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An insight into the reversal of rotation in the Hecht vault

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In a Hecht vault the direction of somersault rotation is reversed during horse contact. Gymnasts contribute actively to this reversal using muscle-generated shoulder torques during this contact phase. There is also the possibility that the way in which the gymnast contacts the horse may contribute to the reversal of somersault rotation by creating naturally occurring total body rotations at horse impact. To investigate this, a two-segment model was used to simulate an instantaneous inelastic impact during which internally generated shoulder torque was constrained to zero. The simplicity of the model used in the simulation provided valuable insight into the role that the preflight trajectory plays in the reversal of total body rotation at horse impact. It was found using realistic takeoff conditions from the board, that over half of the reversal of rotation could be produced by a suitable preflight trajectory.

Keywords: Simulation; Model; Hecht; Vault; Gymnastics

1. Introduction

The men's compulsory vault in the 1996 Olympics in Atlanta is the Hecht vault. The rules governing the Hecht vault stipulate: (a) that at the time of hand contact with the horse, the gymnast's body (shoulders to feet) must be in a layout position at an angle of not less than 20° above the horizontal; (b) that the gymnast, while maintaining the layout position, must reverse the direction of his body's forward rotation about the transverse axis, and land feet first; and (c) that the gymnast must travel at least 2.5 m past the far end of the horse before landing. By today's standards in competitive gymnastics, where multiple twists and somersaults are commonplace, the Hecht vault would appear to be a 'simple' vault as it requires no twisting action and only a somersault displacement of approximately 110° during the postflight from the horse. However, the Hecht vault does provide a unique challenge to the competitor and coach as it requires a change in the direction of the gymnast's preflight angular velocity during the time of contact with the horse. An illustrative sequence of the Hecht vault is shown in Fig. 2.

To date, the primary focus of the takeoff from the board, and the ensuing preflight trajectory has been on enhancing the somersault rotation of the handspring somersault (Cheetham, 1982; Dainis, 1979, 1981; Gervais, 1994; Takei, 1988, 1989), Tsukahara (Brüggemann, 1979; Dillman et al., 1985; Zou et al., 1988, 1989), and Yurchenko (Kwon et al., 1990) vaults. These vaults, which are used in present day competition, are characterized by a short preflight and a large upward vertical velocity of the mass centre at initial horse contact. Such a preflight allows the high angular velocity in preflight to continue into postflight. The Hecht vault on the other hand requires a complete reversal of the body's angular momentum about the mass centre during horse contact. As a consequence, it might be expected that the preflight and horse contact phases associated with the Hecht vault would have to be quite different from those of the somersault vaults. It is clear that the active muscular torques generated by the gymnast around the shoulder joints during horse contact contribute to the reversal of rotation. The role of the preflight trajectory in the Hecht vault is less clear. It may be possible that a suitable preflight trajectory onto the horse could make a contribution to this reversal of rotation independent of the muscular torques exerted by the gymnast.

Computer simulation is a tool that has the potential to increase our understanding of human movement. The primary purpose of computer simulation is to increase the understanding of a particular phenomenon, not to simply replicate it. Too often researchers are discouraged from using computer simulation as they are under the mistaken belief that to be useful, simulation models must replicate the human body down to the last anatomical detail. Recently, Alexander (1992) and Hubbard (1993) have warned against the pitfalls inherent in complex models. Hubbard (1993) states that researchers commonly fall into the trap of making their mathematical models so complex that their results are essentially uninterpretable as the effects of any one independent variable is inextricably intertwined with many others. Both Hubbard and Trinkle (1984) and Alexander (1990) have shown that very simple one and two-segment models can aid in the understanding of human movement. Both advocate the use of the simplest possible model capable of capturing the essence of the task being studied.

The objective of this study was to examine, using a simple mathematical model, the role that the preflight trajectory plays in the reversal of rotation in the Hecht vault. The intention was to gain an understanding of how body position and trajectory at impact facilitates the reversal of rotation rather than attempting to quantify the impact forces.

