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Adaptive Guidance and Control of Uncertain Lunar Landers in Terminal Landing Phases

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Abstract A novel double-loop guidance and control strategy for under-actuated lunar landers in terminal landing phases is developed by using adaptive nonlinear control approach in this study. To derive the main thrust input and inner-loop desired attitude trajectory, the outer-loop position tracking guidance law is firstly developed based on the hyperbolic tangent functions to guarantee the constrained thrust and singularity avoidance of the desired attitudes. Then, to avoid the complicated analytic-derivative computing of the desired attitude trajectory, a stable second-order filter is employed to generate the command attitude trajectory for the inner-loop attitude motion. Finally, an adaptive attitude tracking controller is designed by combining the barrier Lyapunov function and the backstepping technique to get rid of the singularities of the

Euler angles-based attitude kinematic Jacobian matrix. In addition, tuning rules for designing parameters in guidance law and attitude controller are derived based on the Lyapunov analysis, and the pose tracking errors in the closed-loop system ultimately converge to the small neighborhoods of the origin. An example is simulated to verify the effectiveness of the proposed control design approach.

Keywords Lunar lander; guidance and control; adaptive control; nonlinear control

1 Introduction

Lunar lander is generally launched by a rocket to the geocentric parking orbit. After driving into the geocentric parking orbit, the lunar lander will communicate with the command center on the Earth and depart the geocentric parking orbit at the predefined time and location. After the acceleration of the thruster, the lunar lander can fly to the space between the Earth and the Moon, then the flight of the lunar lander is completely determined by the Moon's gravitation due to no atmosphere around the Moon. When the lunar lander is close to the selenocentric orbit, the thruster of the lunar lander will be worked again so that the lunar lander can enter safely the selenocentric orbit. Then, the lunar lander checks its motion state, communicate with the command center on the Earth and depart the selenocentric orbit at the predefined time and location. The lunar lander will land softly on the Moon's surface by the reverse force of the thruster after the powered descending phase and terminal landing phase [1]. To achieve a lunar lander soft landing on the Moon's surface, the guidance and control system plays a key role in both the descending and the landing phases, during which the lander should departure from the selenocentric orbit, decelerate from an initial orbital velocity to a low touch-down velocity, and maneuver to the vertical landing attitude [2].

Lunar landing control design is a challenging mission because of the lunar lander's under-actuated and coupled pose dynamics. Many scientists and engineers focused on the guidance and control system design of the lunar landing operations in the past years. A guidance and control method based on the fixation-point inheritance was proposed in [3] so that the space-

craft can safely descend toward the target point on the celestial body. An optimal nonlinear control law for lunar soft landing was proposed in [4] based on the artificial neural networks and Pontryagin's maximum principle. A barrier function-based guidance law was introduced in [5] to generate the real-time trajectory for a lunar lander subject to the constraints of the control inputs. A line-of-sight homing technology was applied to achieve high-precision lunar landing in [6]. In [7], an explicit guidance law was designed for the powered descent phase and the gravity-turn guidance law was designed for the terminal landing phase. A lunar landing experiment of investigating the final approach and terminal descent phases was reported in [8] based on the ground-based motion simulators. Then, the effects of the control power and inceptor sensitivity were analyzed in [9], while the effects of the control power and guidance cues were shown in [10]. The lunar landing missions were achieved in [3]–[7], but only the open-loop guidance law was developed without considering the three-axes attitude motion of the lunar landers, so that the high-precision control performance cannot be guaranteed well in practice. Moreover, in spite of the analysis on the effects of many factors in lunar landing handling qualities, the explicit guidance and control algorithms were not presented in [8]–[10]. Recently, many closed-loop guidance and control methods were proposed to improve the response performance. An optimal closed-loop control solution to the nonlinear problem of the lunar landing mission has been developed in [11], where the perturbation of the spacecraft mass and the perturbation of the gravitational acceleration were considered. In [12], the high-order sliding mode control system was studied to enhance the robustness properties of a lunar landing vehicle during the approach phase of the powered descent. In [13], an optimal controller which minimizes the fuel consumption and the flight time was designed by using a time-scaling transformation and based on the control-parametrization method to calculate an approximated optimal feedback gain matrix. A class of multiple sliding surface-based nonlinear finite-time guidance algorithms for the asteroid descent and landing was proposed in [14]. A lunar lander demonstrator used for developing and demonstrating lunar landing technologies was developed in [15], but the explicit design procedures were not shown. An existing guidance law developed for missiles was modified in [16] to be applied for the fine braking phase of the lunar powered descent. In [17], a nonsingular terminal sliding mode-based finite time controller was

developed for the soft landing on asteroids. In [18], three lunar landing control techniques were developed based on the various combinations of the terminal and programmed control methods. In [19], an optic-flow estimation-based nonlinear trajectory tracking method was applied to the lunar landing systems. To avoid the constraints on the thrust magnitude and look-angle in the terminal landing phase, a multi-constrained suboptimal guidance method based on the model predictive static programming technique was presented in [20]. A hybrid guidance law for lunar landers was formulated in [21] to incorporate multiple guidance algorithms under a single coherent framework, so that the hybrid guidance law autonomously switches between multiple guidance algorithms to offer the highest landing performance. However, the coupled attitude motion of the lunar lander was ignored in the proposed guidance algorithm. In view of the strong couplings in the under-actuated pose dynamics of the lunar landers shown in [2], a virtual dual-quaternion landing trajectory augmented force-torque sequential control law was developed in [22]. This provides a different insight of the guidance and control system design for the lunar landers, since the lunar lander was treated as the well-known vertical take-off and landing vehicles in [2] and [22]. In [23], a quadratic regularization-based fuel-optimal controller was proposed for the soft lunar landings. However, when the 3-2-1 Euler angles are used in the attitude kinematics of the lunar landers, the computational singularity of the inverse Jacobian cannot be avoided directly in the inner-loop attitude control design if the pitch angle exceeds the inherent constraint of $\pm\pi/2$ as explained in [2][24], but few works were undertaken to deal with this challenging and interesting problem. Thus, it is necessary to develop an advanced guidance and control strategy for lunar landers [24][25].

This study mainly focuses on the development of the guidance and control strategy for the terminal landing phase of a lunar lander. The six-degrees-of-freedom coupled nonlinear pose dynamics of an uncertain lunar lander with four control inputs are developed. Then, a double-loop structure with the outer-loop position design and the inner-loop attitude design is proposed based on an adaptive nonlinear state feedback method. In particular, the hyperbolic tangent functions are employed in the outer-loop guidance law to ensure the bounded thrust and to avoid singularity of the inner-loop desired attitude trajectory, while the barrier Lyapunov function is combined with the backstepping method to derive the inner-loop attitude control law, so that

the inherent constraints of the pitch angle in the kinematic Jacobian is guaranteed. Moreover, a stable second-order command filter is introduced to generate the command attitude trajectory such that the complicated time-derivative computation of the desired attitude trajectory can be avoided. Main contributions and unique features of this work are listed as follows. Firstly, compared with the method of introducing attitude quaternion to indirectly avoid constraints of the Euler angles in [22] and [25], the proposed guidance and control strategy with suitable designing parameters and initial conditions can satisfy the singularity-free requirements in the desired attitude extraction and directly avoid the constraints on the pitch angle attitude. Secondly, compared with the lunar landing system designing methods in [2], [22], and [25], unknown external disturbances and parametric uncertainties in the lunar lander's pose dynamics are considered in the adaptive nonlinear guidance and control system design, where the hyperbolic tangent function is used to design variable structure control for suppression of the bounded external disturbances, while the projection-based adaptive laws are used in the feedforward control to compensate the unknown mass and inertia. Finally, the explicit tuning rules for the designing parameters are derived via Lyapunov analysis, and the ultimately uniformly convergence of the tracking errors for overall system is proved rigorously without ignoring the effects of command filtered errors.

The rest of this paper is organized as follows. The mathematical model of the lunar lander's pose motion is established based on the Euler-angles attitude description in Section 2, and the designing objective of the terminal landing mission is given. In Section 3, the overall system is divided into two subsystems based on a double-loop design strategy, where the outer-loop guidance law is designed before designing the inner-loop attitude controller. In Section 4, the stability of the closed-loop system is analyzed rigourously in the Lyapunov framework. Simulations are presented in Section 5 to verify the effectiveness of the proposed guidance and control strategy, followed by the conclusions in Section 6.

