibration amplitude, ω is the angular velocity, t is the cutting time.

The tool velocity can be expressed as

$$v_t = \dot{x} = 2\pi f a \cos(2\pi f t) \quad (2)$$

The relative cutting velocity v_r with regard to the workpiece at cutting speed V can be obtained as

$$v_r = v_t - (-V) = 2\pi f a \cos(2\pi f t) + V \quad (3)$$

The graphs of evolution of the tool position x , the tool velocity v_t and the relative cutting velocity v_r with time are shown in Fig. 2(a). The maximum vibration velocity of the tool is $v_{tmax} = 2\pi f a$. The VAC process should fulfil the condition $v_{tmax} > V$ for the tool and the workpiece to separate. Therefore, the cutting speed of the separated vibration cutting process should be less than the critical cutting speed v_{cr} , namely:

$$V < v_{cr} = 2\pi f a \quad (4)$$

During the cutting process, the relative cutting velocity is zero ($v_r = 0$) when the tool disengages from the tool-chip interface, where the disengagement instant is t_a (Fig. 2(a)), as follows:

$$2\pi f a \cos(2\pi f t_a) + V = 0 \quad (5)$$

After the tool and the workpiece are separated, the relative cutting velocity is zero again at the instant t_d . But the tool and the workpiece keep separation until the tool-workpiece reengagement moment t_b . The instant t_b is the same moment in the last vibration cycle. Thus, the total area of the shaded part in Fig. 2(a) is zero, which can be expressed as

$$\int_{t_a}^{t_b} (2\pi f a \cos(2\pi f t) + V) dt = 0 \quad (6)$$

In addition, the tool path relative to the workpiece is shown in Fig. 2(b), which presents the relationship of the tool-tip trajectory between CC and UAC when the cutting tool moves relative to the workpiece in both cutting and feed directions. Besides, it also shows the important instants (t_a , t_b , t_c , t_d and t_b') when the tool-tip moves along the path. Furthermore, the relative positions of the tool and the workpiece at different times are illustrated in Fig. 2(c). At instant t_c , the tool's velocity reduces to zero. The relative cutting velocity is zero at the instants t_a and t_d .

In each cycle, the separation time T_S and the cutting time T_C can be expressed as follows:

$$T_S = t_b - t_a \quad (7)$$

$$T_C = T + t_a - t_b \quad (8)$$

Then the tool-workpiece contact ratio r_{nw} is obtained as

$$r_{nw} = \frac{T_C}{T} \quad (9)$$

The separation distance travelled by the tool in a single cycle can be calculated by the hatched area from t_a to t_d . However, the effective cutting length is measured for a period from the tool-workpiece reengagement to their next separation. In a single vibration cycle, D_c is the

Fig. 3. Evolution of effective relative cutting velocity in single cycle of VAC.

cutting distance and D_s is the separation distance between the tool and the workpiece (noncutting distance). They can be calculated from the tool path based on t_a and t_b (Fig. 2). Therefore, the cutting distance D_c and the separation distance D_s between the tool and workpiece in each cycle can be calculated as

$$D_c = \int_{t_b}^{T+t_a} (2\pi f a \cos(2\pi f t) + V) dt = VT \quad (10)$$

$$D_s = - \int_{t_a}^{t_b} (2\pi f a \cos(2\pi f t) + V) dt = a \sin(2\pi f t_b) + V t_b + a - \frac{3}{4} VT \quad (11)$$

The relevant kinematic parameters of VAC can be calculated from the above formulas, where the instants of tool-workpiece separation t_a , their reengagement t_b and that when the relative cutting velocity is equal to the magnitude of the cutting speed V (t_c) are used below as important moments of time in a cycle of VAC.

Evolution of thermo-mechanical behaviour in a cycle of VAC is complex and the cutting mechanism is not clear. In order to reveal the evolution of physical parameters in the VAC process, several important variables are investigated and the modelling theory of orthogonal cutting is utilized. The effect of material's elasticity on chip separation is not considered in this paper. The shear angle and the tool-chip contact length are two important parameters of the primary and secondary shear zones during the cutting process, analysed for a single cycle of vibration in this paper.