2. Methods

The gymnast was modelled as a two-dimensional, two-segment system (Fig.1) consisting of an 'arms' segment (segment 1) and a 'body' segment (segment 2) with mass centres located at A and B respectively. The two segments were linked together at the shoulder, S, by a frictionless pin joint. A shoulder torque generator, C, was added to maintain the relative angle between the body and the arms during both the preflight and postflight phases. During horse contact, external forces (F_{1x} , F_{1y}) were considered to act through the palm of the hand (H). The axis origin for the system was fixed to the takeoff point on the board. This model of the gymnast, albeit simple, was considered appropriate to examine the role that the preflight trajectory plays in facilitating a reversal of rotation during the post flight phase. Parameter values for segment length,

moment of inertia, and mass were determined using the inertia model of Yeadon (1990). Anthropometric measurements were taken on each of 11 elite gymnasts from whom informed consent was obtained following the guidelines of the University of Calgary Ethical Advisory Committee. The 11 inertia sets generated were averaged and then normalized to a body mass of 62.9 kg and a standing height of 1.67 m which correspond to the mean mass and height of gymnasts competing at the 1988 Olympic Games (Takei and Kim, 1990).

Fig. 1. Free body diagram of the two-segment model. The external force on the hands is only present during the time of horse contact.

The mathematical modelling of the Hecht vault was divided into three phases: (a) Phase I – the preflight phase from the board until horse contact; (b) Phase II - horse contact; and (c) Phase III - the postflight phase following horse contact until the time of ground contact. For Phases I and III, there was no external force acting on the hands. Thus for these two phases, six Newtonian equations could be written for the two-segment model:

arm segment:

$$F_{2x} = m_1(a_{CMx/O} + a_{Ax/CMx}), \quad (1)$$

$$F_{2y} = m_1(a_{CMy/O} + a_{Ay/CMy}) + m_1g, \quad (2)$$

$$F_{2x}r_{A/S} \sin \theta_1 - F_{2y}r_{A/S} \cos \theta_1 + C = I_1\alpha_1, \quad (3)$$

body segment:

$$-F_{2x} = m_2(a_{CMx/O} + a_{Bx/CMx}), \quad (4)$$

$$-F_{2y} = m_2(a_{CMy/O} + a_{By/CMy}) + m_2 g, \quad (5)$$

$$F_{2x} r_{S/B} \sin \theta_2 - F_{2y} r_{S/B} \cos \theta_2 - C = I_2 \alpha_2, \quad (6)$$

where:

$\alpha_{CMx/O}$ is the horizontal acceleration of the system's mass centre with respect to the origin, O , of the inertial axis system on the takeoff board,

$\alpha_{Ax/CMx}$ is the horizontal acceleration of the mass centre of the arms respect to the system's mass centre,

$r_{A/S}$ is the distance between the mass centre of the arms (shoulder joint (S)),

α_1 is the absolute angular acceleration of the arm segment (segment 1),

θ_1 is the absolute angle between the arms segment and the right horizontal axis through the shoulder joint,

ω_1 is the absolute angular velocity of the arms segment,

I_1 is the moment of inertia about the mass centre of the arms,

m_1 is the mass of the arms segment,

F_{2x}, F_{2y} are the components of force acting through the shoulder's pin joint

C is the torque generated about the shoulder joint.

Similar notation is used for the body segment Eqs. (4)-(6).

The computer simulation required that separate expressions be derived for α_1 , and α_2 that were functions only of the variables θ_1 , θ_2 , ω_1 , ω_2 . To achieve this, the expressions for F_{1x} , and F_{1y} , from Eqs. (1) and (2) were substituted in Eqs. (3) and (6). Next, the variables $\alpha_{A/CM}$, were expressed in terms of θ_1 , θ_2 , ω_1 , ω_2 using the equations of constraint.

