Online Supplement for “Accelerated Process Optimization for Laser-Based Additive Manufacturing by Leveraging Similar Prior Studies” by “Amir M. Aboutaleb, Linkan Bian, Alaa Elwany, Nima Shamsaei, Scott M. Thompson, Gustavo Tapia”

Appendix A

Since λ'_i 's are assumed independent, their joint distribution would be resulted by multiplying their distribution by each other as follows:

$$\begin{aligned}\pi(\boldsymbol{\lambda}) &= \prod_{i=1}^{n_p} (2\pi\omega_i^2)^{-\frac{1}{2}} \exp\left\{\frac{-1}{2\omega_i^2}(\lambda_i - \beta_i)^2\right\} \\ &= \left(\prod_{i=1}^{n_p} (2\pi\omega_i^2)^{-\frac{1}{2}}\right) \times \exp\left\{\sum_{i=1}^{n_p} \frac{-1}{2\omega_i^2}(\lambda_i - \beta_i)^2\right\} \\ &= \left(\prod_{i=1}^{n_p} (2\pi\omega_i^2)^{-\frac{1}{2}}\right) \times \exp\left\{\frac{-1}{2}\sum_{i=1}^{n_p} \frac{\lambda_i^2}{\omega_i^2} + \sum_{i=1}^{n_p} \frac{\lambda_i\beta_i}{\omega_i^2} - \frac{1}{2}\sum_{i=1}^{n_p} \frac{\beta_i^2}{\omega_i^2}\right\}\end{aligned}$$

Expanded prior distribution of λ'_i 's can be re-arranged according to the terms involving λ'_i 's as follows:

$$\pi(\boldsymbol{\lambda}) \propto \exp\left\{\frac{-1}{2}\sum_{i=1}^{n_p} \frac{\lambda_i^2}{\omega_i^2} + \sum_{i=1}^{n_p} \frac{\lambda_i\beta_i}{\omega_i^2}\right\}$$

Appendix B

Since the design points are generated sequentially, distribution of Y_j depends on prior data $\mathbf{u}$, updated DRPCS's $\boldsymbol{\lambda}$ and the responses from prior batches of experiments in the current study $Y_1, Y_2, \dots, Y_{\lfloor \frac{j-1}{b} \rfloor b}$.

Hence, assuming that distribution of Y_j follows a normal distribution, it can be represented as follows:

$$f_j \left(Y_j \middle| Y_1, Y_2, \dots, Y_{\lfloor \frac{j-1}{b} \rfloor b}, \mathbf{u}, \boldsymbol{\lambda} \right) = (2\pi\sigma_j^2)^{\frac{-1}{2}} \exp \left\{ \frac{-1}{2\sigma_j^2} (Y_j - \mu_j)^2 \right\}$$

$$\mu_j = E[Y_j] = \sum_{i=1}^{n_p} w_{j,i} (u_i + \lambda_i) + \sum_{j'=1}^{\lfloor \frac{j-1}{b} \rfloor b} v_{j,j'} y_{j'}$$

Based on the definition of conditional distribution, joint distribution of Y_j 's can be resulted by multiplying their conditional distributions as follows:

$$\begin{aligned} f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u}) &= \prod_{j=1}^{n_c} f_j \left(Y_j \middle| Y_1, Y_2, \dots, Y_{\lfloor \frac{j-1}{b} \rfloor b}, \mathbf{u}, \boldsymbol{\lambda} \right) \\ &= \prod_{j=1}^{n_c} (2\pi\sigma_j^2)^{\frac{-1}{2}} \exp \left\{ \frac{-1}{2\sigma_j^2} (Y_j - \mu_j)^2 \right\} \\ &= \prod_{j=1}^{n_c} (2\pi\sigma_j^2)^{\frac{-1}{2}} \exp \left\{ \frac{-1}{2\sigma_j^2} \left(Y_j - \sum_{i=1}^{n_p} w_{j,i} u_i - \sum_{i=1}^{n_p} w_{j,i} \lambda_i - \sum_{j'=1}^{\lfloor \frac{j-1}{b} \rfloor b} v_{j,j'} y_{j'} \right)^2 \right\} \end{aligned}$$