2 Problem Statement

To derive the translational dynamics of the lunar lander, an inertial coordinate frame $O_I XYZ$ is formulated with origin O_I attached to the center of the Moon. As shown in Fig. 1, the $O_I XY$

Figure 1: Schematic diagram of terminal landing of the lunar lander.

plane coincides with the Moon's equatorial plane, and the X -axis points from the origin to the line of zero degree longitude; while the Z -axis points to the arctic pole of the Moon. During the terminal landing phase, suppose that the module aims at position O_T with longitude α and latitude β , which is the pre-selected site on the Moon's surface. Then, a target coordinate frame O_Txyz with origin attached to the terminal point O_T is further defined with the x -axis pointing outer space along the direction $O_I O_T$; the z -axis is in the $O_T O_I z$ plane and is perpendicular to $O_I O_T$; while the y -axis can be defined by the right-handed rule. The body-fixed frame $O_Lx_b y_b z_b$ of the lunar lander is defined with the origin in the center of its mass, and unit vectors coincide with the principal axes of inertia. The O_Lx_b axis is along the geometric vertical axis of the lunar lander and the single main thruster is aligned with the O_Lx_b axis. Since the O_Tx axis is perpendicular to the Moon's surface, thus the anticipated landing attitude is considered herein to coincide with target frame O_Txyz . r_l , r_t , and r represent vectors from O_I to O_L , from O_I to O_T , and from O_T to O_L , respectively. Moreover, to facilitate the modeling, assume that the gravity field of the Moon is homogeneous and ignore the Moon's rotation. The pose motion of the lunar lander considered herein is equipped with attitude actuators providing torques about three mutually perpendicular axes in the $O_Lx_b y_b z_b$ frame and one thruster only providing propulsion force along the O_Lx_b axis. The orientation of the lunar lander with respect to the target frame

O_Txyz is described by the Euler angles. Then, the pose dynamics of the lunar lander can be modeled as [2]

$$\begin{cases} \dot{\mathbf{r}} = \mathbf{v} \\ m\dot{\mathbf{v}} = -\frac{m\mu}{\|\mathbf{r}_t + \mathbf{r}\|^3}(\mathbf{r}_t + \mathbf{r}) + fR^T\mathbf{e}_1 + \mathbf{d} \\ \dot{\boldsymbol{\vartheta}} = W(\boldsymbol{\vartheta})\boldsymbol{\omega} \\ J\dot{\boldsymbol{\omega}} = -S(\boldsymbol{\omega})J\boldsymbol{\omega} + \mathbf{u} + \mathbf{w} \end{cases} \quad (1)$$

where $\mathbf{r}$ and $\mathbf{v}$ are the position and the velocity of the origin O_L with respect to O_T , respectively; $\mathbf{r} = \mathbf{r}_l - \mathbf{r}_t$, $\mathbf{r}_t = [r_t, 0, 0]^T$; $\mathbf{e}_1 = [1, 0, 0]^T$; μ is the Moon's gravitational constant; m is the mass of the lunar lander; f is the control thrust of the lunar lander represented in $O_Lx_by_bz_b$; $\mathbf{d}$ is the perturbing force, accounting for unmodeled/unknown forces (e.g., gravity field inaccuracies, solar radiation pressure, and three-body perturbations); $\boldsymbol{\vartheta} = [\gamma, \theta, \psi]^T$, while ψ , θ and γ denote Euler angles revolving about the z_b -axis, y_b -axis and x_b -axis, respectively; $\boldsymbol{\omega}$ denotes the angular velocity of the lunar lander represented in $O_Lx_by_bz_b$; J , $\mathbf{u}$, and $\mathbf{w}$ are the lunar lander's momentum of inertia, control torque, and perturbing torque represented in $O_Lx_by_bz_b$, respectively; rotational matrix from O_Txyz to $O_Lx_by_bz_b$ is expressed as

$$R = \begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ -c\gamma s\psi + s\gamma s\theta c\psi & c\gamma c\psi + s\gamma s\theta s\psi & c\theta s\gamma \\ s\gamma s\psi + s\gamma c\theta c\psi & -s\gamma c\psi + s\gamma s\theta c\psi & c\theta c\gamma \end{bmatrix}.$$

$$W(\boldsymbol{\vartheta}) = \begin{bmatrix} 1 & t\theta s\gamma & t\theta c\gamma \\ 0 & c\gamma & -s\gamma \\ 0 & \frac{s\gamma}{c\theta} & \frac{c\gamma}{c\theta} \end{bmatrix},$$

with $s \triangleq \sin$, $c \triangleq \cos$, $t \triangleq \tan$; and $S(\mathbf{v})$ denotes the skew-symmetric matrix for any vector $\mathbf{v} = [v_1, v_2, v_3]^T$ as $S(\mathbf{v}) = [0, -v_3, v_2; v_3, 0, -v_1; -v_2, v_1, 0]$.

Remark 1 According to Fig. 1, it should be noted that the propulsion unit is fixed to the lunar lander and the single main thruster is aligned with the O_Lx_b axis. This results in the couplings between position and attitude dynamics. Moreover, the translational and rotational motions of the lunar lander perform a 6-DOF coupled dynamic system with only four-independent control inputs f and $\mathbf{u}$. This is a complex nonlinear under-actuated system, which is difficult to be handled by directly using traditional linear control methods.

Assumption 1 In model (1), it is assumed that the mass m and the inertial parameter $\boldsymbol{\delta}_J = [J_{11}, J_{12}, J_{13}, J_{22}, J_{23}, J_{33}]^T$ are uncertain and slowly varying, but they are bounded by $0 < m_0 \leq m \leq \bar{m}$ and $\|\boldsymbol{\delta}_J\| \leq \bar{\lambda}$ with known constants $m_0, \bar{m}$, and $\bar{\lambda}$. The perturbing force and torque are bounded by $\|\mathbf{d}\| \leq \bar{d}$ and $\|\mathbf{w}\| \leq \bar{w}$ with known constants $\bar{d}$ and $\bar{w}$.

For under-actuated pose dynamical model of the lunar lander under Assumption 1, the objective of this paper is to synthesize an advanced guidance and control strategy such that the tracking errors of the pose and velocities ultimately converge to tunable small neighborhoods of the origin.

3 Guidance and Control Design

Define a vector $\mathbf{s} = \mathbf{v} + \lambda \mathbf{r}$ with a constant $\lambda > 0$, then the time derivative of $\mathbf{s}$ based on (1) is

$$m\dot{\mathbf{s}} = m\dot{\mathbf{v}} + m\lambda\dot{\mathbf{r}} = m \left[\lambda \mathbf{v} - \frac{\mu(\mathbf{r}_t + \mathbf{r})}{\|\mathbf{r}_t + \mathbf{r}\|^3} \right] + fR^T \mathbf{e}_1 + \mathbf{d} \quad (2)$$

Based on the estimation of the mass and the hyperbolic tangent function in the framework of Lyapunov theory, the bounded adaptive position controller of the lunar lander is designed by

$$\mathbf{f}_c = \hat{m} \left[-k_1 \tanh(\mathbf{s}) - k_1 \tanh(\mathbf{v}) + \frac{\mu(\mathbf{r}_t + \mathbf{r})}{\|\mathbf{r}_t + \mathbf{r}\|^3} \right] - \bar{d} \tanh(l_1 \mathbf{a}_1) \quad (3)$$

where $\tanh(\mathbf{x}) \triangleq [\tanh(x_1), \tanh(x_2), \tanh(x_3)]^T$ for any vector $\mathbf{x} = [x_1, x_2, x_3]^T$; k_1 and l_1 are the positive constant gains, and $\mathbf{a}_1 \triangleq \tanh(\mathbf{s}) + \tanh(\mathbf{v}) + \frac{\lambda}{k_1} \mathbf{v}$. In the position controller (3), the estimation of mass $\hat{m}$ is derived from a projection-based adaptive law

$$\dot{\hat{m}} = \begin{cases} k_m a_2 & \text{if } m_0 \leq m \leq \bar{m}, \text{ if } \hat{m} = m_0 \text{ and } a_2 > 0, \\ & \text{if } \hat{m} = \bar{m} \text{ and } a_2 < 0; \\ 0 & \text{if } \hat{m} = m_0 \text{ and } a_2 \leq 0, \text{ if } \hat{m} = \bar{m} \text{ and } a_2 \geq 0. \end{cases} \quad (4)$$

with $k_m > 0$ and $a_2 \triangleq \lambda \tanh(\mathbf{s})^T \mathbf{v} - \frac{\mu \mathbf{a}_1^T (\mathbf{r}_t + \mathbf{r})}{\|\mathbf{r}_t + \mathbf{r}\|^3}$.