When the relative cutting velocity calculated with Eq. (3) is negative, the tool's rake face is not in contact with the chip, i.e. there is no effective cutting. However, although the relative cutting velocity is positive from the moment t_d to t_b , the tool and the workpiece are still in the separated stage. Therefore, when the tool-workpiece reengagement occurs in the instant t_b , the cutting starts at a relatively high velocity. The effective relative cutting velocity v_{er} is defined as the level of relative cutting velocity when the effective cutting happens in a cycle of VAC as

$$v_{er} = \begin{cases} v_r, & 0 \leq t < t_a \\ 0, & t_a \leq t \leq t_b \\ v_r, & t_b < t \leq T \end{cases} \quad (12)$$

As shown in Fig. 3, the relative cutting velocity begins to decrease at the highest relative cutting velocity $2\pi f a + V$. At the instant t_a , when the tool and workpiece are separated, the distance between them gradually decreases until the reengagement instant t_b , and then the tool continues to cut the workpiece until the relative cutting velocity reaches the maximum again.

In this paper, the effective relative cutting velocity is employed to analyse the tool-chip contact length and the shear angle in a single vibration cycle. A study of the difficult-to-cut material Ti6Al4V is chosen,

Fig. 4. Evolution of transient shear angle in single cycle of VAC.

with vibration frequency of 20,220 Hz and amplitude of 7.7 μm . In the analysis and modelling parts, the cutting speed is 20 m/min, the feed rate is 0.1 mm/rev and the rake angle of the cutting tool is 6.5°; they are used in Sections 3 to 5. The machining parameters used in the experiments are presented in Section 6.

3. Shear angle in vibration-assisted cutting

As an important parameter of the cutting process, the shear angle affects the shear direction of the primary shear zone and the distribution of forces in the shear zone. The calculation of the shear angle in the VAC process is also divided into several stages.

When the relative cutting velocity reaches its maximum, the shear angle attains the maximum value $\phi_{\max}$. With the reduction of the relative cutting velocity, the shear angle gradually decreases. Similar trends were observed in CC before [21]. The important reason for the decrease of shear angle is that the decrease of relative cutting velocity increases the coefficient of friction, and then raises the friction angle. During this stage, the transient shear angle is ϕ_{tc} . From t_a to t_b , the tool and workpiece separate and approach each other, but they are not in contact. However, the shear angle reflects a geometric variable of the shear zone. After the tool-workpiece separation, the shear angle is considered as the geometric angle ϕ_a , which is a feature of the separation state. Therefore, the shear angle is calculated for the state when the tool is about to leave the workpiece in the tool-workpiece separation stage. At the instant t_b , when the tool-workpiece reengages, the shear angle increases instantaneously due to the sharply increased relative cutting velocity, increasing subsequently slowly to the maximum value because of the increase in the relative cutting velocity. Thus, the transient shear angle is expressed as

$$\phi_t = \begin{cases} \phi_{tc}, & 0 \leq t < t_a \\ \phi_a, & t_a \leq t < t_b \\ \phi_{tc}, & t_b \leq t \leq T \end{cases} \quad (13)$$

In the orthogonal cutting process, the shear angle satisfies the following relation:

$$\phi = A_1 + A_2(\alpha - \lambda) \quad (14)$$

where the material constants A_1 and A_2 were identified as 61° and -1 from experiments [21]. The apparent friction angle λ can be obtained as $\tan^{-1}\mu_a$, which is discussed in Section 4.

The transient shear angle ϕ_{tc} in a vibration cycle can be calculated through above model as shown in Fig. 4.

4. Tool-chip frictional behaviour in vibration-assisted cutting

At the tool-chip interface, the tool-chip frictional behaviour is also

very important for analysis of the VAC process, including the coefficient of friction and the tool-chip contact length. Generally, the tool-chip interface is divided into sticking and sliding zones. The coefficient of friction of the whole contact zone is called as the *apparent (global) coefficient of friction*, and that of the sliding zone is termed the *sliding coefficient of friction*.