If we denote $\delta_j = Y_j - \sum_{i=1}^{n_p} w_{j,i} u_i - \sum_{j'=1}^{\lfloor \frac{j-1}{b} \rfloor b} v_{j,j'} y_{j'}$, $f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u})$ can be re-arranged as follows:

$$f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u}) = \prod_{j=1}^{n_c} (2\pi\sigma_j^2)^{\frac{-1}{2}} \exp \left\{ \frac{-1}{2\sigma_j^2} \left[\left(\sum_{i=1}^{n_p} w_{j,i} \lambda_i \right)^2 + \delta_j^2 - 2\delta_j \sum_{i=1}^{n_p} w_{j,i} \lambda_i \right] \right\}$$

Expanded proportional form of this formula is as follows:

$$f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u}) \propto \exp \left\{ \frac{-1}{2} \sum_{j=1}^{n_c} \frac{1}{\sigma_j^2} \left(\sum_{i=1}^{n_p} w_{j,i} \lambda_i \right)^2 + \sum_{j=1}^{n_c} \frac{\delta_j}{\sigma_j^2} \sum_{i=1}^{n_p} w_{j,i} \lambda_i - \frac{1}{2} \sum_{j=1}^{n_c} \left(\frac{\delta_j}{\sigma_j} \right)^2 \right\}$$

By factoring λ_i 's out we can reorganize proportional formula of $f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u})$ as follows:

$$f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u}) \propto \exp \left\{ \frac{-1}{2} \left(\sum_{i=1}^{n_p} \lambda_i^2 \sum_{j=1}^{n_c} \frac{w_{j,i}^2}{\sigma_j^2} + 2 \sum_{i \neq k} \lambda_i \lambda_k \sum_{j=1}^{n_c} \frac{w_{j,i} w_{j,k}}{\sigma_j^2} \right) + \sum_{i=1}^{n_p} \lambda_i \sum_{j=1}^{n_c} \frac{w_{j,i} \delta_j}{\sigma_j^2} - \frac{1}{2} \sum_{j=1}^{n_c} \left(\frac{\delta_j}{\sigma_j} \right)^2 \right\}$$

Proportional formula of $f(\mathbf{Y}|\boldsymbol{\lambda}, \mathbf{u})$ can be rearranged according to the terms involving λ_i 's as follows:

$$f(\mathbf{Y}|\boldsymbol{\lambda}) \propto \exp \left\{ \frac{-1}{2} \left(\sum_{i=1}^{n_p} \lambda_i^2 \sum_{j=1}^{n_c} \frac{w_{j,i}^2}{\sigma_j^2} + 2 \sum_{i \neq k} \lambda_i \lambda_k \sum_{j=1}^{n_c} \frac{w_{j,i} w_{j,k}}{\sigma_j^2} \right) + \sum_{i=1}^{n_p} \lambda_i \sum_{j=1}^{n_c} \frac{w_{j,i} \delta_j}{\sigma_j^2} \right\}$$

Appendix C

By multiplying the expanded form of $f(\mathbf{Y}|\boldsymbol{\lambda})$ and $\pi(\boldsymbol{\lambda})$, the posterior distribution of λ_i using the Bayesian formula would be resulted as follows:

$$g(\boldsymbol{\lambda}|\mathbf{y}) \propto \exp \left\{ \frac{-1}{2} \sum_{i=1}^{n_p} \frac{\lambda_i^2}{\omega_i^2} + \sum_{i=1}^{n_p} \frac{\lambda_i \beta_i}{\omega_i^2} - \frac{1}{2} \left(\sum_{i=1}^{n_p} \lambda_i^2 \sum_{j=1}^{n_c} \frac{w_{j,i}^2}{\sigma_j^2} + 2 \sum_{i \neq k} \lambda_i \lambda_k \sum_{j=1}^{n_c} \frac{w_{j,i} w_{j,k}}{\sigma_j^2} \right) + \sum_{i=1}^{n_p} \lambda_i \sum_{j=1}^{n_c} \frac{w_{j,i} \delta_j}{\sigma_j^2} \right\}$$