Based on the proposed position controller $\mathbf{f}_c$ in (3), one has to find the actual thrust f and the inner-loop desired attitude trajectory defined by the rotational matrix R_d from the guidance law of the lunar lander. In fact, motivated by the guidance law of the VTOL vehicles [25], the

guidance law of the lunar lander can be obtained from

$$fR_d^T \mathbf{e}_1 = f \begin{bmatrix} \cos \theta_d \cos \psi_d \\ \cos \theta_d \sin \psi_d \\ -\sin \theta_d \end{bmatrix} = \mathbf{f}_c \quad (5)$$

where $fR_d^T \mathbf{e}_1$ is the desired direction of the thrust described by a rotational matrix R_d with the desired Euler angles θ_d and ψ_d . Furthermore, based on the fact $\|R_d^T \mathbf{e}_1\| = 1$, the actual thrust can be calculated by the norm operations on both sides of the guidance law (5) as

$$f\|R_d^T \mathbf{e}_1\| = f = \|\mathbf{f}_c\| \quad (6)$$

Furthermore, from the guidance law (5) and the actual thrust $f = \|\mathbf{f}_c\|$, one has

$$\|\mathbf{f}_c\| \begin{bmatrix} \cos \theta_d \cos \psi_d \\ \cos \theta_d \sin \psi_d \\ -\sin \theta_d \end{bmatrix} = \hat{m} \left[-k_1 \tanh(\mathbf{s}) - k_1 \tanh(\mathbf{v}) + \frac{\mu(\mathbf{r}_t + \mathbf{r})}{\|\mathbf{r}_t + \mathbf{r}\|^3} \right] - \bar{d} \tanh(l_1 \mathbf{a}_1) \triangleq \begin{bmatrix} f_{c1} \\ f_{c2} \\ f_{c3} \end{bmatrix}$$

then the desired pitch angle θ_d and roll angle ψ_d for the inner-loop attitude motion are extracted by

$$\begin{cases} \theta_d = \arcsin\left(\frac{-f_{c3}}{\|\mathbf{f}_c\|}\right), & \theta_d \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \psi_d = \arctan\left(\frac{f_{c2}}{f_{c1}}\right), & \psi_d \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \end{cases} \quad (7)$$

Without loss of generality, the desired yaw attitude γ_r is freely set by the designer.

To avoid the computational singularity of the ψ_d in (7), the constraint $f_{c1} \neq 0$ should be guaranteed. Actually, since $\|\mathbf{r}\| \ll \|\mathbf{r}_t\|$, $\|\mathbf{r}_t + \mathbf{r}\|^2 \leq 2(\|\mathbf{r}_t\|^2 + \|\mathbf{r}\|^2) \leq 4\|\mathbf{r}_t\|^2$, and $\frac{\mu\|\mathbf{r}_t + \mathbf{r}\|}{\|\mathbf{r}_t + \mathbf{r}\|^3} = \frac{\mu}{\|\mathbf{r}_t + \mathbf{r}\|^2} \geq \frac{\mu}{4\|\mathbf{r}_t\|^2}$ and

$$f_{c1} = \hat{m} \left[-k_1 \tanh(s_1) - k_1 \tanh(v_1) + \frac{\mu(r_{t1} + r_1)}{\|\mathbf{r}_t + \mathbf{r}\|^3} \right] - \bar{d} \tanh(l_1 a_{11}) > m_0 \left(-2k_1 + \frac{\mu}{4\|\mathbf{r}_t\|^2} - \frac{\bar{d}}{m_0} \right)$$

where r_{t1}, r_1, s_1, v_1 , and a_{11} are the first entry of vectors $\mathbf{r}_t, \mathbf{r}, \mathbf{s}, \mathbf{v}$, and $\mathbf{a}_1$, respectively. Then, to ensure the condition $f_{c1} > 0$, the parameter k_1 in (3) should satisfy $k_1 < \frac{1}{2} \left(\frac{\mu}{4\|\mathbf{r}_t\|^2} - \frac{\bar{d}}{m_0} \right)$.

To facilitate the inner-loop attitude controller design, the time derivatives of the desired attitude $\boldsymbol{\vartheta}_d = [\gamma_r, \theta_d, \psi_d]^T$ are difficult to calculate directly. Thus, the command filter [26] is employed here to generate its virtual derivative signals:

$$\begin{cases} \dot{\mathbf{x}}_1 = \mathbf{x}_2 \\ \dot{\mathbf{x}}_2 = -2\xi \omega_n \mathbf{x}_2 - \omega_n^2 (\mathbf{x}_1 - \boldsymbol{\vartheta}_d) \end{cases} \quad (8)$$

with the initial states $\mathbf{x}_1(0) = \boldsymbol{\vartheta}_d(0)$ and $\mathbf{x}_2(0) = \mathbf{0}$, where ξ and ω_n are the damping ratio and frequency, respectively. Therefore, the command attitude signals are derived from this filter as $\boldsymbol{\vartheta}_c = \mathbf{x}_1$, $\dot{\boldsymbol{\vartheta}}_c = \mathbf{x}_2$, and $\ddot{\boldsymbol{\vartheta}}_c = -2\xi\omega_n\mathbf{x}_2 - \omega_n^2(\mathbf{x}_1 - \boldsymbol{\vartheta}_d)$.

Remark 2 Since the under-actuated property of the lunar lander, the double-loop structure control strategy is employed to design the control system. The inner-loop desired attitude is calculated from the complicated outer-loop guidance law, but the time derivatives of the desired attitude are also used in the inner-loop attitude tracking subsystem, and the explicit expression of these time derivatives are different to compute in practice, so the command filter is employed to calculate the approximations of the true time derivatives for the desired attitude to design the inner-loop controller. Furthermore, since the command filter is a second-order linear system, the response performance for this linear system can be regulated by the damping ratio and natural frequency. Moreover, the damping ratio and natural frequency should be chosen properly in practice to guarantee the fast tracking performance of the command filter.

Let the attitude control system track $\boldsymbol{\vartheta}_c$ rather than track the $\boldsymbol{\vartheta}_d$ directly. In view of $\boldsymbol{\vartheta}_c$, $\dot{\boldsymbol{\vartheta}}_c$, and $\ddot{\boldsymbol{\vartheta}}_c$, the commanded angular velocity $\boldsymbol{\omega}_c$ and commanded angular acceleration $\dot{\boldsymbol{\omega}}_c$ can be computed by

$$\boldsymbol{\omega}_c = W^{-1}(\boldsymbol{\vartheta}_c)\dot{\boldsymbol{\vartheta}}_c, \quad \dot{\boldsymbol{\omega}}_c = W^{-1}(\boldsymbol{\vartheta}_c)[\ddot{\boldsymbol{\vartheta}}_c - \dot{W}(\boldsymbol{\vartheta}_c)\boldsymbol{\omega}_c] \quad (9)$$

where

$$W^{-1}(\boldsymbol{\vartheta}_c) = \begin{bmatrix} 1 & 0 & -s\theta_c \\ 0 & c\gamma_c & s\gamma_c c\theta_c \\ 0 & -s\gamma_c & c\gamma_c c\theta_c \end{bmatrix},$$

$$\dot{W}(\boldsymbol{\vartheta}_c) = \begin{bmatrix} 0 & w_a & w_b \\ 0 & -\dot{\gamma}_c s\gamma_c & -\dot{\gamma}_c c\gamma_c \\ 0 & w_c & w_d \end{bmatrix},$$

$$\text{and } w_a = \frac{\dot{\theta}_c s\gamma_c + \dot{\gamma}_c c\gamma_c \tan \theta_c}{c^2 \theta_c}, w_b = \frac{\dot{\theta}_c s\gamma_c - \dot{\gamma}_c s\gamma_c \tan \theta_c}{c^2 \theta_c}, w_c = \frac{\dot{\gamma}_c c\gamma_c c\theta_c + \dot{\theta}_c s\theta_c s\gamma_c}{c^2 \theta_c}, w_d = \frac{-\dot{\gamma}_c s\gamma_c c\theta_c + \dot{\theta}_c c\theta_c s\gamma_c}{c^2 \theta_c}.$$

Then define the attitude tracking error $\boldsymbol{\vartheta}_e = [\gamma_e, \theta_e, \psi_e]^T = \boldsymbol{\vartheta} - \boldsymbol{\vartheta}_c$ and a new scalar $\theta_b = \pi/2 - |\theta_c| > 0$. If $\theta_e < \theta_b$, then $|\theta| \leq |\theta_c| + |\theta_e| < \pi/2$, where θ is defined in (1), thus the non-singularity of the Jacobian matrix $W(\boldsymbol{\vartheta})$ in (1) can be guaranteed. Therefore, the attitude tracking system with pitch constraint can be designed by combining backstepping technique

and barrier Lyapunov function.