The relationship between the apparent coefficient of friction μ_a and the sliding coefficient of friction μ_{sl} at the tool-chip interface was derived by Bai et al. [21] as

$$\mu_{sl} = \frac{\tau_{sy}}{P_0} \frac{1}{\left[1 - \frac{1}{\xi} \left(\frac{\mu_a P_0}{\tau_{sy}} - 1\right)\right]^\xi} \quad (15)$$

where the yield shear stress of the material τ_{sy} is equal to the shear stress at the exit of the primary shear zone. And the pressure on the tool tip P_0 can also be calculated by the shear stress at the exit of the primary shear zone. The distribution of shear stress in the primary shear zone is evaluated through the non-equidistant shear zone model reported in [21]. The pressure-distribution exponent ξ is assumed as 2 according to Childs et al. [22]. In our proposed model, the apparent coefficient of friction μ_a and the sliding coefficient of friction μ_{sl} are finally determined by several iterations for a given initial value of μ_a in Section 5.

The tool-chip contact length is the key variable for the tool-chip interface. It defines the length of the interaction between the tool's rake face and the workpiece and the zone of heat exchange in the rake face, which can be utilized to determine other parameters in the secondary shear zone.

In VAC, the tool-chip contact length changes with the periodic separation of the tool's rake face and the chip. Therefore, this section analyses the transient tool-chip contact length in VAC. In a single vibration cycle of VAC, the tool has no contact with the chip at the stage when the effective relative cutting velocity is zero; the transient tool-chip contact length is also zero. When the relative cutting velocity is larger than the cutting speed, the transient tool-chip contact length L_{tc} can be calculated with the conventional theory of orthogonal cutting. When the relative cutting velocity is equal to the cutting speed, the tool-chip contact length at this instant is considered to be the unique value L_{cc} determined by the cutting speed. The tool-chip contact length is assumed to be positively related to the relative cutting velocity because of the tool is about to separate from the workpiece. Thus, the transient tool-chip contact length in a cycle can be obtained as

$$L_{ct} = \begin{cases} L_{tc}, & v_r > V \\ L_{cc}, & v_r = V \\ L_{cc} \frac{v_r}{V}, & 0 < v_r < V \\ 0, & v_r = 0 \end{cases} \quad (16)$$

The transient tool-chip contact length is piecewise modelled according to the nominal cutting velocity V because the cutting state changes at velocities above or below it. The essential reason is that the condition when the relative cutting velocity equals to the nominal cutting velocity ($v_r = V$) is critical for the start of separation between the tool and the workpiece. To further clarify the characteristics of this piecewise function, the expression of the transient tool-chip contact length can be rewritten for time interval as follows:

$$L_{ct} = \begin{cases} L_{tc}, & 0 \leq t < t_c \\ L_{cc} \frac{v_r}{V}, & t_c \leq t < t_a \\ 0, & t_a \leq t < t_b \\ L_{tc}, & t_b \leq t \leq T \end{cases} \quad (17)$$

According to the modelling theory for the secondary deformation

Fig. 5. Evolution of transient tool-chip contact length in single cycle of VAC.

zone in an orthogonal cutting process [21], the tool-chip contact length can be calculated as

$$L_c = t_u \frac{\sin(\phi + \lambda - \alpha)}{\sin\phi \cos\lambda} \left(\frac{2}{3} + \frac{1 + 2(\pi/4 - \phi)}{3 \tan(\phi + \lambda - \alpha)} \right) \quad (18)$$

where t_u is the undeformed chip thickness. ϕ , λ and α are the shear angle, apparent friction angle of the tool-chip interface and the tool rake angle, respectively. When the relative cutting velocity is greater than or equal to the cutting speed, the tool-chip contact lengths L_{tc} and L_{cc} can be obtained from Eq. (18) as the values of tool-chip contact length L_c at different instants.

The transient tool-chip contact length in a vibration cycle calculated with Eq. (17) is presented in Fig. 5. Since the relative cutting velocity gradually decreases from $t = 0$ to the instant t_c , the tool-chip contact length increases slightly, as also concluded by Bai et al. [21]. The main reason for this is that the decrease of the relative cutting velocity leads to slowly growing tool-chip friction and then increases the friction angle, resulting in a slight increase in the transient tool-chip contact length. Subsequently, due to the further reduction of the relative cutting velocity, the tool and the workpiece are about to separate. This unsteady process is different from the previous stage, where the tool-chip contact length decreases sharply due to the physical separation. From t_a to t_b , the tool and workpiece are completely separated, and the tool-chip contact

length is zero. After the instant t_b , the tool reengages with the workpiece, and the contact length slowly decreases as the higher relative cutting velocity continues to increase. The reason here is opposite to that in the cutting process from $t = 0$ to t_c . At the instant, the transient tool-chip contact length reaches the minimum in the cutting stage (from t_b to $T + t_a$) due to the peak of effective relative cutting velocity (Fig. 3). After that moment (T), the cutting process enters the next vibration cycle and the transient tool-chip contact length starts to increase as the relative cutting velocity decreases from T to the instant $T + t_c$.