Trying to factoring out the λ 's terms, the formula above can be re-arranged as follows:

$$g(\boldsymbol{\lambda}|\mathbf{y}) \propto \exp \left\{ \frac{-1}{2} \left(\sum_{i=1}^{n_p} \lambda_i^2 \left(\sum_{j=1}^{n_c} \frac{w_{j,i}^2}{\sigma_j^2} + \frac{1}{\omega_i^2} \right) + 2 \sum_{i \neq k} \lambda_i \lambda_k \sum_{j=1}^{n_c} \frac{w_{j,i} w_{j,k}}{\sigma_j^2} \right) + \sum_{i=1}^{n_p} \lambda_i \left(\sum_{j=1}^{n_c} \frac{w_{j,i} \delta_j}{\sigma_j^2} + \frac{\beta_i}{\omega_i^2} \right) \right\}$$

Appendix D

Here we show the steps of matrix multiplications as follows:

$$\boldsymbol{\lambda}^T \boldsymbol{\Lambda} \boldsymbol{\lambda} = (\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_{n_p})_{1 \times n_p} \times \begin{pmatrix} \Lambda_{1,1} & \Lambda_{1,2} & \cdots & \Lambda_{1,n_p} \\ \Lambda_{2,1} & & \ddots & \Lambda_{2,n_p} \\ \vdots & & & \vdots \\ \Lambda_{n_p,1} & \Lambda_{n_p,2} & \cdots & \Lambda_{n_p,n_p} \end{pmatrix}_{n_p \times n_p} \times \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{n_p} \end{pmatrix}_{n_p \times 1}$$

$$\begin{aligned}
&= (\lambda_1 \Lambda_{1,1} + \lambda_2 \Lambda_{2,1} + \dots + \lambda_{n_p} \Lambda_{n_p,1} \quad \lambda_1 \Lambda_{1,2} + \lambda_2 \Lambda_{2,2} + \dots + \lambda_{n_p} \Lambda_{n_p,2} \quad \dots \quad \lambda_1 \Lambda_{1,n_p} + \lambda_2 \Lambda_{2,n_p} + \dots + \lambda_{n_p} \Lambda_{n_p,n_p})_{1 \times n_p} \\
&\times \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_{n_p} \end{pmatrix}_{n_p \times 1} \\
&= (\lambda_1^2 \Lambda_{1,1} + \lambda_2 \lambda_1 \Lambda_{2,1} + \dots + \lambda_{n_p} \lambda_1 \Lambda_{n_p,1}) + (\lambda_1 \lambda_2 \Lambda_{1,2} + \lambda_2^2 \Lambda_{2,2} + \dots + \lambda_{n_p} \lambda_2 \Lambda_{n_p,2}) + \dots + \\
&(\lambda_1 \lambda_{n_p} \Lambda_{1,n_p} + \lambda_2 \lambda_{n_p} \Lambda_{2,n_p} + \dots + \lambda_{n_p}^2 \Lambda_{n_p,n_p})
\end{aligned}$$

Now $\boldsymbol{\lambda}^T \mathbf{\Lambda} \boldsymbol{\lambda}$ can be summarized and represented by its elements as follows:

$$\boldsymbol{\lambda}^T \mathbf{\Lambda} \boldsymbol{\lambda} = \sum_{i=1}^{n_p} \lambda_i^2 \Lambda_{i,i} + \sum_{i \neq k} \lambda_i \lambda_k \Lambda_{i,k}$$