Step 1: Define $\rho = \theta_e/\theta_b$. Clearly, if $|\rho| < 1$ then $|\theta| < \pi/2$ can be guaranteed. Thus the time derivative of ρ is

$$\dot{\rho} = \frac{1}{\theta_b} \dot{\theta}_e - \frac{\dot{\theta}_b}{\theta_b^2} \theta_e \quad (10)$$

Define Lyapunov function

$$V_a = \frac{1}{2}(\gamma_e^2 + \log \frac{1}{1-\rho^2} + \psi_e^2) \quad (11)$$

Then the time derivative of V_a is

$$\begin{aligned} \dot{V}_a &= \gamma_e \dot{\gamma}_e + \frac{\rho \dot{\rho}}{1-\rho^2} + \psi_e \dot{\psi}_e \\ &= \gamma_e \dot{\gamma}_e + \frac{\rho}{\theta_b(1-\rho^2)} [\dot{\theta}_e - \frac{\dot{\theta}_b}{\theta_b} \theta_e] + \psi_e \dot{\psi}_e \\ &= [\gamma_e, \frac{\rho}{\theta_b(1-\rho^2)}, \psi_e] (\dot{\boldsymbol{\vartheta}} - \dot{\boldsymbol{\vartheta}}_c - \boldsymbol{\eta}) \end{aligned} \quad (12)$$

where $\boldsymbol{\eta} = [0, (\dot{\theta}_b/\theta_b)\theta_e, 0]^T$, and

$$\dot{\theta}_b = \begin{cases} \dot{\theta}_c & \theta_c < 0 \\ 0 & \theta_c = 0 \\ -\dot{\theta}_c & \theta_c > 0 \end{cases}, \quad \ddot{\theta}_b = \begin{cases} \ddot{\theta}_c & \theta_c < 0 \\ 0 & \theta_c = 0 \\ -\ddot{\theta}_c & \theta_c > 0 \end{cases}.$$

Design virtual input

$$\boldsymbol{\alpha}_v = -k_2 \boldsymbol{\vartheta}_e + \dot{\boldsymbol{\vartheta}}_c + \boldsymbol{\eta} \quad (13)$$

where $k_2 > 0$. Then, one has

$$\dot{\boldsymbol{\vartheta}}_c + \boldsymbol{\eta} = \boldsymbol{\alpha}_v + k_2 \boldsymbol{\vartheta}_e \quad (14)$$

Substituting (14) into (12) yields

$$\dot{V}_a = -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) + \boldsymbol{h}^T \boldsymbol{\omega}_e \quad (15)$$

where $\boldsymbol{h} = [\gamma_e, \frac{\theta_e}{\theta_b^2 - \theta_e^2}, \psi_e]^T$, $\boldsymbol{\omega}_e = \dot{\boldsymbol{\vartheta}} - \boldsymbol{\alpha}_v$.

Step 2: Denote $P = W^{-1}(\boldsymbol{\vartheta})$. From the attitude kinematics equation in (1), we have

$$\dot{\boldsymbol{\omega}} = P \ddot{\boldsymbol{\vartheta}} + \dot{P} \dot{\boldsymbol{\vartheta}} \quad (16)$$

where $\dot{P} = -P \dot{W}(\boldsymbol{\vartheta}) P$.

Because of the mass uncertainty, the inertial matrix J in attitude dynamics is also an uncertain constant symmetric matrix. Therefore, the inertial uncertainties should be considered in the control torque design. Consider the lunar lander's inertial matrix as

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix}.$$

Then introducing a linear operator for any $\boldsymbol{\chi} = [\chi_1, \chi_2, \chi_3]^T$ as

$$L(\boldsymbol{\chi}) = \begin{bmatrix} \chi_1 & \chi_2 & \chi_3 & 0 & 0 & 0 \\ 0 & \chi_1 & 0 & \chi_2 & \chi_3 & 0 \\ 0 & 0 & \chi_1 & 0 & \chi_2 & \chi_3 \end{bmatrix}$$

yields $J\boldsymbol{\chi} = L(\boldsymbol{\chi})\boldsymbol{\delta}_J$, where $\boldsymbol{\delta}_J = [J_{11}, J_{12}, J_{13}, J_{22}, J_{23}, J_{33}]^T$.

Define the estimation $\hat{\boldsymbol{\delta}}_J$ for $\boldsymbol{\delta}_J$ and denote the estimation error as $\tilde{\boldsymbol{\delta}}_J = \hat{\boldsymbol{\delta}}_J - \boldsymbol{\delta}_J$. Then, choose a Lyapunov function

$$V_b = V_a + \frac{1}{2}\boldsymbol{\omega}_e^T P^T J P \boldsymbol{\omega}_e + \frac{1}{2\varepsilon}\tilde{\boldsymbol{\delta}}_J^T \tilde{\boldsymbol{\delta}}_J \quad (17)$$

where $\varepsilon > 0$. Thus, from (1), (15), and (16), the time derivative of V_b is calculated as

$$\begin{aligned} \dot{V}_b &= -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) + \mathbf{h}^T \boldsymbol{\omega}_e \\ &\quad + \boldsymbol{\omega}_e^T P^T J (\dot{P}\boldsymbol{\omega}_e + P\dot{\boldsymbol{\omega}}_e) + \frac{1}{\varepsilon}\tilde{\boldsymbol{\delta}}_J^T \dot{\tilde{\boldsymbol{\delta}}}_J \\ &= -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) + \mathbf{h}^T \boldsymbol{\omega}_e \\ &\quad + \boldsymbol{\omega}_e^T P^T J [\dot{P}(\dot{\boldsymbol{\vartheta}} - \boldsymbol{\alpha}_v) + P(\ddot{\boldsymbol{\vartheta}} - \dot{\boldsymbol{\alpha}}_v)] + \frac{1}{\varepsilon}\tilde{\boldsymbol{\delta}}_J^T \dot{\tilde{\boldsymbol{\delta}}}_J \\ &= -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) + \mathbf{h}^T \boldsymbol{\omega}_e \\ &\quad + \boldsymbol{\omega}_e^T P^T [-S(\boldsymbol{\omega})J\boldsymbol{\omega} + \mathbf{u} + \mathbf{w} - J\dot{P}\boldsymbol{\alpha}_v - JP\dot{\boldsymbol{\alpha}}_v] + \frac{1}{\varepsilon}\tilde{\boldsymbol{\delta}}_J^T \dot{\tilde{\boldsymbol{\delta}}}_J \end{aligned} \quad (18)$$

Since $-S(\boldsymbol{\omega})J\boldsymbol{\omega} - J\dot{P}\boldsymbol{\alpha}_v - JP\dot{\boldsymbol{\alpha}}_v = Y\boldsymbol{\delta}_J$ with $Y = -S(\boldsymbol{\omega})L(\boldsymbol{\omega}) - L(\dot{P}\boldsymbol{\alpha}_v) - L(P\dot{\boldsymbol{\alpha}}_v)$, then

$$\begin{aligned} \dot{V}_b &= -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) + \boldsymbol{\omega}_e^T P^T (W^T \mathbf{h} \\ &\quad + Y\boldsymbol{\delta}_J + \mathbf{u} + \mathbf{w}) + \frac{1}{\varepsilon}\tilde{\boldsymbol{\delta}}_J^T \dot{\tilde{\boldsymbol{\delta}}}_J \end{aligned} \quad (19)$$

Design the attitude controller

$$\mathbf{u} = -k_3 W^T \boldsymbol{\omega}_e - W^T \mathbf{h} - Y \hat{\boldsymbol{\delta}}_J - \bar{w} \tanh(l_2 P \boldsymbol{\omega}_e) \quad (20)$$

and the adaptive law

$$\dot{\hat{\boldsymbol{\delta}}}_J = \begin{cases} \varepsilon(Y^T P \boldsymbol{\omega}_e - \varepsilon_0 \hat{\boldsymbol{\delta}}_J) & \|\hat{\boldsymbol{\delta}}_J\| \leq \bar{\lambda}; \\ & \|\hat{\boldsymbol{\delta}}_J\| = \bar{\lambda}, \hat{\boldsymbol{\delta}}_J^T Y^T P \boldsymbol{\omega}_e > 0; \\ \varepsilon(Y^T P \boldsymbol{\omega}_e - \varepsilon_0 \hat{\boldsymbol{\delta}}_J) + \varepsilon \boldsymbol{\zeta} & \|\hat{\boldsymbol{\delta}}_J\| = \bar{\lambda}, \hat{\boldsymbol{\delta}}_J^T Y^T P \boldsymbol{\omega}_e \leq 0. \end{cases} \quad (21)$$

where $k_3 > 0$, $l_2 > 0$, $\varepsilon > 0$, $\varepsilon_0 > 0$, $\boldsymbol{\zeta} = \frac{\hat{\boldsymbol{\delta}}_J^T Y^T P \boldsymbol{\omega}_e}{\|\hat{\boldsymbol{\delta}}_J\|^2} \hat{\boldsymbol{\delta}}_J$.

