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Analysis of mutually incoherent symmetrical illumination for electronic speckle pattern shearing interferometry

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5 **Analysis of mutually incoherent symmetrical illumination for**
6 **electronic speckle pattern shearing interferometry**

7

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3 The authors designed a speckle shearing interferometer using symmetrical
4 mutually incoherent illumination, in an effort to provide measurements of in-plane
5 strain. It is presented the analysis of the sensitivity to displacement and strain of this
6 interferometer, together with the analysis of the optical phase extraction of the
7 resultant fringe pattern. This interferometer is an improvement on previous designs as
8 it provides information of the in-plane strain separated from components of the
9 displacement. Experimental results show the fringe patterns in support of the
10 theoretical analysis.

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14 Keywords: Speckle, Interferometry, Strain, ESPI, ESPSI

1 1. INTRODUCTION

2 Optical metrology techniques based on speckle provide measurements of
3 displacement and strain. They bring the advantages of being a whole-field
4 measurement, non-contact and in real-time [1]. Displacement and strain can be
5 divided into their orthogonal components, the Out-Of-Plane (OOP) component and
6 the two X and Y In-Plane (IP) components. Electronic Speckle Pattern Interferometry
7 (ESPI) is used for the measurement of displacement in the three orthogonal axes,
8 allowing the independent extraction of the two IP and the one OOP displacement
9 components [2].

10 Electronic Speckle Pattern Shearing Interferometry (ESPSI) provides a
11 measurement of the first spatial derivatives of the displacement, these being related to
12 mechanical strain [3,4]. This measurement is of great practical interest since
13 engineering designs can generally cope with microscopic displacements of their
14 components, but must avoid unwanted accumulation of strain in the mechanisms.
15 Several speckle shearing interferometers can discretely measure the OOP spatial
16 derivative component [4-8]. However, this is not the case for discrete measurement of
17 the in-plane spatial derivative components, where ESPSI has technical difficulties.

18 The in-plane spatial derivative terms have been extracted using aperture based
19 designs and Fourier plane analysis [9] but these systems have yet to be demonstrated
20 as real-time instruments. Alternatively, extraction of the in-plane spatial derivative
21 terms has been demonstrated using sequential measurement, changing illumination
22 angles between each data set, [4, 10-12]. Whilst this approach does work, if the
23 deformation of the object under study changes between each of the sequential
24 measurements, as it is often the case with real-life engineering problems, then the

1 technique will not produce the correct results. A solution to this issue has been
2 proposed [13], with a shearing interferometer utilising three wavelengths for
3 illumination, but the complexity of the system should perhaps not be underestimated.

4 Direct measurements of in-plane strain components have been presented but
5 under special circumstances, such as plane stress or plane strain conditions [14].
6 Further work has suggested that the use of dual or simultaneous illumination
7 wavefronts may allow direct analysis of in-plane components for arbitrary objects
8 [15].

9 Speckle interferometers using more than one illuminating beam generate
10 complex interference patterns whose analysis requires careful understanding of the
11 optical properties of speckle, as well as analysis of the geometry of object and image.
12 The standard notation used to describe speckle shearing interferometer output [1, 2],
13 reduces the expression for the interference of the optical wavefronts at the observation
14 plane, to a cosine expression that contains the addition of phase delays. However, this
15 does not take into account the relative spatial correlations of the speckle patterns
16 scattered from different incident beams or scattered from different areas of the object,
17 separated by the lateral shear. Furthermore, for a dual beam system, the standard
18 notation used for speckle shearing interferometry does not provide indicators for
19 which wavefronts or which illuminating beams interact, after the lateral shift of the
20 images, or after the absolute value subtraction of the image patterns.

21 The approach taken in our work has been to modify the expression describing
22 the speckle interference pattern so that it is separated into several intensity terms, each
23 one labelled according to which illuminating wavefront contributes to it. These labels

1 take into account the polarisation state of each wavefront, in order to indicate which
2 ones cause interference and which others will just add together their intensities.

3 Prior work has used the notation described above to deal with dual beam
4 mutually coherent speckle shearing interferometers [16, 23]. The same notation is
5 utilised here to study the possibility of a dual beam mutually incoherent illumination
6 interferometer for deformation analysis. The theoretical analysis for the new
7 interferometer is presented in this work, together with the initial experimental results
8 that support the predictions.