By comparing the formula above with the final form of $g(\boldsymbol{\lambda}|\mathbf{y})$ in Appendix B, we can see the similarity between resulted terms in $\boldsymbol{\lambda}^T \mathbf{\Lambda} \boldsymbol{\lambda}$ and the coefficients of λ 's terms in $g(\boldsymbol{\lambda}|\mathbf{y})$. Hence, elements of $\mathbf{\Lambda}$ can be presented as follows:

$$\Lambda_{i,k} = \begin{cases} \sum_{j=1}^{n_c} \frac{w_{j,i}^2}{\sigma_j^2} + \frac{1}{\omega_i^2} & i = k \\ \sum_{j=1}^{n_c} \frac{w_{j,i} w_{j,k}}{\sigma_j^2}, & i \neq k \end{cases}$$

Moreover, by comparing canonical form of multivariate normal distribution, $h_2(\mathbf{z}|\boldsymbol{\eta}, \mathbf{\Lambda}) \propto$

$\exp \left\{ a + \boldsymbol{\eta}^T \mathbf{z} - \frac{1}{2} \mathbf{z}^T \mathbf{\Lambda} \mathbf{z} \right\}$, and $g(\boldsymbol{\lambda}|\mathbf{y})$ in Appendix B we can see that elements of $\boldsymbol{\eta}$ can be represented as follows:

$$\eta_i = \sum_{j=1}^{n_c} \frac{w_{j,i} \delta_j}{\sigma_j^2} + \frac{\beta_i}{\omega_i^2}$$

Appendix E

The improvement in number of experimental runs using our method, compared to SMED, is highlighted in red.

Table 1. Prior data with negative DRPCS.

Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	SMED Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Our Method Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	2000	0.05	0.03	0.7295	1	44	2000	0.05	0.03	0.7295
2	45	400	0.15	0.05	0.7268	2	45	400	0.15	0.05	0.7268
3	40	400	0.1	0.05	0.7831	3	40	400	0.1	0.05	0.7831
4	40	400	0.15	0.04	0.7328	4	40	400	0.15	0.04	0.7328
5	40	400	0.05	0.05	0.9692	5	40	400	0.05	0.05	0.9692
6	44	600	0.1	0.05	0.7295	6	44	600	0.1	0.05	0.7295
7	41	400	0.05	0.05	0.9725	7	41	400	0.05	0.05	0.9725
8	42	400	0.05	0.05	0.9754	8	42	400	0.05	0.05	0.9754
9	43	400	0.05	0.05	0.9780	9	43	400	0.05	0.05	0.9780
10	44	400	0.05	0.05	0.9805	10	44	400	0.05	0.05	0.9805
11	43	600	0.05	0.05	0.9016	11	43	600	0.05	0.05	0.9016
12	45	400	0.05	0.05	0.9827	12	45	400	0.05	0.05	0.9827
13	40	800	0.05	0.05	0.7831	13	40	800	0.05	0.05	0.7831
14	44	800	0.05	0.05	0.8164	14	44	800	0.05	0.05	0.8164
15	40	400	0.05	0.04	0.9909	15	40	400	0.05	0.04	0.9909
16	40	600	0.05	0.05	0.8806	16	45	400	0.05	0.04	0.9967
17	41	400	0.05	0.04	0.9924						
18	42	400	0.05	0.04	0.9937						
19	43	400	0.05	0.04	0.9948						
20	44	400	0.05	0.04	0.9958						
21	42	600	0.05	0.04	0.9477						
22	45	400	0.05	0.04	0.9967						

Improvement = 27.27%

Table 2. Prior data with smaller DRPCS (compared to Table 9).