Substituting (20) and (21) into (19) yields

$$\begin{aligned} \dot{V}_b = & -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) - k_3 \boldsymbol{\omega}_e^T \boldsymbol{\omega}_e - \boldsymbol{\omega}_e^T P^T Y \tilde{\boldsymbol{\delta}}_J \\ & + \boldsymbol{\omega}_e^T P^T \mathbf{w} - \bar{w} \boldsymbol{\omega}_e^T P^T \tanh(l_2 P \boldsymbol{\omega}_e) + \frac{1}{\varepsilon} \tilde{\boldsymbol{\delta}}_J^T \dot{\hat{\boldsymbol{\delta}}}_J \end{aligned} \quad (22)$$

Since $\boldsymbol{\omega}_e^T P^T \mathbf{w} \leq \bar{w} \|P \boldsymbol{\omega}_e\|_1$ and

$$\|P \boldsymbol{\omega}_e\|_1 - \boldsymbol{\omega}_e^T P^T \tanh(l_2 P \boldsymbol{\omega}_e) \leq \frac{3\kappa}{l_2}$$

where $\kappa = 0.2785$ and $\|P \boldsymbol{\omega}_e\|_1$ denotes the 1-norm of $P \boldsymbol{\omega}_e$. Thus, if $\|\hat{\boldsymbol{\delta}}_J\| \leq \bar{\lambda}$ or if $\|\hat{\boldsymbol{\delta}}_J\| = \bar{\lambda}$ and $\hat{\boldsymbol{\delta}}_J^T Y^T P \boldsymbol{\omega}_e > 0$, then substituting (21) into (22) and using the fact

$$\tilde{\boldsymbol{\delta}}_J^T \hat{\boldsymbol{\delta}}_J = \tilde{\boldsymbol{\delta}}_J^T (\tilde{\boldsymbol{\delta}}_J + \boldsymbol{\delta}_J) = \frac{1}{2}(\|\tilde{\boldsymbol{\delta}}_J\|^2 - \|\boldsymbol{\delta}_J\|^2 + \|\tilde{\boldsymbol{\delta}}_J + \boldsymbol{\delta}_J\|^2)$$

lead to

$$\begin{aligned} \dot{V}_b \leq & -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) - k_3 \boldsymbol{\omega}_e^T \boldsymbol{\omega}_e \\ & - \varepsilon_0 \tilde{\boldsymbol{\delta}}_J^T \hat{\boldsymbol{\delta}}_J + \frac{3\bar{w}\kappa}{l_2} \\ \leq & -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) - k_3 \boldsymbol{\omega}_e^T \boldsymbol{\omega}_e \\ & - \frac{\varepsilon_0}{2} \|\tilde{\boldsymbol{\delta}}_J\|^2 + \frac{\varepsilon_0}{2} \|\boldsymbol{\delta}_J\|^2 + \frac{3\bar{w}\kappa}{l_2} \end{aligned} \quad (23)$$

If $\|\hat{\boldsymbol{\delta}}_J\| = \bar{\lambda}$ and $\hat{\boldsymbol{\delta}}_J^T Y^T P \boldsymbol{\omega}_e \leq 0$, then substituting (21) into (22) yields

$$\begin{aligned} \dot{V}_b \leq & -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) - k_3 \boldsymbol{\omega}_e^T \boldsymbol{\omega}_e + \frac{3\bar{w}\kappa}{l_2} \\ & - \frac{\varepsilon_0}{2} \|\tilde{\boldsymbol{\delta}}_J\|^2 + \frac{\varepsilon_0}{2} \|\boldsymbol{\delta}_J\|^2 + \frac{\tilde{\boldsymbol{\delta}}_J^T \hat{\boldsymbol{\delta}}_J}{\|\hat{\boldsymbol{\delta}}_J\|^2} \hat{\boldsymbol{\delta}}_J^T Y^T P \boldsymbol{\omega}_e \end{aligned} \quad (24)$$

Figure 2: Double-loop structure of the lunar landing control system.

Since $\|\hat{\delta}_J\| = \bar{\lambda}$ and $\|\delta_J\| \leq \bar{\lambda}$, then

$$\begin{aligned}\tilde{\delta}_J^T \hat{\delta}_J &= (\hat{\delta}_J - \delta_J)^T \hat{\delta}_J = \frac{1}{2}(\|\hat{\delta}_J\|^2 + \|\hat{\delta}_J - \delta_J\|^2 - \|\delta_J\|^2) \\ &= \frac{1}{2}(\bar{\lambda}^2 + \|\hat{\delta}_J - \delta_J\|^2 - \|\delta_J\|^2) \geq 0\end{aligned}$$

Thus, if $\|\hat{\delta}_J\| = \bar{\lambda}$ and $\hat{\delta}_J^T Y^T P \omega_e \leq 0$, then $\frac{\tilde{\delta}_J^T \hat{\delta}_J}{\|\hat{\delta}_J\|^2} \hat{\delta}_J^T Y^T P \omega_e \leq 0$. Therefore, the proposed adaptive law ensures that (23) always holds.

The double-loop structure of the lunar landing control system is shown in Fig. 2. In the position loop, the guidance law $fR^T e_1$ can be designed firstly to guarantee the convergence of the landing position. Then the thrust f and desired attitude ϑ_d are computed from the guidance law. Furthermore, based on the second-order linear command filter, the attitude tracking control input u can be designed from the filtered command attitude ϑ_c to ensure the convergence of the attitude tracking error. Thus, the position tracking objective and terminal landing mission are achieved ultimately.

Remark 3 It is noted that there are two kinds of singularities in this study, the computational singularity of desired attitude command and the computational singularity of the inner-loop attitude tracking. While the guidance law is proposed based on the outer-loop position tracking control for the lunar lander, the lander is under-actuated such that the planar position tracking should be controlled by the inner-loop pitch and roll attitude motions, thus the desired attitude commands should be generated from the outer-loop position tracking laws. However, since the trigonometric function in the direction cosine matrix between desired attitude frame and

the lander's body-fixed frame, the desired attitude command should be calculated by solving the trigonometric function shown in (7). Therefore, the computational singularity of desired attitude command would be avoided. Secondly, in the inner-loop attitude tracking control, the Euler angle attitude description is used to denote the attitude errors. In order to ensure that the Jacobian $W(\vartheta)$ of attitude error kinematics is always nonsingular, the value of pitch angle should always satisfies $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ in the attitude tracking missions, then the barrier-Lyapunov-based state constraint control design method is employed to avoid this computational singularity.

4 Stability Analysis

In this section, we will prove that the ultimate bounds of the pose and velocities of the lunar lander can be made sufficiently small by tuning controller parameters, specially the constraint $|\theta(t)| < \pi/2$ is guaranteed if $|\theta(0)| < \pi/2$, and all signals of closed-loop system always bounded.

Theorem 1 Consider the inner-loop attitude motion of the lunar lander in terminal landing phase under Assumption 1, if the initial pitch angle satisfies $|\theta(0)| < \pi/2$ and the initially commanded pitch angle satisfies $\min\{-\pi/2, \theta(0)/2 - \pi/4\} < \theta_c(0) < \max\{\pi/2, \theta(0)/2 + \pi/4\}$, then the proposed adaptive nonlinear attitude controller (20) can guarantee that the pitch angle always satisfies $|\theta(t)| < \pi/2$ for all $t \geq 0$, while the attitude error ϑ_e , angular velocity error ω_e , and estimated error $\tilde{\delta}_J$ ultimately converge to small neighborhoods of the origin.

Proof According to (17), (23), and the fact $\log \frac{1}{1-\rho^2} \leq \frac{\rho^2}{1-\rho^2}$, we know

$$\begin{aligned} \dot{V}_b &\leq -k_2(\gamma_e^2 + \frac{\rho^2}{1-\rho^2} + \psi_e^2) - k_3\omega_e^T\omega_e - \frac{\varepsilon_0}{2}\tilde{\delta}_J^T\tilde{\delta}_J + \varpi \\ &\leq -k_2(\gamma_e^2 + \log \frac{1}{1-\rho^2} + \psi_e^2) - \frac{k_3}{\bar{\sigma}}\omega_e^TP^TJP\omega_e \\ &\quad - \frac{\varepsilon_0}{2}\tilde{\delta}_J^T\tilde{\delta}_J + \varpi \leq -k_bV_b + \varpi \end{aligned}$$

where $\varpi = \frac{\varepsilon_0}{2}\|\tilde{\delta}_J\|^2 + \frac{3\bar{w}\kappa}{l_2}$, $\bar{\sigma}$ denotes the maximum eigenvalue of matrix P^TJP , $k_b = \min\{2k_2, \frac{2k_3}{\bar{\sigma}}, \varepsilon\varepsilon_0\}$.