9

10 **2. THEORETICAL ANALYSIS**

11 The proposed interferometer uses two mutually incoherent and symmetrically incident
12 beams, beams Left (L) and Right (R), to illuminate the object, Figure 1. The lateral
13 shifting of the images is performed by means of a Michelson design with two non-
14 perpendicular mirrors (A and B), and the optical axis of the CCD camera bisects the
15 angle made by the two laser illumination beams. It should also be noted that, as would
16 be expected with a Michelson based optical system, the theoretical development has
17 many initial similarities with existing speckle pattern interferometry theory [1, 23],
18 although modified and expanded to take into account the nature of simultaneous
19 incoherent illumination.

20 The analysis of the interferometer starts with the labelling of each component
21 of the optical set-up: if we denote by LA, LB, RA, RB the amplitudes, phase included,
22 of the wavefronts arriving to a point (x,y) on the image plane, the intensity registered

1 by the sensors of the camera will be given by eq. [1], where $LA = \|LA\| \cdot e^{i\phi_{LA}}$ and

2 $LA^* = \|LA\| \cdot e^{-i\phi_{LA}}$:

3
$$I(x, y) = (LA + LB) \cdot (LA + LB)^* + (RA + RB) \cdot (RA + RB)^* \quad [1]$$

4 The beams L and R are not coherent between themselves and thus can not
5 make interference with each other. Different experimental conditions can simulate this
6 case: set-ups with two beams with their polarisations orthogonal to each other, as
7 shown in Figure 1, set-ups where each beam comes from a different laser source, or
8 finally set-ups with two beams equally polarised but with a difference of optical path
9 bigger than the coherence length of the laser source. The case of two beams with
10 different wavelengths is not considered on this analysis.

11 In eq. [1], (x,y) represents any point on the CCD camera sensing area, $I(x,y)$ is
12 the intensity of light registered at that point, and the * sign represents the conjugated
13 wave. The product [1] results in eq. [2]:

14
$$I(x, y) = LA \cdot LA^* + LB \cdot LB^* + LA \cdot LB^* + LB \cdot LA^* + \quad [2]$$
$$RA \cdot RA^* + RB \cdot RB^* + RA \cdot RB^* + RB \cdot RA^*$$

15 where the terms of the addition $LA \cdot LA^*$, $LB \cdot LB^*$, $RA \cdot RA^*$ and $RB \cdot RB^*$ represent
16 the intensity of the beams LA, LB, RA, RB respectively, eq. [3]:

17
$$LA \cdot LA^* = \|LA\| \cdot e^{i\phi_{LA}} \cdot \|LA\| \cdot e^{-i\phi_{LA}} = \|LA\|^2 \quad [3]$$

18

19 [INSERT FIGURE 1 ABOUT HERE]

20

1 where $\|LA\|$ is the *modulus* of the amplitude of the electric field associated to the light
2 beam LA, ϕ_{LA} represents its phase and $\|LA\|^2$ its intensity. The phase ϕ_{LA} is a random
3 value that depends on the roughness of the surface on that particular point of the
4 object. Though this is a random value, it will remain constant as long as the surface is
5 not affected or eroded.

6 Applying similar calculations to the rest of the terms in eq [2], the result will
7 be equations [4], [5] and [6]:

$$8 \quad LB \cdot LB^* = \|LB\| \cdot e^{i\phi_{LB}} \cdot \|LB\| \cdot e^{-i\phi_{LB}} = \|LB\|^2 \quad [4]$$

$$9 \quad RA \cdot RA^* = \|RA\| \cdot e^{i\phi_{RA}} \cdot \|RA\| \cdot e^{-i\phi_{RA}} = \|RA\|^2 \quad [5]$$

$$10 \quad RB \cdot RB^* = \|RB\| \cdot e^{i\phi_{RB}} \cdot \|RB\| \cdot e^{-i\phi_{RB}} = \|RB\|^2 \quad [6]$$

11 Hence, LA and RA represent the light wavefronts scattered from point (x,y) on
12 the object plane, scattered from the incident beams L and R respectively and reflected
13 by mirror A, making incidence onto point (x,y) of the CCD plane. Analogously, LB
14 and RB will be the light wavefronts scattered from point (x+ δx ,y) on the object plane,
15 reflected by mirror B (tilted) and incident onto the same point (x,y) on the CCD plane.
16 The tilting on mirror B introduces the lateral shear δx .

