**Online Supplement for “An Approach for Analyzing and Managing Flexibility in Engineering
Systems Design Based on Decision Rules and Multistage Stochastic Programming”**

by Cardin, M.-A., Xie, Q., Ng, T.S., Wang, S. and Hu, J. (2016)

Table I: Summary of notation

Variable	Definition	Variable	Definition
T	Number of time periods in the planning horizon	ξ_t	Realization of uncertainty / Amount of food waste collected in year t
η_t	Amount of other organic waste collected in year t	x_t	Option chosen for in time t
π	Discount rate	ξ	A scenario of uncertainty
Ω	Set of all possible uncertainty scenarios	ξ^k	k^{th} scenario of uncertainty
p^k	Probability of scenario k	X_t	Set of feasible options in period t
x	Option sequence	X	Set of all feasible option sequences
δ	Decision rule	θ	Set of parameters of the decision rule
δ_θ	Decision rule with parameter θ	$\mathcal{D}$	Set of all mappings from Ω to X
$\xi_{[t]}$	The history of the uncertainty realization up to period t	$r_t(\delta_\theta(\xi_{[t]}^k), k)$	Profit function in period t in scenario k
$\delta_\theta(\xi)$	The sequence of options chosen following the decision rule	$\delta_\theta(\xi_{[t]}^k)$	The option chosen at time t in scenario k
$r(\delta_\theta(\xi), \xi)$	Total profit function	Δ	A subset of $\mathcal{D}$
δ^*	The optimal decision rule	θ^k	A replication of the decision rule parameters in scenario k
$\bar{\theta}$	The average of θ^ξ over its K replications	λ	Lagrange multipliers
λ^k	Lagrange multipliers for scenarios k	$L(\delta_\theta, \lambda)$	Lagrangian of the original problem
$D(\lambda)$	Optimal value of the Lagrangian relaxation problem with the Lagrange multiplier λ	Z^{LD}	Optimal value of the Lagrangian dual problem
t_i	Step size in i th iteration in the gradient	r_t	Profit function

	method		
R_t	Revenue function	C_t	Cost function
P_t	The penalty function due to the capacity shortage	f_t	Capacity shortage of AD
f'_t	Capacity shortage of gasifier	n	Number of jumps from each node
$\xi_{t+1,j}$	Food waste amount in node order j of the following n nodes at time interval $t + 1$	p_j	Probability of the state in node order j
α	The severity level of the current capacity	β	The number of modules expanded each time
o_u	Capacity of the unit module	e_t^k	Binary variables indicating whether to expand in year t in scenario k
h_t^k	The amount of capacity to be added	ε	The number of modules initially installed in AD
ε'	The capacity of gasifier initially installed	x_M	The maximum number of modules allowed to be installed in AD
y_M	The maximum capacity allowed to be installed in gasifier	x_f	The capacity of the baseline inflexible design
y_t	Capacity of gasifier in year t	$\gamma_{1t}, \gamma_{2t}, \gamma_{3t}$	Coefficients of the linear decision rule for gasifier
T_t	Tipping fee function of hybrid WTE system	R_{At}	Revenue function of AD in hybrid WTE system
R_{Gt}	Revenue function of gasifier in hybrid WTE system	C_{At}	Cost function of AD in hybrid WTE system
C_{Gt}	Cost function of gasifier in hybrid WTE system	S_t	Set of state variables in period t
$R_t(S_{t-1})$	Function of total reward starting from period t until the last period	$\phi_t(\cdot)$	Basis function in the linear value function approximation in ADP
ρ_t	Vector of weights of in the linear value function approximation in ADP	H	Number of iterations in LSPI algorithm
I	Number of sample paths generated in each iteration in LSPI algorithm		

* The variables with superscript k correspond to the variables in scenario k .

Table II: List of assumptions for WTE system

Parameters	Value	Definition
T	9	Time span of the system considered
p	70%	Purity ratio of the food waste feedstock
τ	10%	Residue ratio of the AD process
z_1	S\$22,469/tpd*	Revenue from electricity generation per tpd food waste processed in AD unit
z_2	S\$1,336/tpd	Revenue from compost per tpd waste processed in AD unit
z_3	S\$28,105/tpd	Revenue from tipping fee per tpd waste collected
z_4	S\$700/tpd	Transportation cost per tpd food waste collected
z_5	S\$75,000/tpd	Capacity installation cost per unit AD capacity
z_6	S\$816/tpd	Land rental cost per unit capacity installed
z_7	S\$204/tpd	Land rental cost per unit capacity reserved
z_8	S\$675/tpd	Labor cost per unit capacity of AD unit
z_9	S\$225/tpd	Maintenance cost per unit capacity of AD unit
z_{10}	S\$28,105/tpd	Disposal cost per tpd waste disposed in landfill
π	8%	Discount rate
o_u	30 tpd	Capacity of an unit module
p'	70%	Purity ratio of the other organic waste feedstock
τ'	20%	Residue ratio of the gasifier
z_{11}	S\$62,678/tpd	Revenue from electricity generation per tpd waste processed in the gasifier
z_{12}	S\$5,840/tpd	Total of labor, admin, maintenance cost per tpd waste treated in the gasifier
z_{13}	S\$2,920/tpd	Cost of the RDF process per tpd of waste treated in the gasifier

z_{14}	S\$96,970/tpd	Capital cost of per tpd capacity of the gasifier
----------	---------------	--

*tons per day (tpd) is used to as the unit of waste amount and the capacity of the AD plant.

Table III: Parameters of the main uncertainty drivers in the case study

Parameters	Definition	Food waste	Other organic waste
μ	Annual growth rate	14.1%	6.0%
σ	Volatility	16.4%	4.1%
ξ_0/η_0	Waste amount in year 0	191 tpd	2,823 tpd