Thus, from the comparison principle, we obtain

$$V_b(t) \leq [V_b(0) - \frac{\varpi}{k_b}]e^{-k_bt} + \frac{\varpi}{k_b}, \forall t \geq 0$$

Furthermore, we have

$$\log \frac{1}{1-\rho^2} \leq 2V_b(t) \leq 2[V_b(0) - \frac{\overline{\omega}}{k_b}]e^{-k_b t} + \frac{2\overline{\omega}}{k_b} \quad (25)$$

$$\gamma_e^2 + \psi_e^2 \leq 2V_b(t) \leq 2[V_b(0) - \frac{\overline{\omega}}{k_b}]e^{-k_b t} + \frac{2\overline{\omega}}{k_b} \quad (26)$$

$$\|\omega_e\|^2 \leq \frac{2}{\sigma_0} V_b(t) \leq \frac{2}{\sigma_0} [V_b(0) - \frac{\overline{\omega}}{k_b}]e^{-k_b t} + \frac{2\overline{\omega}}{\sigma_0 k_b} \quad (27)$$

$$\|\tilde{\mathbf{d}}_J\|^2 \leq 2\varepsilon V_b(t) \leq 2\varepsilon [V_b(0) - \frac{\overline{\omega}}{k_b}]e^{-k_b t} + \frac{2\varepsilon \overline{\omega}}{k_b} \quad (28)$$

where σ_0 denotes the minimum eigenvalue of matrix $P^T J P$. Taking exponentials on both sides of (25) and some computations yields $|\rho| \leq \sqrt{1 - e^{-2[V_b(0) - \frac{\overline{\omega}}{k_b}]e^{-k_b t} - \frac{2\overline{\omega}}{k_b}}}$. Furthermore, $|\theta_e| \leq \theta_b \sqrt{1 - e^{-2[V_b(0) - \frac{\overline{\omega}}{k_b}]e^{-k_b t} - \frac{2\overline{\omega}}{k_b}}}$. Thus, $-\theta_b(t) < \theta_e(t) < \theta_b(t)$ is guaranteed for all $t \geq 0$ as long as $-\theta_b(0) < \theta_e(0) < \theta_b(0)$. Then, we have $|\theta(t)| = |\theta_e(t) + \theta_c(t)| \leq |\theta_e(t)| + |\theta_c(t)| < |\theta_b(t)| + |\theta_c(t)| < \pi/2$ for all $t \geq 0$. Since $\theta_c(0)$ satisfies $\min\{-\pi/2, \theta(0)/2 - \pi/4\} < \theta_c(0) < \max\{\pi/2, \theta(0)/2 + \pi/4\}$ such that $-\theta_b(t) < \theta_e(t) < \theta_b(t)$ for all $t \geq 0$, then $-\pi/2 < \theta(t) < \pi/2$ is guaranteed for all $t \geq 0$. Moreover, from (25)-(28), we know that $\theta_e(t)$, $\gamma_e(t)$, $\psi_e(t)$, $\omega_e(t)$, and $\tilde{\mathbf{d}}_J(t)$ are bounded signals, while $\lim_{t \rightarrow \infty} |\theta_e(t)| \leq \theta_b \sqrt{1 - e^{-\frac{2\overline{\omega}}{k_b}}}$, $\lim_{t \rightarrow \infty} |\gamma_e(t)| \leq \sqrt{\frac{2\overline{\omega}}{k_b}}$, $\lim_{t \rightarrow \infty} |\psi_e(t)| \leq \sqrt{\frac{2\overline{\omega}}{k_b}}$, $\lim_{t \rightarrow \infty} \|\omega_e(t)\| \leq \sqrt{\frac{2\overline{\omega}}{\sigma_0 k_b}}$, and $\lim_{t \rightarrow \infty} \|\tilde{\mathbf{d}}_J(t)\| \leq \sqrt{\frac{2\varepsilon \overline{\omega}}{k_b}}$. Thus the ultimate bounds of attitude tracking errors can be tuned sufficiently small by choosing suitable controller parameters $k_2, k_3, l_2, \varepsilon$, and ε_0 .

Theorem 2 For the command filter (8), the error between the filter output and input is sufficiently small by setting proper values of the damping ratio ξ and sufficiently large ω_n , that is

$$\|\boldsymbol{\vartheta}_c - \boldsymbol{\vartheta}_d\| \leq \frac{\|\dot{\boldsymbol{\vartheta}}_d(0)\|}{\omega_n \sqrt{1-\xi^2}} + \frac{\sup_{0 \leq \tau \leq t} \{\|\ddot{\boldsymbol{\vartheta}}_d(\tau)\| + 2\xi \omega_n \|\dot{\boldsymbol{\vartheta}}_d(\tau)\|\}}{\xi \omega_n^2 \sqrt{1-\xi^2}}$$

Proof Define the error between the input and output of the command filter as $\tilde{\boldsymbol{\vartheta}}_1 = \boldsymbol{\vartheta}_c - \boldsymbol{\vartheta}_d$, $\tilde{\boldsymbol{\vartheta}}_2 = \dot{\boldsymbol{\vartheta}}_c - \dot{\boldsymbol{\vartheta}}_d$. Then,

$$\begin{cases} \dot{\tilde{\boldsymbol{\vartheta}}}_1 = \tilde{\boldsymbol{\vartheta}}_2 \\ \dot{\tilde{\boldsymbol{\vartheta}}}_2 = -2\xi \omega_n \tilde{\boldsymbol{\vartheta}}_2 - \omega_n^2 \tilde{\boldsymbol{\vartheta}}_1 - (\ddot{\boldsymbol{\vartheta}}_d + 2\xi \omega_n \dot{\boldsymbol{\vartheta}}_d) \end{cases} \quad (29)$$

Since $\tilde{\boldsymbol{\vartheta}}_1(0) = \mathbf{x}_1(0) - \boldsymbol{\vartheta}_d(0) = \mathbf{0}$, then the solution of the linear system (29) is

$$\begin{aligned} \tilde{\boldsymbol{\vartheta}}_1(t) &= \frac{e^{-\xi \omega_n t}}{\omega_n \sqrt{1-\xi^2}} \sin(\omega_n \sqrt{1-\xi^2} t) \tilde{\boldsymbol{\vartheta}}_2(0) \\ &\quad + \int_0^t \frac{e^{-\xi \omega_n (t-\tau)}}{\omega_n \sqrt{1-\xi^2}} \sin(\omega_n \sqrt{1-\xi^2} (t-\tau)) [\ddot{\boldsymbol{\vartheta}}_d(\tau) + 2\xi \omega_n \dot{\boldsymbol{\vartheta}}_d(\tau)] d\tau \end{aligned}$$

Thus, from $\mathbf{x}_1(0) = \boldsymbol{\vartheta}_d(0)$, $\mathbf{x}_2(0) = \mathbf{0}$, and $\|\tilde{\boldsymbol{\vartheta}}_2(0)\| = \|\mathbf{x}_2(0) - \dot{\boldsymbol{\vartheta}}_d(0)\| = \|\dot{\boldsymbol{\vartheta}}_d(0)\|$, we know

$$\begin{aligned}\|\tilde{\boldsymbol{\vartheta}}_1(t)\| &\leq \frac{\tilde{\boldsymbol{\vartheta}}_2(0)}{\omega_n \sqrt{1-\xi^2}} + \frac{\sup_{0 \leq \tau \leq t} \{\|\ddot{\boldsymbol{\vartheta}}_d(\tau)\| + 2\xi \omega_n \|\dot{\boldsymbol{\vartheta}}_d(\tau)\|\}}{\xi \omega_n^2 \sqrt{1-\xi^2}} \\ &= \frac{\dot{\boldsymbol{\vartheta}}_d(0)}{\omega_n \sqrt{1-\xi^2}} + \frac{\sup_{0 \leq \tau \leq t} \{\|\ddot{\boldsymbol{\vartheta}}_d(\tau)\| + 2\xi \omega_n \|\dot{\boldsymbol{\vartheta}}_d(\tau)\|\}}{\xi \omega_n^2 \sqrt{1-\xi^2}}\end{aligned}$$

Thus, by setting proper values of the damping ratio ξ and sufficiently large ω_n , the error $\|\tilde{\boldsymbol{\vartheta}}_1(t)\|$ between the filter output and input is sufficiently small.

Theorem 3 Consider the outer-loop position motion of the lunar lander in terminal landing phase under Assumption 1, if the parameters are chosen such that the condition $3\bar{d}\kappa < m_0\eta_s l_1$ with $\eta_s = \frac{3-\sqrt{5}}{2}\min\{k_1, \lambda\}$ is satisfied, then the proposed nonlinear position controller (3) with adaptive law (4) can guarantee that the estimated mass satisfies $m_0 \leq \hat{m}(t) \leq \bar{m}$ if $m_0 \leq \hat{m}(0) \leq \bar{m}$ for all $t \geq 0$, while the position error $\mathbf{r}$ and velocity error $\mathbf{v}$ ultimately converge to small neighborhoods of the origin.