17 The rest of the terms in I(x,y) can be calculated using the same notation for the
18 amplitude and phase, eq. [7] and eq. [8]:

$$19 \quad \begin{aligned} RA \cdot RB^* + RB \cdot RA^* &= \|RA\| \cdot \|RB\| \cdot e^{i\phi_{RA}} \cdot e^{-i\phi_{RB}} + \|RA\| \cdot \|RB\| \cdot e^{-i\phi_{RA}} \cdot e^{i\phi_{RB}} = \\ &= \|RA\| \cdot \|RB\| \cdot \{e^{i(\phi_{RA} - \phi_{RB})} + e^{i(\phi_{RB} - \phi_{RA})}\} \\ &= \|RA\| \cdot \|RB\| \cdot 2 \cdot \cos(\phi_{RA} - \phi_{RB}) \end{aligned} \quad [7]$$

$$1 \quad LALB^* + LBLA^* = \|LA\| \cdot \|LB\| \cdot 2 \cdot \cos(\phi_{LA} - \phi_{LB}) \quad [8]$$

2 The final result is expressed by eq. [9]:

$$3 \quad I(x, y) = \|LA\|^2 + \|LB\|^2 + \|RA\|^2 + \|RB\|^2 \\ + \|RA\| \cdot \|RB\| \cdot 2 \cdot \cos(\phi_{RA} - \phi_{RB}) \quad [9] \\ + \|LA\| \cdot \|LB\| \cdot 2 \cdot \cos(\phi_{LA} - \phi_{LB})$$

4 This equation represents the intensity of light at the arbitrary point (x,y) on the
5 CCD camera sensing plane, *before* the object undergoes any alteration or deformation.

6 We assume that the object deformation or displacement is smaller than the
7 average speckle grain size, an assumption generally made for all the correlation
8 speckle interferometers. Hence the intensity of each beam at the point (x,y) will be the
9 same *before and after* the changes on the object, as expressed by eq. [10]:

$$10 \quad \|LA\|_{\text{after}} = \|LA\|_{\text{before}} \quad [10]$$

11 As previously discussed, the intensity and phase of light arising from mirrors
12 A and B are different due to the different angle of illumination, the image shift
13 introduced by mirror B and the different deformations experimented by the object on
14 points (x,y) and (x+δx,y):

$$15 \quad \|LA\| \neq \|RA\| \neq \|LB\| \neq \|RB\| \quad [11]$$

$$16 \quad \phi_{RA} \neq \phi_{RB} \quad [12]$$

$$17 \quad (\phi_{LA} - \phi_{LB})^{\text{Before}} \neq (\phi_{LA} - \phi_{LB})^{\text{After}} \quad [13]$$

18

[INSERT TABLE 1 ABOUT HERE]

The phase changes introduced by the deformation or displacement of the object will change the recorded intensity pattern. Both in-plane and out-of-plane displacements introduce phase changes that modify the resultant speckled pattern of intensity. According to Table 1, the intensity registered after the displacement will be expressed by eq. [14]:

$$\begin{aligned}
 I(x, y)_{\text{CCD}}^{\text{After}} = & \|LA\|^2 + \|LB\|^2 + \|RA\|^2 + \|RB\|^2 \\
 & + \|RA\| \cdot \|RB\| \cdot 2 \cdot \text{Cos} \left[\begin{aligned} & \left[\phi_{RA} + \frac{2\pi}{\lambda} (-u \cdot \text{Sin}\theta - w \cdot (1 + \text{Cos}\theta)) \right] - \\ & \left[\phi_{RB} + \frac{2\pi}{\lambda} (-(u + \delta u) \cdot \text{Sin}\theta - (w + \delta w) \cdot (1 + \text{Cos}\theta)) \right] \end{aligned} \right] \\
 & + \|LA\| \cdot \|LB\| \cdot 2 \cdot \text{Cos} \left[\begin{aligned} & \left[\phi_{LA} + \frac{2\pi}{\lambda} (u \cdot \text{Sin}\theta - w \cdot (1 + \text{Cos}\theta)) \right] - \\ & \left[\phi_{LB} + \frac{2\pi}{\lambda} ((u + \delta u) \cdot \text{Sin}\theta - (w + \delta w) \cdot (1 + \text{Cos}\theta)) \right] \end{aligned} \right]
 \end{aligned} \quad [14]$$

This last expression for the intensity may be rewritten as in eq. [15]:

$$\begin{aligned}
 I(x, y)_{\text{CCD}}^{\text{After}} = & \|LA\|^2 + \|LB\|^2 + \|RA\|^2 + \|RB\|^2 \\
 & + \|LA\| \cdot \|LB\| \cdot 2 \cdot \text{Cos} \left[\phi_{LA} - \phi_{LB} + \underbrace{\frac{2\pi}{\lambda} (-\delta u \cdot \text{Sin}\theta + \delta w (1 + \text{Cos}\theta))}_{\Delta_1} \right] \\
 & + \|RA\| \cdot \|RB\| \cdot 2 \cdot \text{Cos} \left[\phi_{RA} - \phi_{RB} + \underbrace{\frac{2\pi}{\lambda} (\delta u \cdot \text{Sin}\theta + \delta w (1 + \text{Cos}\theta))}_{\Delta_2} \right]
 \end{aligned} \quad [15]$$