The constraint equations between the system's centre of mass and the individual segments' centres of gravity are a direct consequence of the friction-less pin joint through the shoulders that holds the two segments together. The choice of the system's centre of mass as the reference point was based on the ease of calculating the trajectory of the mass centre from the well-known equations of projectile motion. The constraint equations are:

$$\mathbf{r}_{B/CM} = \left(\frac{-m_1}{m_1 + m_2} \right) (\mathbf{r}_{S/B} + \mathbf{r}_{A/S}), \quad (7)$$

$$\mathbf{r}_{A/CM} = \left(\frac{m_2}{m_1 + m_2} \right) (\mathbf{r}_{S/B} + \mathbf{r}_{A/S}), \quad (8)$$

where $\mathbf{r}$, with the appropriate/subscript, designates a relative location vector (Fig. 1).

Differentiating these two equations twice with respect to time results in the following relationship:

$$\begin{aligned} \mathbf{a}_{B/CM} = \left(\frac{-m_1}{m_1 + m_2} \right) & \left(\boldsymbol{\alpha}_2 \times \mathbf{r}_{S/B} + \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{S/B}) \right. \\ & \left. + \boldsymbol{\alpha}_1 \times \mathbf{r}_{A/S} + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{A/S}) \right). \end{aligned} \quad (9)$$

$$\begin{aligned} \mathbf{a}_{A/CM} = \left(\frac{m_2}{m_1 + m_2} \right) & \left(\boldsymbol{\alpha}_2 \times \mathbf{r}_{S/B} + \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{S/B}) \right. \\ & \left. + \boldsymbol{\alpha}_1 \times \mathbf{r}_{A/S} + \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{A/S}) \right). \end{aligned} \quad (10)$$

These vector expressions for $\mathbf{a}_{B/CM}$, and $\mathbf{a}_{A/CM}$ were expanded and substituted into the two equations developed previously. The outcome was that the expressions for α_1 , and α_2 , were now only dependent on the five variables θ_1 , θ_2 , ω_1 , ω_2 and C since $\alpha_{CMx/O}$ and $\alpha_{CMy/O}$ are known constants of values 0 and - 9.81 respectively. The software package MATHEMATICA (Wolfram Research, Inc., 1991) was used to generate these lengthy equations for α_1 and α_2 so as to reduce the possibility of bookkeeping errors.

Phase II required a separate set of equations from that of the two aerial phases, since the distal end of the arms segment (i.e. hands) experiences an external impulsive force during horse contact. In order to determine the ability of a suitable preflight trajectory to produce a naturally occurring reversal of rotation during contact with the horse, the muscular torque generator at the shoulder joint was constrained to zero during horse contact. As the actual magnitude of reversed rotation was not under investigation for this particular study, the impact with the horse was considered to be inelastic, thus reducing the velocity of the hands to zero immediately after impact. During impact, the arms segment is under compression since the radial velocity of the mass centre of this segment is reduced to zero. This results in large impulsive reaction forces from the horse acting on the hands. Impulsive reactions also act at the shoulder joint connecting the two segments. The assumption that the velocity of the hands is reduced to zero permitted a constraint relationship to be expressed between the point where the hands came in contact with the horse and the mass centres of the two segments. Two independent equations for α_1 and α_2 in terms of θ_1 , θ_2 , ω_1 and ω_2 were formed using a similar approach to that outlined previously for the aerial phases. Separate equations for the forces on the hands were also determined so that the time of horse release could be identified as the time-step where the vertical hand force component dropped to zero or below.

The brief moment (Δt) of initial impact with the horse was treated separately from the rest of Phase II. A separate set of equations to handle this brief event were formulated using the impulse-momentum relationship of Newton's Second Law.