5. Analytical modelling and parameter prediction

At present, there is no established theory for unsteady-state cutting. Scholars usually use the finite-element method or adopt the steady-state cutting theory but consider the main factors to modify the model. This paper selected the latter. The evolution of important variables in UAC of Ti6Al4V presented using the finite-element method in Bai et al. [23] demonstrated that the strain, strain rate and temperature fluctuated with the tool vibration. Thus, there is reason to believe that these variables in the VAC process change at high frequency. In addition, the steady-state cutting theory used in this paper is mainly the non-equidistant shear zone model, and the unsteady process affects the deflection of the primary shear band significantly. In fact, the experimental results below show that the shear angle changes by about $4 \sim 6^\circ$ in VAC. Thus, the deflection angle is very small compared to the size of a long narrow shear band. It is considered that the unsteady-state process has little influence on the use of the shear-zone theory. Therefore, the chip-formation theory of orthogonal cutting is adopted in this paper.

Based on the above analysis of the transient tool-chip contact length and the transient shear angle, the VAC process is modelled by combining it with the chip-formation theory of orthogonal cutting. The flow chart for calculation of the process parameters for VAC is shown in Fig. 6. The non-equidistant shear zone model is adopted for the primary shear zone, where a straight band of thickness h is divided in proportion k . The main shear plane AB, the inlet shear plane CD and the outlet shear plane EF are parallel. Two coordinate systems are defined: $\mathcal{R} = \{o, x, y, z\}$ and $\mathcal{R}_s = \{o, x_s, y_s, z_s\}$ — for the tool-workpiece movement system and the material shear flow system. This model can be used to obtain the levels of strain, strain rate, temperature and shear stress in the primary shear zone. They are transferred to the secondary shear zone (SSZ) in order to calculate the chip thickness and pressure on the tool tip. Then, a sticking-sliding zone model is utilized at the tool-chip interface to

Fig. 6. Flow chart for calculation of process parameters for VAC.

Fig. 7. Evolution of cutting forces (a) and coefficient of friction (b) in single cycle of VAC.

predict the apparent and sliding coefficients of friction, the tool-chip contact length and the length of the sticking zone, the equivalent strain together with average temperature. Another coordinate system $\mathcal{R}_c = \{0, x_c, y_c, z_c\}$ is defined for the chip flow system. More details are available in our previous work [21]. Thus, the transient shear angle and the tool-chip contact length can be determined. The shear angle ϕ is selected, when the shear stress at the tool-chip interface τ_{int} (calculated based on the resultant force at the primary shear zone) equals the shear flow stress at the tool-chip interface k_{int} (corresponds to the strain, strain rate and temperature at the tool-chip interface). The transient shear angle ϕ_{ic} and ϕ_a are the values of shear angle ϕ at different times. This analytical model for the VAC process can be used to predict the dominant parameters such as cutting force, friction coefficient, etc., as well as

the secondary parameters such as shear strain, etc.

Thus, the cutting force (F_c) and the thrust force (F_t) can be expressed as

$$F_c = \begin{cases} \frac{wt_u \tau_{AB} \cos(\lambda - \alpha)}{\sin(\phi_{ic}) \cos(\phi_{ic} + \lambda - \alpha)}, & 0 \leq t < t_a \\ 0, & t_a \leq t < t_b \\ \frac{wt_u \tau_{AB} \cos(\lambda - \alpha)}{\sin(\phi_{ic}) \cos(\phi_{ic} + \lambda - \alpha)}, & t_b \leq t \leq T \end{cases} \quad (19)$$

Fig. 8. Effect of processing parameters on transient shear angle in single cycle of VAC: (a) cutting speed; (b) feed rate; (c) vibration amplitude.