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	45	400	0.15	0.05	0.7268	1	45	400	0.15	0.05	0.7268
2	41	800	0.15	0.03	0.9496	2	41	800	0.15	0.03	0.9496
3	44	800	0.15	0.03	0.8406	3	44	800	0.15	0.03	0.8406
4	40	400	0.15	0.03	0.8199	4	40	400	0.15	0.03	0.8199
5	42	800	0.15	0.03	0.9067	5	42	800	0.15	0.03	0.9067
6	40	800	0.1	0.03	0.7328	6	40	800	0.1	0.03	0.7328
7	40	600	0.15	0.03	0.7281	7	40	600	0.15	0.03	0.7281
8	41	600	0.15	0.04	0.9496	8	40	400	0.05	0.05	0.9692
9	43	600	0.15	0.04	0.8708	9	42	400	0.05	0.05	0.9754
10	40	400	0.15	0.04	0.7328	10	43	400	0.05	0.05	0.9780
11	42	600	0.15	0.04	0.9067	11	44	400	0.05	0.05	0.9805
12	40	400	0.05	0.05	0.9692	12	45	400	0.05	0.05	0.9827
13	42	400	0.05	0.05	0.9754	13	44	600	0.05	0.05	0.9078
14	43	400	0.05	0.05	0.9780	14	41	400	0.05	0.05	0.9725
15	44	400	0.05	0.05	0.9805	15	40	800	0.05	0.05	0.7831
16	45	400	0.05	0.05	0.9827	16	40	400	0.05	0.04	0.9909
17	44	600	0.05	0.05	0.9078	17	41	400	0.05	0.04	0.9924
18	41	400	0.05	0.05	0.9725	18	42	400	0.05	0.04	0.9937
19	40	800	0.05	0.05	0.7831	19	43	400	0.05	0.04	0.9948
20	40	400	0.05	0.04	0.9909	20	44	400	0.05	0.04	0.9958
21	41	400	0.05	0.04	0.9924	21	45	400	0.05	0.04	0.9967
22	42	400	0.05	0.04	0.9937						
23	43	400	0.05	0.04	0.9948						
24	44	400	0.05	0.04	0.9958						
25	45	400	0.05	0.04	0.9967						

Improvement = 16%

Table 3. Prior data with positive and negative DRPCS.

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	44	600	0.15	0.04	0.8406	2	44	600	0.15	0.04	0.8406
3	45	400	0.15	0.05	0.7268	3	45	400	0.15	0.05	0.7268
4	40	400	0.15	0.03	0.8199	4	44	1200	0.1	0.03	0.8406
5	44	1200	0.1	0.03	0.8406	5	45	400	0.15	0.03	0.8602
6	44	800	0.1	0.03	0.7554	6	44	600	0.1	0.03	0.8528
7	40	400	0.15	0.04	0.7328	7	40	400	0.1	0.03	0.9381
8	44	600	0.15	0.03	0.7294	8	43	400	0.1	0.03	0.9521
9	45	400	0.05	0.05	0.9827	9	45	400	0.1	0.03	0.9597
10	44	400	0.05	0.05	0.9805	10	44	400	0.1	0.03	0.9560
11	42	400	0.05	0.05	0.9754	11	42	400	0.1	0.03	0.9477
12	43	400	0.05	0.05	0.9780	12	41	400	0.1	0.03	0.9431
13	40	400	0.05	0.05	0.9692	13	40	400	0.05	0.03	0.9999
14	44	600	0.05	0.05	0.9078						
15	41	400	0.05	0.05	0.9725						
16	40	800	0.05	0.05	0.7831						
17	40	600	0.05	0.05	0.8806						
18	41	400	0.05	0.04	0.9924						
19	40	400	0.05	0.04	0.9909						
20	42	400	0.05	0.04	0.9937						
21	43	400	0.05	0.04	0.9948						
22	44	400	0.05	0.04	0.9958						
23	45	400	0.05	0.04	0.9967						

Improvement = 43.4%

Table 4. Prior data with positive DRPCS.