Arms

$$m_1 v_{1xi} + (F_{1x,avg} + F_{2x,avg}) \Delta t = m_1 v_{1xf}, \quad (11)$$

$$m_1 v_{1yi} + (F_{1y,avg} + F_{2y,avg}) \Delta t - m_1 g \Delta t = m_1 v_{1yf}, \quad (12)$$

$$I_1 \omega_{1i} + T_1 \Delta t = I_1 \omega_{1f}, \quad (13)$$

body

$$m_2 v_{2xi} - F_{2x,avg} \Delta t = m_2 v_{2xf}, \quad (14)$$

$$m_2 v_{2yi} - F_{2y,avg} \Delta t - m_2 g \Delta t = m_2 v_{2yf}, \quad (15)$$

$$I_2 \omega_{2i} + T_2 \Delta t = I_2 \omega_{2f}, \quad (16)$$

where the subscripts i and f refer to the initial and final values of the brief impulse interval, u is the velocity of the specified segment's mass centre, F_{avg} is the average of the specified force component over the brief time interval, and T_1 and T_2 are the average external torques acting on the two respective segments over the same brief time interval:

$$T_1 = -F_{1x,avg} r_{A/H} \sin \theta_1 + F_{1y,avg} r_{A/H} \cos \theta_1 + F_{2x,avg} r_{A/S} \sin \theta_1 - F_{2y,avg} r_{A/S} \cos \theta_1 + C_{avg}, \quad (17)$$

$$T_2 = +F_{2x,avg} r_{B/S} \sin \theta_2 - F_{2y,avg} r_{B/S} \cos \theta_2 - C_{avg}. \quad (18)$$

From the equations of constraint, where it is assumed that the hands do not slip during horse contact, we have:

$$v_{1xf} = \omega_{1f} r_{A/H} \sin \theta_{1f}, \quad (19)$$

$$v_{1yf} = -\omega_{1f} r_{A/H} \cos \theta_{1f}. \quad (20)$$

$$v_{2xf} = \omega_{1f} r_{S/H} \sin \theta_{1f} + \omega_{2f} r_{B/S} \sin \theta_{2f}. \quad (21)$$

$$v_{2yf} = -\omega_{1f} r_{S/H} \cos \theta_{1f} - \omega_{2f} r_{B/S} \cos \theta_{2f}. \quad (22)$$

Substituting Eqs. (17)-(22) into Eqs. (11)-(16), results in six equations for nine unknowns (θ_{1f} , θ_{2f} , F_{1x} , F_{1y} , F_{2x} , F_{2y} , ω_{1f} , ω_{2f} , C_{avg}). Since it was the angle and not the actual magnitude of impact that was of primary importance for an understanding of the preflight trajectory's role in the reversal phenomenon, the impact duration was constrained to a very short period of time (1/1000 s). Under this impact condition, the

angular displacements of the two segments can be assumed to change very little. Thus the angular displacements after impact may be approximated by their values immediately prior to impact. This, along with setting the average shoulder torque (C_{avg}) to zero during this brief interval, reduced the unknowns from nine to six (F_{1x} , F_{1y} , F_{2x} , F_{2y} , ω_{1f} , ω_{2f}), which meant that the system of equations defining the model's impact with the horse was now determinate. The new updated values for the angular velocities of the two segments resulting from this collision event were then used as the initial values for the beginning of any subsequent horse contact phase that might result. The time of horse release was determined by the vertical reaction on the hands falling to zero. At this time the angular momentum about the mass centre was calculated from the angular velocities of arms and body and a common angular velocity was determined using the conservation of angular momentum at the beginning of the postflight phase.

The exact time of horse contact of the hands with the horse was determined in Phase I using a nonlinear interpolation approach, as the regular time-step interval of 1/60 s could not be expected to coincide exactly with the instant of horse contact. The interpolation procedure consisted of fitting a second order polynomial to the vertical location curve produced by the hands during the three frames prior to the time that the hands reached the level of the top of the horse.

$$y_I = a \times [I \times \Delta t]^2 + b \times [I \times \Delta t] + c, \quad (23)$$

$$y_{I-1} = a \times [(I-1) \times \Delta t]^2 + b \times [(I-1) \times \Delta t] + c, \quad (24)$$

$$y_{I-2} = a \times [(I-2) \times \Delta t]^2 + b \times [(I-2) \times \Delta t] + c, \quad (25)$$

where I refers to the current frame number, Δt is the time interval (1/60 s), and y_I is the vertical location of the hands at the approximate instant of horse contact. The terms $y_{I=1}$ and $y_{I=2}$ refer to the vertical displacement of the hands one and two time-steps prior to the approximate instant of horse contact. The coefficients a , b , and c can be determined from these three equations. A very good estimate of the time t of initial horse contact can then be determined using the quadratic equation:

$$1.20 = at^2 + bt + c, \quad (26)$$

where 1.20 m is the known height of the horse above board level.