Fig. 9. Comparison of shear angle in CC and VAC for various process parameters: (a) cutting speed; (b) feed rate; (c) vibration amplitude.

$$F_t = \begin{cases} \frac{wt_u \tau_{AB} \sin(\lambda - \alpha)}{\sin(\phi_{ic}) \cos(\phi_{ic} + \lambda - \alpha)}, & 0 \leq t < t_a \\ 0, & t_a \leq t < t_b \\ \frac{wt_u \tau_{AB} \sin(\lambda - \alpha)}{\sin(\phi_{ic}) \cos(\phi_{ic} + \lambda - \alpha)}, & t_b \leq t \leq T \end{cases} \quad (20)$$

where the cutting width w is 1 mm, the undeformed chip thickness t_u is numerically equal to the feed rate. The shear stress τ_{AB} at the primary shear plane AB (Fig. 6) is obtained from the non-equidistant shear-zone model [21].

The evolution of cutting force and the tool-chip coefficient of friction in a single cycle of VAC is shown in Fig. 7.

As seen from the force curve (Fig. 7(a)), the main cutting (F_c) and thrust (F_t) forces are zero in the stage of separation of the tool and the workpiece. At the beginning of the vibration cycle (from 0 to t_c), the cutting force increases slowly due to the decrease of the relative cutting velocity, which is consistent with the results reported for CC [21]. From t_c to t_a , the cutting forces decrease sharply due to the separation of the tool and the workpiece, and then decrease slowly because of the increase of the relative cutting velocity after the instant t_b . In addition, the tool-workpiece contact ratio $r_{tw} \ll 1$; thus, many researchers concluded that the average cutting force in VAC is lower than that of CC. Actually, comparison of the main cutting and thrust forces in CC with the average cutting forces in VAC shows clearly that the VAC reduces the average cutting forces significantly.

The changed trends of apparent (μ_a) and sliding (μ_s) coefficients of friction (Fig. 7(b)) are similar to that of cutting forces, but the growth rate of the sliding coefficient of friction ($\Delta\mu_s$) is more significant than

that of the apparent one ($\Delta\mu_a$) at the stage when the relative cutting velocity reduces. The main reason is that the sticking zone near the tool tip at the tool-chip interface is not sensitive to the cutting speed; thus, the apparent (global) coefficient of friction is less affected by the variation of the relative cutting velocity than the sliding coefficient of friction.

Besides, the sliding coefficient of friction is always slightly higher than the apparent one, corresponding to the reported results [21]. It also can be found that the average coefficient of friction is less than the coefficient of friction in CC.

However, the parameters of VAC are very difficult to measure in a single period of vibration as its vibration frequency is normally very high. Therefore, the important parameters — shear angle in VAC — is analyzed and validated in the next section.

6. Prediction and validation of shear angle in vibration-assisted cutting

6.1. Prediction of shear angle for various process parameters

The transient shear angles are predicted for various processing parameters based on the developed model. The predictions were implemented for the cutting speed of 15 m/min, 20 m/min, 25 m/min, 30 m/min and the feed rate of 0.08 mm/rev, 0.10 mm/rev, 0.12 mm/rev, 0.14 mm/rev for vibration amplitude of 4.7 μm, 6.2 μm, 7.7 μm, 9.3 μm; the vibration frequency was 20,220 Hz. The effect of different processing parameters on the transient shear angle in a single cycle of VAC is shown in Fig. 8. Overall, the trends for the shear angle under various machining parameters are similar, with the main change caused by the

Fig. 10. Experimental work: (a) sample preparation; (b) measurement of shear angle.

separation of tool from the workpiece in the cycle. For different cutting speeds, the transient shear angle increase slightly with the increase in the speed in the tool-workpiece cutting stage. Meanwhile, the tool-workpiece separation time was shortened (i.e. the tool-workpiece contact ratio r_{tw} increased) as the cutting speed increased (Fig. 8(a)). For different feed rates, the increase in the rate also caused a slight increase in the shear angle at the cutting stage, but the feed rate did not affect the separation time between the tool and the workpiece (Fig. 8(b)). The increase in the amplitude also improved the transient shear angle at the cutting stage. Besides, the effect of the amplitude on the absolute value of shear angle was greater than that of the cutting speed and the feed rate, mainly because the variation of amplitude had a more significant effect on the relative cutting velocity. In addition, the tool-workpiece separation time was extended (i.e. the tool-workpiece contact ratio r_{tw} decreased) as the amplitude increased (Fig. 8(c)).