Proof Consider a function $V_m = \frac{1}{2k_m}\hat{m}^2$, the time derivative of which is calculated as $\dot{V}_m = \frac{1}{k_m}\hat{m}\dot{\hat{m}}$. According to (4), if $m_0 \leq \hat{m} \leq \bar{m}$, the estimated mass $\hat{m}$ is bounded obviously; if $\hat{m} = m_0$ and $a_2 > 0$, then $\dot{V}_m = m_0 a_2 > 0$, thus $\hat{m}$ is increasing; if $\hat{m} = \bar{m}$ and $a_2 < 0$, then $\dot{V}_m = m_0 a_2 < 0$, thus $\hat{m}$ is decreasing; if $\hat{m} = m_0$ and $a_2 \leq 0$, or if $\hat{m} = \bar{m}$ and $a_2 \geq 0$, then $\dot{V}_m = m_0 a_2 = 0$, thus $\hat{m}$ is invariant. Therefore, the estimated mass under the proposed adaptive law (4) satisfies $m_0 \leq \hat{m}(t) \leq \bar{m}$ if $m_0 \leq \hat{m}(0) \leq \bar{m}$ for all $t \geq 0$.

Choose a function

$$V_c = m \left[\frac{\lambda}{2k_1} \mathbf{v}^T \mathbf{v} + \int_0^s \tanh(\boldsymbol{\tau})^T d\boldsymbol{\tau} + \int_0^v \tanh(\boldsymbol{\tau})^T d\boldsymbol{\tau} \right] + \frac{\tilde{m}^2}{2k_m}$$

where $\tilde{m} = \hat{m} - m$ is the estimated error. Since the inner-loop attitude dynamics are faster than outer-loop position dynamics, while the inner-loop attitude tracking errors and command filtered errors can be respectively tuned sufficiently small by setting large k_b and ω_n from Theorems 1 and 2, then the attitude tracking errors can be neglected in the outer-loop position tracking system in view of the assumption of time-scale separation [28]. This means that $f(R - R_d)^T \mathbf{e}_1 = 0$ can be treated for the outer-loop position dynamics under the results of The-

orems 1 and 2. Thus, from (1) and (2), the time derivative of V_c is

$$\begin{aligned}\dot{V}_c &= \frac{m\lambda}{k_1} \mathbf{v}^T \dot{\mathbf{v}} + m \tanh(\mathbf{s})^T \dot{\mathbf{s}} + m \tanh(\mathbf{v})^T \dot{\mathbf{v}} + \frac{1}{k_m} \tilde{m} \dot{\hat{m}} \\ &= m\lambda \tanh(\mathbf{s})^T \mathbf{v} + \mathbf{a}_1^T \left[-\frac{m\mu}{\|\mathbf{r}_t + \mathbf{r}\|^3} (\mathbf{r}_t + \mathbf{r}) + \mathbf{f}_c + \mathbf{d} \right] + \frac{1}{k_m} \tilde{m} \dot{\hat{m}}\end{aligned}\quad (30)$$

Substituting (3) into (30) yields

$$\begin{aligned}\dot{V}_c &= -\hat{m}k_1 \|\tanh(\mathbf{s}) + \tanh(\mathbf{v})\|^2 - \hat{m}\lambda \tanh(\mathbf{v})^T \mathbf{v} \\ &\quad + \mathbf{a}_1^T \mathbf{d} - \bar{\mathbf{d}} \mathbf{a}_1^T \tanh(l_1 \mathbf{a}_1) + \tilde{m} \left(\frac{1}{k_m} \dot{\hat{m}} - a_2 \right)\end{aligned}\quad (31)$$

In view of the adaptive law (4), if $\dot{\hat{m}} = k_m a_2$, then $\tilde{m}(\frac{1}{k_m} \dot{\hat{m}} - a_2) = 0$; if $\dot{\hat{m}} = 0$, then $\tilde{m}(\frac{1}{k_m} \dot{\hat{m}} - a_2) = -\tilde{m}a_2 = (m - \hat{m})a_2 \leq 0$ under the conditions $\hat{m} = m_0(\bar{m})$ and $a_2 \leq 0 (a_2 \geq 0)$. Furthermore, according to the fact

$$\mathbf{a}_1^T \mathbf{d} - \bar{\mathbf{d}} \mathbf{a}_1^T \tanh(l_1 \mathbf{a}_1) \leq \bar{d} [\|\mathbf{a}_1\|_1 - \mathbf{a}_1^T \tanh(l_1 \mathbf{a}_1)] \leq \frac{3\bar{d}\bar{\kappa}}{l_1}$$

with $\|\mathbf{d}\| \leq \bar{d}$ in Assumption 1, we know that

$$\begin{aligned}\dot{V}_c &\leq -\hat{m}k_1 \|\tanh(\mathbf{s}) + \tanh(\mathbf{v})\|^2 - \hat{m}\lambda \tanh(\mathbf{v})^T \mathbf{v} + \frac{3\bar{d}\bar{\kappa}}{l_1} \\ &\leq -m_0 \tanh(\mathbf{z})^T \Gamma \tanh(\mathbf{z}) + \frac{3\bar{d}\bar{\kappa}}{l_1}\end{aligned}$$

where $\mathbf{z} = [\mathbf{s}^T, \mathbf{v}^T]^T$, $\Gamma = \begin{bmatrix} k_1 I_3 & k_1 I_3 \\ k_1 I_3 & (k_1 + \lambda) I_3 \end{bmatrix}$, and I_3 is the 3×3 identity matrix. Since there exists an invertible matrix $\Lambda = \begin{bmatrix} I_3 & I_3 \\ O_3 & I_3 \end{bmatrix}$ with 3×3 null matrix O_3 , so that $\Lambda^{-T} \Gamma \Lambda^{-1} = \text{diag}\{k_1 I_3, \lambda I_3\}$, then

$$\dot{V}_c \leq -m_0 \eta_s \|\tanh(\mathbf{z})\|^2 + \frac{3\bar{d}\bar{\kappa}}{l_1} \quad (32)$$

where $\frac{3-\sqrt{5}}{2}$ in η_s is the minimum eigenvalue of the transformation matrix $\Lambda^T \Lambda$. If the parameters satisfy $\frac{3\bar{d}\bar{\kappa}}{l_1} < m_0 \eta_s$, then $\frac{3\bar{d}\bar{\kappa}}{m_0 \eta_s l_1} < 1$. Thus, from (32), if $\|\tanh(\mathbf{z})\| > \sqrt{\frac{3\bar{d}\bar{\kappa}}{m_0 \eta_s l_1}}$, then $\dot{V}_c < 0$, which means that $\tanh(\mathbf{z})$ ultimately converges to

$$\Omega_1 = \left\{ \tanh(z_i) \mid |\tanh(z_i)| < \sqrt{\frac{3\bar{d}\bar{\kappa}}{m_0 \eta_s l_1}}, i = 1, 2, \dots, 6 \right\}.$$

Thus, $\mathbf{z}$ ultimately converges to a tunable set

$$\Omega_2 = \left\{ z_i \mid |z_i| < \text{atanh} \left(\sqrt{\frac{3\bar{d}\bar{\kappa}}{m_0 \eta_s l_1}} \right), i = 1, 2, \dots, 6 \right\}.$$

Furthermore, from the definition of vectors $\mathbf{z}$ and $\mathbf{s}$, we know that $\mathbf{r}$ and $\mathbf{v}$ ultimately converge to tunable sets

$$\Omega_3 = \left\{ r_i \mid |r_i| \leq \frac{2}{\lambda} \operatorname{atanh} \left(\sqrt{\frac{3\bar{d}\kappa}{m_0\eta_s l_1}} \right), i = 1, 2, 3 \right\},$$

$$\Omega_4 = \left\{ v_i \mid |v_i| \leq \operatorname{atanh} \left(\sqrt{\frac{3\bar{d}\kappa}{m_0\eta_s l_1}} \right), i = 1, 2, 3 \right\}.$$

Therefore, the position error $\mathbf{r}$ and velocity error $\mathbf{v}$ ultimately converge to small neighborhoods of the origin by choosing sufficiently large l_1 , k_1 , and λ under $3\bar{d}\kappa < m_0\eta_s l_1$.