Beams L and R are incoherent with each other and thus the interference terms do not include crossed terms containing both. This reduces the number of interference terms and simplifies the resultant moiré inter-crossing of speckle fringe patterns. That

1 represents an improvement from the design with two mutually coherent symmetrical
2 beams [23].

3 The images recorded before and after the deformation of the object are
4 subtracted in intensity pixel by pixel. The resultant image will have dark speckle
5 fringes on the places of the object where $I^{\text{before}}(x,y) \neq I^{\text{after}}(x,y)$. In order for this to
6 happen the two interference terms in eq. [15], Δ_1 and Δ_2 , must be equal to an even
7 number of times π :

$$8 \quad \Delta_1 = \frac{2\pi}{\lambda} (-\delta u \cdot \sin\theta + \delta w(1 + \cos\theta)) = 2 \cdot m_1 \cdot \pi \quad [16]$$

$$9 \quad \Delta_2 = \frac{2\pi}{\lambda} (\delta u \cdot \sin\theta + \delta w(1 + \cos\theta)) = 2 \cdot m_2 \cdot \pi \quad [17]$$

10 where m_1 and m_2 are integer numbers and represent the fringe order. As the lateral
11 shearing δx between the images is small, these expressions may be rewritten in a
12 differential form as:

$$13 \quad \Delta_1 = \frac{2\pi}{\lambda} \cdot \left[-\left(\frac{\partial u}{\partial x}\right) \cdot \delta x \cdot \sin\theta + \left(\frac{\partial w}{\partial x}\right) \cdot \delta x \cdot (1 + \cos\theta) \right] = 2 \cdot m_1 \cdot \pi \quad [18]$$

$$14 \quad \Delta_2 = \frac{2\pi}{\lambda} \cdot \left[\left(\frac{\partial u}{\partial x}\right) \cdot \delta x \cdot \sin\theta + \left(\frac{\partial w}{\partial x}\right) \cdot \delta x \cdot (1 + \cos\theta) \right] = 2 \cdot m_2 \cdot \pi \quad [19]$$

15 Only on the points (x,y) of the image where both conditions, equations [18]
16 and [19], are simultaneously satisfied the result of the image subtraction will provide
17 a dark fringe. As the two equations must be satisfied simultaneously, their addition
18 and subtraction must be a condition as well for the observation of the fringes. After
19 some calculation, the final results are:

$$2 \cdot \left(\frac{\partial w}{\partial x} \right) \cdot \delta x \cdot (1 + \cos \theta) = m' \lambda \quad [20]$$

$$2 \cdot \left(\frac{\partial u}{\partial x} \right) \cdot \delta x \cdot \sin \theta = m'' \lambda \quad [21]$$

The resultant pattern is a moiré-like pattern resultant from the intersection of two patterns: the fringe pattern of the pure out-of-plane strain, eq. [20], inter-crossing with the fringe pattern of the pure in-plane strain, eq. [21]. The in-plane component of the strain is completely separated from the out-of-plane component, in a moiré-like pattern. The extraction of the out-of-plane component of the strain can be done independently by means of a second camera and source recording only the out-of-plane component, in a typical perpendicular illumination speckle shearing set-up.

The fact that a shearing interferometer results in a moiré fringe pattern is not new in itself, with authors having previously presented photographic based systems using Fourier analysis [17, 18], to examine out-of-plane and in-plane terms. However, the intention in our work here is to explore optical configurations which result directly in real-time in-plane derivative systems, with the potential for quantitative evaluation.

15

3. OPTICAL PHASE EXTRACTION

Optical phase information extraction provides the tool for eventual generation of quantified data from the instrumentation. By introducing a controlled optical phase shift within the interferometer, it is possible to perform the phase stepping [19] of the resultant speckle pattern and fringe pattern.

1 There are two possible ways to perform phase stepping in this interferometer:
2 introducing a phase step in one of the illuminating beams or introducing a phase step
3 within the Michelson shearing-head.