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	44	2000	0.05	0.03	0.7295	2	44	2000	0.05	0.03	0.7295
3	44	1800	0.05	0.04	0.8406	3	45	400	0.15	0.05	0.7268
4	45	400	0.15	0.05	0.7268	4	45	400	0.15	0.03	0.8602
5	44	1400	0.05	0.05	0.8098	5	44	1200	0.1	0.03	0.8406
6	44	1200	0.1	0.03	0.8406	6	44	1400	0.05	0.05	0.8098
7	44	1400	0.05	0.04	0.7261	7	45	400	0.1	0.03	0.9597
8	45	400	0.15	0.03	0.8602	8	42	400	0.1	0.03	0.9477
9	45	400	0.1	0.03	0.9597	9	44	400	0.1	0.03	0.9560
10	43	400	0.1	0.03	0.9521	10	40	400	0.1	0.03	0.9381
11	40	400	0.1	0.03	0.9381	11	44	600	0.1	0.03	0.8528
12	44	400	0.1	0.03	0.9560	12	43	400	0.1	0.03	0.9521
13	42	400	0.1	0.03	0.9477	13	41	400	0.1	0.03	0.9431
14	44	600	0.1	0.03	0.8528	14	40	400	0.05	0.03	0.9999
15	41	400	0.1	0.03	0.9431						
16	43	400	0.05	0.03	0.9998						

Improvement = 12.5%

Table 5. More prior data points.

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	45	400	0.15	0.05	0.7268	1	45	400	0.15	0.05	0.7268
2	45	400	0.05	0.05	0.9827	2	45	400	0.05	0.05	0.9827
3	44	400	0.05	0.05	0.980	3	44	400	0.05	0.05	0.980
4	42	400	0.05	0.05	0.9754	4	42	400	0.05	0.05	0.9754
5	43	400	0.05	0.05	0.9780	5	43	400	0.05	0.05	0.9780
6	40	400	0.05	0.05	0.9692	6	40	400	0.05	0.05	0.9692
7	44	600	0.05	0.05	0.9078	7	44	600	0.05	0.05	0.9078
8	41	400	0.05	0.05	0.9725	8	41	400	0.05	0.05	0.9725
9	40	800	0.05	0.05	0.7831	9	40	800	0.05	0.05	0.7831
10	40	600	0.05	0.05	0.8806	10	40	400	0.05	0.04	0.9909
11	40	400	0.05	0.04	0.9909	11	41	400	0.05	0.04	0.9924
12	41	400	0.05	0.04	0.9924	12	42	400	0.05	0.04	0.9937
13	42	400	0.05	0.04	0.9937	13	43	400	0.05	0.04	0.9948
14	43	400	0.05	0.04	0.9948	14	44	400	0.05	0.04	0.9958
15	44	400	0.05	0.04	0.9958	15	45	400	0.05	0.04	0.9967
16	45	400	0.05	0.04	0.9967	Improvement = 6.25%					

Appendix F

Although there is not any universal guideline for choosing an appropriate charge function, the performance of the proposed method and SMED is examined given charge functions $q(\mathbf{s}) = (1 - \gamma(\mathbf{s}))^\gamma$ for $\gamma = 1, 3, 5$. This charge function is suggested by (Dasgupta 2007). The simulations of this part are applied to multiple scenarios: negative DRPCS, positive and negative DRPCS, and positive DRPCS. The results are summarized in Tables 14-22. Note that we used the charge function with $\gamma = 3$ in Section 3.1.1 and 3.1.2 to investigate the effects of DRPCS. Tables 17-19 are identical to Tables 9, 11, and 12, respectively. We include them in this section for the convenience of the reader.

a. Simulations for $\gamma = 1$

Table 6. Performance of SMED and our method with negative DRPCS and $\gamma=1$.

SMED						Our Method					
Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	2000	0.05	0.03	0.7295	1	44	2000	0.05	0.03	0.7295
2	45	400	0.15	0.05	0.7268	2	45	400	0.15	0.05	0.7268
3	40	400	0.05	0.05	0.9692	3	40	400	0.05	0.05	0.9692
4	40	400	0.15	0.03	0.8199	4	40	400	0.15	0.03	0.8199
5	40	600	0.05	0.05	0.8806	5	40	600	0.05	0.05	0.8806
6	45	400	0.05	0.05	0.9827	6	45	400	0.05	0.05	0.9827
7	44	400	0.05	0.05	0.9805	7	44	400	0.05	0.05	0.9805
8	44	1000	0.05	0.05	0.7443	8	44	1000	0.05	0.05	0.7443
9	40	400	0.05	0.04	0.9909	9	40	400	0.05	0.04	0.9909
10	43	400	0.05	0.05	0.9780	10	43	400	0.05	0.05	0.9780
11	42	400	0.05	0.04	0.9937	11	42	400	0.05	0.04	0.9937
12	45	400	0.05	0.04	0.9967	12	45	400	0.05	0.04	0.9967

Table 7. Performance of SMED and our method with positive and negative DRPCS and $\gamma=1$.