In the VAC process, the shear angle always changes in each cycle. However, the level of the maximum shear angle is considered to retain and transfer to the chip as a result of material's flow and rapid cooling of the chip. In this paper, the effective shear angle in a cycle of VAC was defined as the maximum shear angle for prediction. The average shear angle can be calculated from the mean transient shear angle in a cycle. Here, VAC is the special process — ultrasonically assisted cutting because the selected frequency exceeds 20 kHz. The prediction of effective shear angle and average shear angle with various process parameters are shown in Fig. 9. Obviously, the average shear angle in CC is equal to the effective shear angle because the shear angle is regarded constant during the cutting process. In general, for different machining parameters, the effective shear angle of VAC is larger than that of CC, while the average shear angle of VAC is smaller than the average shear angle (effective shear angle) of CC. The main reason for this is the fact that the maximum relative cutting speed in VAC is higher than the cutting speed in CC, leading to a larger effective shear angle in VAC. Also, the tool-workpiece separation during the VAC process results in a smaller average shear angle than that in CC.

With the increase in the cutting speed (Fig. 9(a)), the effective shear angle in VAC slightly increases, and it gradually increases in CC. There are several reasons for this. First, the contribution of the cutting-speed improvement compared to the relative cutting speed in VAC is less than that in CC. Second, our previous results [21] indicate that the shear angle increases with an increase in the cutting speed and it is obvious that the shear angle changes more in the low-speed region (cutting speed in CC) than in the high-speed region (relative cutting speed in VAC). In addition, Fig. 9(a) also demonstrates that the average shear angle gradually increases in VAC. Actually, the growing cutting speed leads to the increases in the duty ratio and the effective relative cutting velocity in vibration-assisted cutting, which results in the increase of the average shear angle. With the increase in the feed rate (Fig. 9 (b)), the effective

shear angle in both VAC and CC and the average shear angle in VAC increase slightly. In Fig. 9(c), the amplitude of zero corresponds to the CC process. When the vibration amplitude increases in VAC, the effective shear angle grows gradually while the average shear angle decreases. This is a result of a more significant effect of the increase of amplitude on the increase of the relative cutting speed, so the effective shear angle increases. The increase of amplitude also increases the separation time of the tool and the workpiece; thus, the average shear angle in VAC decreases.

6.2. Experimental work and validation of shear angle

The parameters of VAC, such as transient tool-chip contact length and transient shear angle, currently cannot be measured in a single period of vibration as vibration frequency in VAC is normally very high. However, the proposed model indicates that these transient parameters are interrelated and ultimately reflected in material's removal in shear and chip formation, and the effective shear angle is the most representative variable that can be measured. Therefore, the experimental validations of shear angle in VAC are performed.

In order to validate the predicted levels of the shear angle, the orthogonal cutting experiment with VAC was performed. The difficult-to-cut material Ti6Al4V was selected and the process parameters were the same as in the developed model. As shown in Fig. 10, the ultrasonically assisted cutting device was fixed on the lathe, consisting of the piezoelectric transducer and the concentrator. A cemented-carbide cutting tool was attached to the bottom of concentrator. The workpiece was machined with equidistant grooves, with the cutting edge was parallel to the cylindrical surface of the workpiece so that the orthogonal cutting process was performed corresponding to the 2D plane state. The tool feed is along the radial direction of the workpiece. The obtained chips were spiral columnar, helpful for sample preparation (Fig. 10(a)) and observation. The obtained chips were grinded and polished after hot inlaying, then observed under the metallographic microscope after etching. The shear angle was measured (Fig. 10(b)) for each group of chip samples. In the experiments, a chip-segmentation frequency can be evaluated as 2~4 kHz for the used cutting speed (15~30 m/min) [21]. That is, the vibration frequency (~20 kHz) is much larger than the chip-segmentation frequency. Thus, it is assumed that the variation of shear angle in a vibration cycle does not interfere with the formation of adiabatic shear bands. Therefore, the effective shear angle in VAC can be related to the serrated chip morphology instead of the shear angle at random moment. To measure the shear angle (Fig. 10(b)), it is necessary to draw a straight line (l) along the adiabatic shear band, and draw a tangent (m) passing through the intersection of this line and the back surface of the chip, then build a line (n) perpendicular to the tangent line through the intersection. Thus, the shear angle is determined by the

Fig. 11. Typical comparison of chip morphology and shear angle in CC (a) and VAC (b).