During the two aerial phases of the vault, the angle between the body and the arms that was established at the instant of takeoff, was kept constant by applying the necessary shoulder torque (C) as determined by Eq. (27). This equation was obtained by substituting Eqs. (1) and (2) and Eq. (10) into Eq (3), and then applying the constraints $\omega_1 = \omega_2$, and $\alpha_1 = \alpha_2 = 0$.

$$\begin{aligned}
C = & - \left(\left(m_1 \left(a_{CMx/O} + K \left(-\omega_2^2 r_{B/S} \cos \theta_2 - \omega_1^2 r_{A/S} \cos \theta_1 \right) \right) \right) r_{A/S} \sin \theta_1 \right. \\
& - \left(m_1 \left(a_{CMy/O} + K \left(-\omega_2^2 r_{B/S} \sin \theta_2 - \omega_1^2 r_{A/S} \sin \theta_1 \right) \right) \right. \\
& \left. \left. + m_1 g \right) r_{A/S} \cos \theta_1 \right), \tag{27}
\end{aligned}$$

where $K = m_1/(m_1 + m_2)$.

The forward Euler iterative scheme was used to drive the simulation of the model:

$$\theta(I+1) = \theta(I) + \omega(I) \times \Delta t, \tag{28}$$

$$\omega(I+1) = \omega(I) + \alpha(I) \times \Delta t, \tag{29}$$

Where

$\theta(I)$ is the absolute angle of the segment at the present time-step I,

$\theta(I+1)$ is the absolute angle of the segment at the next time-step.

Similar notation is used for the absolute angular velocity (ω).

This iterative method proceeded by first calculating the α value for each of the two segments (i.e. arms and body) using the current values of θ_1 , θ_2 , ω_1 , ω_2 for the present time-step. The present α and ω values were then used to determine the new θ and ω values for the next time step. This procedure was repeated until the vault was completed.

The mathematical simulation commenced at the instant of takeoff from the board, and terminated when either the distal end of the arms (i.e. hands), or the distal end of the body (i.e. feet) contacted the floor. The time step for the simulation was set at 1/60 s so as to agree with the common operating rate of most video cameras. The required input parameters, which could be altered for each vault simulation, consisted of the following initial conditions at board takeoff: (a) the angular positions of both segments; (b) the angular velocity of the rigid system; (c) the horizontal and vertical components of velocity of the system's mass centre; and (d) the distance that the board was positioned from the horse.

To examine the many different combinations of 'plausible' takeoff conditions that might possibly lead to a successful Hecht vault using only the induced rotations produced from horse impact, the constrained optimization approach of Powell (Press et al., 1992) was used. The Powell optimization method uses a zero order search method which has the advantage that it does not require the gradient of the objective function to be known. To use the Powell subroutine, the computer source code for the Hecht vault was implemented as a function that could be minimized.