4 If a phase-step is introduced in one of the illuminating beams, this will not
5 result in a phase stepping of the speckle fringe pattern: the two illuminating beams are
6 already incoherent to each other and an additional phase shift between them can not
7 introduce any phase difference that will arise to a change in the speckle pattern. Hence
8 the only option is introducing the phase step within the Michelson shearing-head.

9 In order to do that, one of the mirrors must be pushed forward and backwards
10 by means of a piezo-electric actuator, introducing a phase-shift within the laterally
11 sheared images. To analyse the effect of this type of phase-shift it can be assumed for
12 the sake of argument that the piezo actuator is fitted to mirror B. The phase-shift
13 introduced results in ϕ_{LB} changing into $\phi_{LB}+\phi'$ and ϕ_{RB} into $\phi_{RB}+\phi'$, generating a
14 speckle intensity change after the displacement or deformation of the object, as
15 described by eq. [22]:

$$\begin{aligned}
 I(x,y)_{CCD}^{After} = & \|LA\|^2 + \|LB\|^2 + \|RA\|^2 + \|RB\|^2 \\
 & + \|RA\| \cdot \|RB\| \cdot 2 \cdot \cos \left[\begin{aligned} & \left[\phi_{RA} + \frac{2\pi}{\lambda} (-u \cdot \sin\theta - w \cdot (1 + \cos\theta)) \right] - \\ & \left[(\phi_{RB} + \phi') + \frac{2\pi}{\lambda} ((u + \delta u) \cdot \sin\theta - (w + \delta w) \cdot (1 + \cos\theta)) \right] \end{aligned} \right] \\
 & + \|LA\| \cdot \|LB\| \cdot 2 \cdot \cos \left[\begin{aligned} & \left[\phi_{LA} + \frac{2\pi}{\lambda} (u \cdot \sin\theta - w \cdot (1 + \cos\theta)) \right] - \\ & \left[(\phi_{LB} + \phi') + \frac{2\pi}{\lambda} ((u + \delta u) \cdot \sin\theta - (w + \delta w) \cdot (1 + \cos\theta)) \right] \end{aligned} \right]
 \end{aligned} \quad [22]$$

17 This expression leads to eq. [23]:

$$\begin{aligned}
I(x, y)_{\text{CCD}}^{\text{After}} &= \|LA\|^2 + \|LB\|^2 + \|RA\|^2 + \|RB\|^2 \\
&+ \|LA\| \cdot \|LB\| \cdot 2 \cdot \cos \left[\underbrace{\phi_{LA} - \phi_{LB} - \phi' + \frac{2\pi}{\lambda} (-\delta u \cdot \sin \theta + \delta w (1 + \cos \theta))}_{\Delta_1'} \right] \\
&+ \|RA\| \cdot \|RB\| \cdot 2 \cdot \cos \left[\underbrace{\phi_{RA} - \phi_{RB} - \phi' + \frac{2\pi}{\lambda} (\delta u \cdot \sin \theta + \delta w (1 + \cos \theta))}_{\Delta_2'} \right]
\end{aligned} \quad [23]$$

where the interference terms Δ_1' and Δ_2' include now the phase shift ϕ' , equations [24] and [25]:

$$\Delta_1' = \frac{2\pi}{\lambda} \cdot \left[-\left(\frac{\partial u}{\partial x}\right) \cdot \delta x \cdot \sin \theta + \left(\frac{\partial w}{\partial x}\right) \cdot \delta x \cdot (1 + \cos \theta) \right] - \phi' = 2 \cdot m_1 \cdot \pi \quad [24]$$

$$\Delta_2' = \frac{2\pi}{\lambda} \cdot \left[\left(\frac{\partial u}{\partial x}\right) \cdot \delta x \cdot \sin \theta + \left(\frac{\partial w}{\partial x}\right) \cdot \delta x \cdot (1 + \cos \theta) \right] - \phi' = 2 \cdot m_2 \cdot \pi \quad [25]$$

where m_1 , m_2 are integer numbers that indicate the fringe order. After some calculations these two equations lead to:

$$2 \cdot \left(\frac{\partial w}{\partial x}\right) \cdot \delta x \cdot (1 + \cos \theta) - \frac{\phi'}{2 \cdot \pi} = m' \cdot \lambda \quad [26]$$

$$2 \cdot \left(\frac{\partial u}{\partial x}\right) \cdot \delta x \cdot \sin \theta - \frac{\phi'}{2 \cdot \pi} = m'' \cdot \lambda \quad [27]$$

The resultant pattern is a moiré-like pattern obtained from the intersection of two patterns: the phase shifted fringe pattern of the pure out-of-plane strain, eq. [26], inter-crossing with the phase shifted fringe pattern of the pure in-plane strain, eq. [27].