Fig. 12. Comparison of theoretical and experimental results for effective shear angle in CC and VAC with various process parameters: (a) cutting speed; (b) feed rate; (c) vibration amplitude.

angle between the vertical line (n) and the line passing through the shear band (l).

The experiments were performed for the cutting speed of 15 m/min, 20 m/min, 25 m/min, 30 m/min and the feed rate of 0.08 mm/rev, 0.10 mm/rev, 0.12 mm/rev, 0.14 mm/rev with vibration amplitude of 4.7 μm, 6.2 μm, 7.7 μm, 9.3 μm; the vibration frequency was 20,220 Hz. The effective shear angles in CC and VAC were obtained based on the above measuring method. A typical comparison of chip morphology and shear angle in CC and VAC is shown in Fig. 11. The cutting speed and the feed rate were set as 20 m/min and 0.14 mm/rev, respectively. A vibration amplitude was 7.7 μm for VAC. Apparently, the use of ultrasonic vibration in the cutting process improved the effective shear angle. In addition, the size of sawtooth chip and the number of serrated chips also

show some differences. It seems that VAC improves the chip segmentation.

The effective shear angles for CC and VAC were measured in the experiments for different process parameters as shown in Fig. 12. The effective shear angle in a cycle of VAC was defined as the retained shear angle in the chip from the shear-angle graph. The error bar reflects the scatter in measurements of the shear angle at three different chip locations on the same sample. Apparently, the effective shear angle in VAC for different process parameters was indeed higher than that in CC. For different cutting speeds, the effective shear angle in CC increased gradually (Fig. 12(a)). The effective shear angle of in VAC increased slowly with the cutting speed. For various feed rates, the effective shear angle in both VAC and CC increased slowly (Fig. 12(b)). The effective

shear angle of VAC grows significantly as the vibration amplitude increased (Fig. 12(c)). Overall, the theoretical results are in good agreement with the experiments. However, better matching of the theoretical and experimental results was achieved for CC, while the theoretical values of VAC are slightly higher than the experimental values. The reason could be that final shear angle of chip in the VAC process is not exactly the maximum value, and some shear angles less than the maximum value were retained under some conditions.

7. Conclusions

In this paper, the transient shear angle and the tool-chip contact length of one-dimensional VAC were analysed, and the process parameters in VAC were predicted based on the orthogonal cutting theory. In addition, the results predicted for the shear angle were validated with the orthogonal cutting experiment in both CC and VAC. Conclusions from this study can be drawn as follows:

- (a) The evolution of the transient shear angle in a vibration cycle can be divided into three stages. The instantaneous shear angle reaches its maximum value when the relative cutting velocity of the tool and workpiece attains the maximum. From the instant that the tool and workpiece separate to the time of their reengagement, the transient shear angle retains the stable value.
- (b) The development of the transient tool-chip contact length in a single vibration cycle can be divided into four stages. When the tool velocity is zero, the transient tool-chip contact length reaches the maximum value. The transient tool-chip contact length vanished during the period from the disengagement to the reengagement of the tool and the workpiece. The variation of the transient tool-chip contact length is small from the instant of tool-workpiece reengagement to the instant when the tool velocity reaches zero in the next cycle.
- (c) For various process parameters, the effective shear angle in VAC is larger than that in CC. However, the average shear angle in VAC is smaller than the average shear angle (effective shear angle) in CC. The increase of vibration amplitude improves the transient shear angle. In addition, the effect of vibration amplitude on shear angle is more significant than that of cutting speed and feed rate.
- (d) The theoretical results for the effective shear angle in CC and VAC are in good agreement with the data of orthogonal cutting experiments.

In this paper, the analysis of the transient shear angle and friction behaviour of VAC can provide the theoretical support for further understanding of the mechanism of VAC and the thermo-mechanical modelling of VAC process. In addition, this paper presents a method and a model with potential to analyze the transient fluctuations of variables in a cycle of VAC that can be used not only for one-dimensional but also multi-dimension vibration-assisted cutting. Although more factors should be considered in multi-dimensional vibration-assisted cutting, the proposed model can provide a reference for further investigations of the mechanisms, underpinning fluctuations of variables. Furthermore, the effectiveness of the model was validated by taking titanium alloy as an example in this paper. However, the proposed approach is applicable for other metallic materials with suitable parameter calibration.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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