The objective function was composed of penalty variables relating to inappropriate behaviour by the model during the simulated vault. For example, if the horizontal velocity of the mass centre of the model exceeded 6.5 m/s horizontally at takeoff from

the board, an exponentially increasing penalty value for a designated variable in the objective function, would accrue. The value of 6.5 m/s was used as an estimate of the upper bound of the horizontal preflight velocity for a Hecht vault (Sprigings and Yeadon, 1994). If the model failed to travel at least 2.5 m past the end of the horse before landing, the value of another penalty variable inside the objective function increased. If the body segment of the gymnast failed to reach at least an angle of 20° above the horizontal at horse contact, another penalty variable increased in value. In all, there were a total of 22 penalty variables that were incorporated within the objective function. To the human mind, a number of these penalty variables would seem unnecessary, such as that which ensures takeoff from the board in the forward direction, but to an optimization scheme which relies purely on brute force to arrive at a minimum, none of these 'obvious' knowns could be assumed, and penalty constraints had to be implemented. Each iteration of the optimization program proceeded until either the feet or the hands reached floor level. Dynamic graphic images of the model on the computer screen provided feedback on the simulation, as well as the effectiveness of the imposed constraints used in the optimization.

To examine the effect that an increase in horizontal takeoff velocity can have on the performance of the vault, optimized simulations were run using higher cut-off thresholds for horizontal takeoff velocity. This was done until a performance complying with F.I.G. (International Gymnastics Federation) specifications was achieved.

To determine whether an even simpler model would have been sufficient to produce the characteristic reversal of rotation of a Hecht vault, the simulation model was reduced to a single rigid segment that represented the complete gymnast. This was done by calculating the magnitude of internally generated shoulder torque needed to maintain a rigid body-aim configuration during horse contact.

To examine the performance of the model under realistic conditions, the mean values of 27 competitive performances of the Hecht (Sprigings and Yeadon, 1994) were calculated and used as input to the model. This provided a means of evaluating whether real-life gymnasts are currently using a preflight trajectory that takes advantage of any naturally occurring reversal of rotation induced by the manner in which contact is made with the horse.

3. Results

For the first optimization search, H65, the penalty threshold for the upper limit of horizontal takeoff velocity was set at 6.5 m/s. Under this condition of constraint, the optimization scheme produced a vault in which the body of the gymnast achieved an angle of 7° above the horizontal at horse contact (Fig. 2). This is short of the required 20° angle. At the instant of initial horse contact, the gymnast's mass centre had a small downward velocity of 0.5 m/s. The horse contact phase lasted only for one time-step; a consequence of the inelastic impact. At horse release, the horizontal and vertical components of velocity of the gymnast's mass centre were 3.7 m/s and 2.3 m/s, respectively. During the early stages of the simulated postflight phase following the reversal of body rotation, the feet of the model contacted the horse. If we assume that a gymnast would split his legs so as to miss the horse during this stage, then he would land with a forward lean of 8° from the vertical at a distance of 1.27 m from the far end

of the horse. Although such a performance falls short of the criteria for a good Hecht, it is remarkable that most of the required reversal of rotation has been achieved in a simulation which did not receive any contribution from the shoulder torque generator. This indicates that for a suitable preflight trajectory the majority of the reversal of rotation is a consequence of the impact with the horse rather than a result of any muscular torques exerted by the gymnast around the shoulder joints.

To achieve a Hecht vault that complied fully with F.I.G. specifications, the simulation results indicated that the horizontal takeoff speed had to reach approximately 8.5 m/s with the takeoff board placed 3.85 m from the horse. While such conditions are certainly unattainable by any gymnast, it does provide us with an insight to the importance of takeoff speed in executing this vault.

The simulation based on the mean values of 27 competitive Hecht performances had horizontal and vertical velocities of 5.6 m/s and 0.9 m/s respectively just prior to horse contact. At this time the body was 2° above the horizontal and had an angular velocity of $195^\circ/\text{s}$ (Table 1). After horse impact without any internal shoulder torque, the simulation produced an angular velocity of $-35^\circ/\text{s}$ in postflight. This is short of the $-104^\circ/\text{s}$ of the competitive data but it should be recognized that the simulated inelastic impact produced more than 70% of the $299^\circ/\text{s}$ reversal of angular velocity produced in competition.

The search for an optimized Hecht using a single-segment model was unsuccessful. In all simulations the single-segment model either failed to reverse the direction of preflight rotation (Fig. 3) or failed to takeoff from the horse. It appears that there are no suitable preflight conditions for a single-segment model which result in anything resembling a Hecht vault.