As a summary, in this interferometer the introduction of phase stepping in one of the illuminating beams does not have any effect on the phase-stepping of the resultant

1 fringe pattern. On the other hand, the introduction of phase stepping within the
2 Michelson shearing head results in a phase shift on both components of the moiré
3 pattern, the out-of-plane strain and the in-plane strain. This property of phase stepping
4 followed by a phase unwrapping process may be used to extract optical phase
5 information, and hence quantitative data, from this interferometer.

4. EXPERIMENTAL RESULTS

The theoretical analysis predicts that the dual beam mutually incoherent interferometer will produce moiré patterns consistent of the inter-crossing of the out-of-plane strain pattern with the in-plane strain pattern. The goal of the experimentation was to demonstrate that the interferometer actually follows that prediction. For that purpose, and using the same specimen and experimental conditions, we first obtained in-plane deformation data by means of in-plane ESPI and numerically differentiated it to obtain a map of in-plane strain. In order to corroborate this data we then applied photoelasticity techniques to obtain a qualitative distribution of in-plane strain across the sample. Finally we obtained the map of out-of-plane strain by means of conventional OOP-ESPSI. The OOP-strain fringe patterns obtained by this technique when overlapped with the IP-strain distribution numerically obtained from the IP-deformation data, produce a moiré fringe pattern that was successfully replicated by means of the new dual beam mutually incoherent ESPI interferometer.

The experiments were performed on a Brazilian Disk specimen, or split cylinder test, under compressive loading. This type of specimen consists of a flat disk compressed along its equator and has traditionally been used with techniques such as photoelasticity, moiré interferometry or holographic interferometry [20-21]. A 75mm diameter and 6mm thick Brazilian disk was manufactured from an AralditeTM sheet, and was compressed using a test rig linked to a hydraulic DH-Budenburg dead-weight tester. This provided forces up to 5000N, with a precision of $\pm 1\text{N}$.

The interferometer used a 50mWatt Nd-YAG laser (wavelength $\lambda=532\text{nm}$), with a coherence length in excess of 10m. The camera used to record the images was a CCD

Pulnix TM-9701 linked to a MuTech Corporation image processing board. Optical phase-stepping was introduced by means of a piezoelectric actuator by Piezo-Systems Jena.

4.1. In plane ESPI analysis of the Brazilian disk

The in-plane deformation map on the Brazilian disk under compression was measured by means of an in-plane ESPI set-up. The set of figures, Figures 2 a), b) and c), shows a set of phase-stepped fringe patterns of horizontal illumination ESPI for the Brazilian disk under a force of 17.5 ± 0.1 Kg-Force (171.6 ± 1 Newton). The first Figure 2.a) corresponds to the subtraction between the reference image and the image of the deformed object, and the other two images, Figure 2.b) and 2.c), correspond to the $\pm 2\pi/3$ phase-stepped fringe patterns respectively.

The fringe located on the left hand side of the disk corresponds to the order $m=0$, the part of the disk pressed against the stop that does not perform any displacement. The fringe pattern indicates that the displacement distributes evenly across the surface of the disk, with an expected increase in the spatial frequency of the fringes at the contact points with the piston (to the right) and with the stop (left of the disk).

Figure 3 shows the wrapped-phase map corresponding to that set of phase-stepped images. Every black-to-white step on that pattern corresponds to a 2π increase in displacement, what in this experiment corresponds to $0.53 \mu\text{m}$ per fringe along the X axis, and results in a total displacement of $4.77 \pm 0.1 \mu\text{m}$ along the X axis. A Cosine transform unwrapping algorithm [22] was applied to the wrapped-phase map and the

1 result is shown on Figure 4. On this image the area of the disk in white colour
2 corresponds to zero displacement, while the area of the image in black corresponds to
3 the maximum displacement of $4.77 \pm 0.1 \mu\text{m}$.

4
5 [figures 2,a,b,c should go about here]

6 [figure 3 should go about here]

7
8 [figure 4 should go about here]

10 11 **4.2. Numerical differentiation of the ESPI in-plane deformation map**

12 The ESPI in-plane deformation maps can be numerically differentiated in order to
13 obtain maps of in-plane strain, $(\partial u / \partial x)$. With this aim the maps of in-plane ESPI
14 obtained with the Brazilian disk were processed in MatLab™, by The Mathworks
15 Inc., at the Wolfson School of Mechanical and Manufacturing Engineering,
16 Loughborough University, and the results are shown on Figure 5. On the figure it can
17 be appreciated how the distribution of in plane strain has an elliptical distribution
18 along a horizontal lobe. This is in agreement with the results of the double beam
19 mutually incoherent illumination interferometer, the results of our photoelasticity
20 experiments, and also in agreement with the results by A. Castro-Montero et al. [21]
21 obtained by holographic interferometry on a Brazilian disk.