The proposed controllers (20) and (3) can guarantee the singularity-free properties of the attitude Jacobian matrix $W(\boldsymbol{\vartheta})$ in (1) and the inner-loop desired attitude signals in (7). The main procedure of the control algorithm is stated as follows. Firstly, given the initial information $\mathbf{r}(0)$, $\mathbf{v}(0)$, $\boldsymbol{\vartheta}(0)$, $\boldsymbol{\omega}(0)$, $\hat{m}(0)$, $\mathbf{r}_t$, $\gamma_r(t)$, $m_0, \bar{m}, \bar{d}$ under the conditions $|\theta(0)| < \pi/2$ and $\min\{-\pi/2, \theta(0)/2 - \pi/4\} < \theta_c(0) < \max\{\pi/2, \theta(0)/2 + \pi/4\}$, calculate the control thrust f from (6) and (4), and compute desired attitude θ_d and ψ_d from (7) with setting tunable parameters l_1 , k_1 , and λ satisfying the conditions $k_1 < \frac{1}{2}(\frac{\mu}{4\|\mathbf{r}_t\|^2} - \frac{\bar{d}}{m_0})$ and $3\bar{d}\kappa < m_0\eta_s l_1$. Secondly, given the initial information satisfies $\mathbf{x}_1(0) = \boldsymbol{\vartheta}_d(0)$ and $\mathbf{x}_2(0) = \mathbf{0}$, calculate the command attitude information $\boldsymbol{\vartheta}_c, \dot{\boldsymbol{\vartheta}}_c, \boldsymbol{\omega}_c$, and $\dot{\boldsymbol{\omega}}_c$ from (8) with setting ξ and $\boldsymbol{\omega}_n$. Thirdly, given the initial information $\hat{\boldsymbol{\delta}}(0)$, $\bar{\lambda}$, and $\bar{w}$, calculate the control torque $\mathbf{u}$ from (20) and (21).

It is noted that the constraint $\min\{-\pi/2, \theta(0)/2 - \pi/4\} < \theta_c(0) < \max\{\pi/2, \theta(0)/2 + \pi/4\}$ should be checked before calculating the control inputs. If this constraint is unsatisfied under the initial conditions, then the new controller parameters l_1 , k_1 , and λ in (3) should be chosen again under the conditions $k_1 < \frac{1}{2}(\frac{\mu}{4\|\mathbf{r}_t\|^2} - \frac{\bar{d}}{m_0})$ and $3\bar{d}\kappa < m_0\eta_s l_1$. If the constraint is still unsatisfied after resetting the controller parameters, then the new desired position $\mathbf{r}_t$ and yaw trajectory $\gamma_r(t)$ can be chosen again.

5 Simulation Example

A numerical simulation of a terminal descent mission is carried out in this section to show the effectiveness of the proposed landing strategy. The inertia matrix of the lunar module is $J = \operatorname{diag}\{10, 10, 10\}(\text{kgm}^2)$ and the mass is $m = 100(\text{kg})$; other parameters are $\mu = 4.90275 \times 10^{12}(\text{m}^3/\text{s}^2)$, $\mathbf{r}_t = 1.7371 \times 10^6(\text{m})$, $\gamma_r = 0(\text{rad})$. The upper bounds of disturbances are $\bar{d} = 0.5$

and $\bar{w} = 0.05$. The bounds of the mass and inertia parameter are set as $m_0 = 90$, $\bar{m} = 110$, $\bar{\lambda} = 17.5$. Initial estimations are set as $\hat{m}(0) = 95$ and $\hat{\delta}_J(0) = [9.8, 0, 0, 9.5, 0, 10]^T$. To validate the performance of the proposed controller, two cases for the different initial states of the lunar lander are considered in simulations, and the controller parameters are chosen properly, where l_1 , k_1 , and λ are chosen based on the rules $k_1 < \frac{1}{2}(\frac{\mu}{4\|\mathbf{r}_t\|^2} - \frac{\bar{d}}{m_0})$ and $3\bar{d}\kappa < m_0\eta_s l_1$, where $\kappa = 0.2785$ and $\eta_s = \frac{3-\sqrt{5}}{2}\min\{k_1, \lambda\}$; $k_2 > 0$, $k_3 > 0$, $l_2 > 0$, $\varepsilon > 0$, and $\varepsilon_0 > 0$ are chosen such that k_b is large enough, where $k_b = \min\{2k_2, \frac{2k_3}{\sigma}, \varepsilon\varepsilon_0\}$; ξ and ω_n are chosen largely for the stable tracking performance of the command filter.

Case 1 The initial position, velocity, attitude and angular velocity of the lunar lander in the target frame O_Txyz are $\mathbf{r}(0) = [100, 200, 150]^T$ (m), $\mathbf{v}(0) = [1, 1.5, 2]^T$ (m/s), $\boldsymbol{\vartheta}(0) = [\pi/4, \pi/4, \pi/4]^T$ (rad), $\boldsymbol{\omega}(0) = [0, 0, 0]^T$ (rad/s), respectively. The controller and filter parameters are set to $\lambda = 0.1$, $k_1 = 0.2$, $k_2 = 1.6$, $k_3 = 5$, $k_m = 0.5$, $\varepsilon = 0.05$, $\varepsilon_0 = 0.02$, $l_1 = l_2 = 2$, $\xi = 1$, $\omega_n = 2$.

Case 2 The initial position, velocity, attitude and angular velocity of the lunar lander in the target frame O_Txyz are $\mathbf{r}(0) = [100, 400, -500]^T$ (m), $\mathbf{v}(0) = [-1.5, -1.5, 2]^T$ (m/s), $\boldsymbol{\vartheta}(0) = [-\pi/6, -\pi/4, \pi/5]^T$ (rad), $\boldsymbol{\omega}(0) = [-0.02, 0.02, -0.02]^T$ (rad/s), respectively. The controller and filter parameters are also chosen as the Case 1.

Figs. 3-6 show the simulation results of the position, velocity, attitude, and angular velocity of the lunar lander under the initial states shown in Case 1. It is seen that the three-dimensional landing position error is ultimately reduced to values smaller than 0.05(m) with the high precision of landing velocity as 2×10^{-3} (m/s). Meanwhile, the three-axes attitude precision is also higher than 1×10^{-3} (rad) with the high precision of angular velocity as 5×10^{-5} (rad/s). Moreover, the transient response time of the position and attitude motions is smaller than 200(s). The control forces and control torques are shown in Fig. 7, where the control forces are always larger than gravity and ensure the controllability of the three-dimensional position motion. Fig. 8 implies that the online adaptive parameters are always bounded. These results illustrate that the lunar landings can be achieved with the good performance of transient and steady-state responses.

Figs. 9-12 show the simulation results of the position, velocity, attitude, and angular velocity of the lunar lander under the larger initial states shown in Case 2. The landing position

Figure 3: Time histories of the lunar lander's three-dimensional positions in Case 1.

Figure 4: Time histories of the lunar lander's three-dimensional velocities in Case 1.

Figure 5: Time histories of the lunar lander's three-axes attitudes in Case 1.

Figure 6: Time histories of the lunar lander's three-axes angular velocities in Case 1.

Figure 7: Time histories of the lunar lander's control inputs in Case 1.

Figure 8: Time histories of the adaptive parameters in Case 1.

Figure 9: Time histories of the lunar lander's three-dimensional positions in Case 2.

Figure 10: Time histories of the lunar lander's three-dimensional velocities in Case 2.

Figure 11: Time histories of the lunar lander's three-axes attitudes in Case 2.

Figure 12: Time histories of the lunar lander's three-axes angular velocities in Case 2.

Figure 13: Time histories of the lunar lander's control inputs in Case 2.

Figure 14: Time histories of the adaptive parameters in Case 2.

error is also smaller than 0.05(m), and the landing velocity error is ultimately smaller than 5×10^{-3} (m/s) such that the soft landing mission can be achieved, so the three-dimensional landing position error and velocity error are acceptable under the larger initial error for the lunar landers. Compared with the simulation results in Case 1, the three-axes attitude precision is still less than 1×10^{-3} (rad) with the angular velocity precision as 5×10^{-5} (rad/s). However, the transient response time of the position and attitude motions is larger than 200(s). The bounded control forces and control torques are shown in Fig. 13, and Fig. 14 implies that the online adaptive parameters are also always bounded. In spite of the larger initial states in Case 2, simulation results illustrate that the lunar landings can still be achieved well by using the same controller parameters with the Case 1. The simulation results for these two cases show that the proposed controller and the proposed tuning rules are effective for the lunar landing systems with different initial states.

6 Conclusions

A novel nonlinear pose controller for an uncertain lunar lander in terminal landing phase is developed based on the double-loop design structure. To avoid the computational singularities in the outer-loop desired attitude extraction, the constrained guidance law is designed based on the hyperbolic tangent function and bounded estimation of the mass. The command filter is employed to compute the command attitude trajectory to avoid the complicated time derivatives of the extracted desired attitude trajectory. Moreover, to avoid the computational singularity of the Euler angles-based inverse Jacobian matrix, the adaptive inner-loop attitude tracking controller is designed based on the barrier Lyapunov function and bounded estimations of the inertial parameter to ensure constrained pitch tracking. In particular, the tuning rules of the designing parameters are derived with stability analysis. It is proved in the Lyapunov framework that the pose and velocities tracking errors of the lunar lander ultimately converge to the small neighborhoods of the origin by choosing suitable designing parameters.

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Declaration of Competing Interest

No conflict of interest.

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