In order to check that the results of the simulation program were not contaminated by programming errors, an independent simulation model was programmed by the coauthor. It was found that the results of the two programs were in agreement.

As with any optimization scheme, there is no guarantee that the minimum arrived at by the POWELL subroutine is the global minimum. A check was performed on the reliability of the determined 'optimal' starting values by entering different starting values into the POWELL optimization scheme to see if the recomputed 'best' values converged on those found in previous runs. The results indicated that the POWELL optimization method was stable for this particular application.

Fig. 2. Optimized simulation for a Hecht vault (H65) using a two-segment model with zero shoulder torque and a horizontal preflight velocity limit of 6.5 m/s. Values shown are for the instant of takeoff from the board.

Fig. 3. Simulation results for a single-segment model. This particular model was identical to model H65 except that rotation at the shoulder joint was prevented during impact. Values shown are for the instant of takeoff from the board and are the same as those used for the two-segment model in Fig. 2.

4. Discussion

A simple two-segment model has been used to simulate the preflight, horse contact, and postflight phases of the Hecht vault in gymnastics. The assumption of a fixed body configuration during the two aerial phases of the vault is conservative in that it results in less postflight rotation for the simulation than if the arms segment of the model is allowed to rotate forwards relative to the body in each aerial phase.

The use of the two-segment model to simulate a gymnast during the contact phase with a vaulting horse provided a means of examining the role that preflight trajectory plays in reversing the rotation in the Hecht vault. This elementary model has not incorporated any elastic elements into either the horse surface or the shoulder joint of the model. As a result, the duration of the contact with the horse was limited to one brief time-step. With such a short duration of horse contact, the impulsive reaction forces are unrealistically large. It was not the intention, however, to obtain estimates of the reaction forces during the contact phase. Rather, the aim was to investigate how body position and trajectory at impact contribute to the required rotation for a Hecht vault. An elastic horse would tend to increase the contact time as well as create a vertical velocity of the hands which will facilitate the reversal of angular momentum about the mass centre. Thus the lack of elastic elements in the present model would be expected to hinder the production of the reverse rotation. As a consequence, the estimate of the contribution of the impact with the horse to the reversal of rotation would be higher if elastic elements were to be introduced into the model.

Although this two-segment model is without viscoelastic elements, it does have the capacity to lose energy during impact. By forcing the velocity of the hands relative to the horse to be zero at impact, the Newtonian equations that model this phase of the vault implicitly assume a zero coefficient of restitution value which results in a maximum loss of energy during impact. This is equivalent to having large viscoelasticity that reduces the relative velocity of the hands and horse to zero and dissipates energy as heat. Admittedly, this method of modelling the impact with the horse results in a more conservative estimate of performance since the energy loss is substantially higher than that observed in a competitive Hecht vault. For example, using the velocity values in Table 1 for the mean competitive Hecht vault, the energy loss during impact is 24%, while the energy loss of the simulation based on the same initial preflight conditions is 41%. This suggests that in reality there are elastic elements in the horse or gymnast that make significant contributions to the performance. Indeed in competitive Hecht vaults the presence of such elastic energy is further supported by the observation that the vertical velocity of the hands as they release the horse is around 4 m/s. However, the lack of elastic elements in this model does not limit the conclusion that suitable preflight conditions produce an impact in which the angular velocity is reversed in direction.

Table 1. Preflight and postflight characteristics of handspring somersault, Hecht, and simulated Hecht

	Handspring somersault ^a	Hecht ^b	Simulated Hecht ^c
$V_x(CM)$ at horse contact [m/s]	5.0	5.6	6.5
$V_y(CM)$ at horse contact [m/s]	2.3	0.9	-0.5
θ_2 at horse contact [°]	212	182	187
ω at horse contact [°/s]	411	195	190
$V_x(CM)$ at horse release [m/s]	3.6	4.2	3.7
$V_y(CM)$ at horse release [m/s]	2.7	2.4	2.3
ω at horse release [°/s]	591	-104	-117

^a Mean values from USA National Championships (Takei and Kim, 1990).