22 [figure 5 should go about here]

4.3. Photoelasticity results

To complement the analysis, photoelasticity results were obtained by means of a reflection polariscope (030-series by Measurements Group Inc., North Carolina, USA) consisting of two polariser/quarter-wave assemblies mechanically coupled for synchronous rotation. This optical set-up produces an isochromatic pattern that highlights the distribution of pure in-plane strain.

The qualitative results of the photoelasticity experiments, figure 6, were in agreement with the distribution of in plane strain previously obtained by means of the numerical differentiation of the in plane deformation maps, figure 5. The in plane strain was concentrated on a central lobe orientated along the direction of the application of the force. This result was also in agreement with the result of the dual beam mutually incoherent illumination interferometer.

[Figures 6 a,b,c,d should go about here]

4.4. Out of plane strain measurements

The out-of-plane component of the strain, $(\partial w / \partial x)$, was analysed independently with the specimen compressed by the same forces and rig used throughout the experiments. The amount of horizontal shearing (δx) was equal to 5mm, and as in the case of ESPI, a three images phase-step system was implemented to obtain quantitative results from the experiments, Figure 7.

[figure 7 should go about here]

The experiment of out-of-plane ESPI with horizontal shearing produced a set of non-equispaced vertical parallel fringes across the surface of the object. The change in spatial frequency of the fringes along the surface of the disk is more evident as the force increases. Figures 8-10 show the wrapped-phase map corresponding to forces of 17.5 ± 0.1 Kg-Force, 20.0 ± 0.1 Kg-Force and 22.5 ± 0.1 Kg-Force respectively. As with all the experiments presented in this work, the force is applied from the right hand side of the image.

In Figure 8 it is possible to count two fringes (the zero order on the left hand side and two more fringes) what indicates a maximum out-of-plane strain of 106.4 ± 10.6 μ strain on the left hand side of the disk. Figure 9 and Figure 10 show a maximum out-of-plane strain of 159.6 ± 10.6 μ strain and 212.8 ± 10.6 μ strain respectively.

[insert figure 8 about here]

[insert figure 9 about here]

[insert figure 10 about here]

4.5. Dual beam mutually incoherent illumination interferometer analysis of the Brazilian Disk

This new interferometer uses two mutually incoherent beams making incidence at opposite angles from the optical axis of the observation system, where a Michelson shearing head is used to introduce the lateral shifting between images. The speckle fringe patterns predicted by the theoretical analysis will consist in the moiré-like inter-crossing of the pure out-of-plane strain fringes with the pure in-plane strain fringes. Moiré fringes will be visible in the zones of the object where the equations for out-of-plane strain and in-plane strain are simultaneously satisfied, equations [20] and [21]. This represents an improvement from an interferometer with double beam coherent illumination [23], where the moiré pattern also included the ESPI in-plane fringes.

The optical set-up was as described in figure 1, with the difference that instead of illuminating with one laser source split into two beams with orthogonal polarisations, we utilised two beams from two different laser sources with the same wavelength. This ensured that no amount of depolarisation could reduce the contrast of the fringes, since the beams were permanently incoherent to each other.

Figure 11 shows the reference image of the Brazilian disk with the two laterally shifted images, $\delta x=10\text{mm}$, overlapping on the image plane. A typical set of results is shown in Figures 12 a) to d).

1 [insert figure 11 about here]

2 [insert figures 12 a,b,c,d about here]

3

4 [insert figure 12 e) about here]

5

1 Figure 12 a) shows the specimen under just enough force to show only out-of-
2 plane strain on the object: the force was not enough to show a visible pattern of in-
3 plane strain and equation [21] for the in-plane strain corresponded to $m=0$ across the
4 whole specimen. The pressure was then very slowly increased on the DH-Budenburg
5 dead-weight rig to show how the in-plane strain fringe comes up into view.

6 Figure 12 b) shows the specimen after the force was increased, and now the
7 central part of the disk begins to show some decorrelation. This means that in the
8 central lobe the in-plane strain has increased up to the point where equation [21]
9 corresponds to $m<1$, still not enough to originate a fringe but enough to produce
10 decorrelation on the moiré pattern.

11 Figure 12 c) shows the specimen after another increase in the force applied. Now
12 the in-plane strain on the specimen is enough to reach $m=1$ in equation [21], what
13 gives rise to an elliptical distribution of in-plane strain at the centre of the disk, as
14 predicted on the previous experiments and shown for indication in Figure 12 e).

15 Figure 12 d) shows how the central lobe in the centre of the specimen is now
16 even bigger ($m\geq 1$), surrounded by a region where the fringes disappear in the moiré
17 ($0<m<1$) and then become visible again, $m=0$.

18 The qualitative set of results confirm the theoretical predictions for speckle
19 shearing interferometer with two mutually incoherent symmetrically incident beams,
20 making it possible for the first time to observe a distribution of pure in plane strain by
21 optical means and in real time.

5. CONCLUSIONS

A variation on a conventional Michelson-shearing interferometer has been presented, in this case with the use of double beam mutually incoherent symmetrical illumination. The theoretical analysis allows the prediction of the fringe patterns resultant from this novel interferometer in both stationary state and in the presence of phase-stepping introduced within the Michelson shearing-head.

A series of experiments were performed on a Brazilian disk specimen utilising conventional speckle shearing techniques, numerical differentiation of ESPI displacement maps and photoelasticity techniques. All the experimental results showed the same consistent results with the sample under the same forces. Furthermore, the dual beam mutually incoherent illumination interferometer did show the predicted distribution of in plane strain fringes, and in agreement with the experimental results from the conventional speckle interferometers.

The novel interferometer can highlight pure in plane strain in one single measurement and in real time, with the advantage that now a single experiment produces a measurement instead of having to perform alternative measurements with in-plane ESPI and conventional ESPSI, since the combination of results from two interferometers always produces an increase in the error of the measurement.

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1 School of Mechanical and Manufacturing Engineering, Loughborough University,
2 and Vicente Moreno, at the Universidade de Santiago de Compostela, Spain.

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4 **-END-**

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Figure 1. *Speckle shearing interferometry with symmetrical mutually incoherent illumination.*

Mirror	Component of the Displacement	Optical Path Change in Beam L	Optical Path Change in Beam R
A	u (X axis)	$+ u \cdot \sin\theta$	$- u \cdot \sin\theta$
A	v (Y axis)	<i>nil</i>	<i>nil</i>
A	w (Z axis)	$- w \cdot (1 + \cos\theta)$	$- w \cdot (1 + \cos\theta)$
B	$u + \delta u$ (X axis)	$+ (u + \delta u) \cdot \sin\theta$	$- (u + \delta u) \cdot \sin\theta$
B	$v + \delta v$ (Y axis)	<i>nil</i>	<i>nil</i>
B	$w + \delta w$ (Z axis)	$- (w + \delta w) \cdot (1 + \cos\theta)$	$- (w + \delta w) \cdot (1 + \cos\theta)$

Table 1. *Optical path changes in double beam interferometer.*

2.a

2.b

2.c

Figures 2 a,b,c). In Plane ESPI with horizontal illumination on the Brazilian disk, Force= 171.6 ± 1 Newton, a) $\phi=0$, b) $\phi=-2\pi/3$, c) $\phi=+2\pi/3$.

Figure 3. *Wrapped phase map corresponding to Force= 171.6 ± 1 Newton.*

Figure 4. *Unwrapped- phase map for Brazilian disk, Force= 171.6 ± 1 Newton.*

Figure 5. *Contour map showing the distribution of pure in plane strain (maximum at the centre, arbitrary units), from the differentiation of the in plane displacement maps.*

Figure 6, a) to d). *Qualitative results from the photoelasticity experiments show a distribution of in-plane strain located inside a horizontal central lobe.*

Figure 7. *Optical set-up for out-of-plane ESPSI, with perpendicular illumination.*

Figure 8. *Wrapped-phase map of out-of-plane ESPSI with horizontal shear, showing fringes corresponding to the out of plane component ($\partial w/\partial x$) of the strain, Force= 17.5 ± 0.1 Kg-Force.*

Figure 9. *Same as previous, Force= 20.0 ± 0.1 Kg-Force.*

Figure 10. *Same as previous, Force= 22.5 ± 0.1 Kg-Force.*

Figure 11. *The reference image consists of two laterally shifted images.*

Figures 12 abcd. *Distribution of in-plane strain is shown in real-time with a double beam mutually incoherent ESPSI interferometer.*

Figure 12 e). *Diagram of the distribution of in-plane strain.*

List of figure captions

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