^b Mean values from Canadian National Championships (Springs and Yeadon, 1994).

^c Optimized simulated Hecht H65 in this study.

The assumption of zero shoulder torque during horse contact is unlikely to reflect competitive technique but was necessary since the aim of this study was to investigate the possibility of reversing rotation without the use of shoulder torque. In addition it is questionable whether appreciable shoulder torque can be exerted for all of the contact phase since the arms reach an angular velocity relative to the body in excess of 400°/s (Springs and Yeadon, 1994) thus limiting the maximum torque generating capabilities of the gymnast.

Even though the model does not use internally generated torques at the shoulders during impact, it can account for many of the characteristics of a Hecht vault. Indeed the preflight characteristics of the optimized simulated Hecht vault (H65) are similar to those of competitive performances of the Hecht (Table 1). The vertical velocity of the mass centre at horse contact in the optimized simulation was -0.5 m/s compared to the values 0.9 m/s and 2.3 m/s for competitive performances of the Hecht and handspring somersault, respectively. During parabolic flight a velocity of -0.5 m/s occurs after the peak when the mass centre is 0.013 m below the peak while a vertical velocity of 0.9 m/s occurs before the peak is reached when the mass centre is 0.041 m below the peak. Thus contact is made with the horse in the simulated Hecht just after the peak of preflight whereas for the competitive Hecht, contact occurs just before the preflight has reached a peak. In both cases horse contact occurs close to the peak of the preflight parabola. The simulated Hecht obtained a postflight angular velocity of 117°/s which exceeded the mean value of 104°/s of the competitive Hechts. The gymnasts in the competition compensated for this lower angular velocity by piking during postflight and by counter-rotating their arms.

The differences between the preflights of handspring somersault and Hecht vaults are understandable from the point of view that there is an optimal preflight trajectory

that facilitates rotation in a particular direction. The optimum technique for a Hecht when there is shoulder torque available may be expected to be somewhat different from the optimum obtained in this study but it is unlikely to be very different since the manner in which the gymnast contacts the horse is responsible for most of the required rotation. Thus the differences between the preflight of the optimized simulated Hecht and the preflights of the competitive performances in Table 1 could be either a consequence of the lack of shoulder torque and elastic elements in the model or a result of less than optimal technique in the competitive performances. Since the performance data in Sprigings and Yeadon (1994) is based on the 1993 Canadian Championships, which was the first competition in which the Hecht was used as the compulsory vault, it is to be expected that the techniques used were less than optimal. Because of this, it is speculated that preflight trajectories used in subsequent competitions will be more similar to the optimized simulated preflight characteristics. In particular it is predicted that the mean horizontal preflight velocity will be greater than 5.6 m/s, and the mean vertical velocity at horse contact will be less than 0.9 m. As a consequence the body angle above the horizontal at horse contact will be greater than 2° and the angular velocity at horse release will be greater than $104^{\circ}/s$.

The hypothetical simulation H65 produced sufficient reversal of rotation to be classified as a Hecht while the simulation based on the mean preflight data of 27 competitive Hecht performances produced more than 70% of the reversal of angular velocity that occurred in the mean competitive performance. It is remarkable that a simple two-segment model employing an inelastic impact with the horse can produce most of the reversal of rotation needed in a Hecht vault. The result that a single-segment model is incapable of producing a Hecht for any possible preflight indicates that it is the freedom at the shoulder joint in the two-segment model together with a suitable preflight trajectory that leads to a naturally induced reversal of rotation. From the results of the simulation in which real-life takeoff values were used, it may be concluded that the contribution to the reversal of rotation arising from the preflight impact angle with the horse, is greater than the contributions from both the elasticity of the horse and gymnast and the torques exerted by the gymnast during horse contact. It is speculated that the appropriate initial conditions at contact will make similar substantial contributions to performance in many other examples of human movement.

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