SMED						Our Method					
Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	45	400	0.15	0.05	0.7268	1	44	2000	0.05	0.03	0.7295
2	44	800	0.15	0.03	0.8406	2	45	400	0.15	0.05	0.7268
3	44	2000	0.05	0.03	0.7295	3	40	400	0.05	0.05	0.9692
4	45	400	0.05	0.05	0.9827	4	40	400	0.15	0.03	0.8199
5	40	400	0.05	0.05	0.9692	5	40	600	0.05	0.05	0.8806
6	44	600	0.05	0.05	0.9078	6	45	400	0.05	0.05	0.9827
7	43	400	0.05	0.05	0.9780	7	44	400	0.05	0.05	0.9805
8	40	1000	0.05	0.05	0.7274	8	44	1000	0.05	0.05	0.7443
9	40	400	0.05	0.04	0.9909	9	40	400	0.05	0.04	0.9909
10	44	400	0.05	0.04	0.9958	10	43	400	0.05	0.05	0.9780
11	45	400	0.05	0.04	0.9967	11	42	400	0.05	0.04	0.9937

Table 8. Performance of SMED and our method with positive DRPCS and $\gamma=1$.

SMED						Our Method					
Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	45	400	0.15	0.05	0.7268	1	45	400	0.15	0.05	0.7268
2	44	2000	0.05	0.03	0.7295	2	44	2000	0.05	0.03	0.7295
3	44	1800	0.05	0.04	0.8406	3	44	1400	0.05	0.05	0.8098
4	44	800	0.15	0.03	0.8406	4	44	800	0.15	0.03	0.8406
5	44	1400	0.05	0.05	0.8098	5	40	400	0.05	0.05	0.9692
6	45	400	0.05	0.05	0.9827	6	40	600	0.05	0.05	0.8806
7	40	400	0.05	0.05	0.9692	7	45	400	0.05	0.05	0.9827
8	44	600	0.05	0.05	0.9078	8	44	400	0.05	0.05	0.9805
9	43	400	0.05	0.05	0.9780	9	40	400	0.05	0.04	0.9909
10	40	400	0.05	0.04	0.9909	10	44	400	0.05	0.04	0.9958
11	44	400	0.05	0.04	0.9958	11	45	400	0.05	0.04	0.9967
12	45	400	0.05	0.04	0.9967						

b. Simulations for $\gamma = 3$.Table 9. Performance of SMED and our method with positive DRPCS and $\gamma=3$ (Same as Table 9).

SMED—Scenario 1						Our Method					
Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	2000	0.05	0.03	0.7295	1	44	2000	0.05	0.03	0.7295
2	45	400	0.15	0.05	0.7268	2	45	400	0.15	0.05	0.7268
3	40	400	0.1	0.05	0.7831	3	40	400	0.1	0.05	0.7831
4	40	400	0.15	0.04	0.7328	4	40	400	0.15	0.04	0.7328
5	40	400	0.05	0.05	0.9692	5	40	400	0.05	0.05	0.9692
6	44	600	0.1	0.05	0.7295	6	44	600	0.1	0.05	0.7295
7	41	400	0.05	0.05	0.9725	7	41	400	0.05	0.05	0.9725
8	42	400	0.05	0.05	0.9754	8	42	400	0.05	0.05	0.9754
9	43	400	0.05	0.05	0.9780	9	43	400	0.05	0.05	0.9780
10	44	400	0.05	0.05	0.9805	10	44	400	0.05	0.05	0.9805
11	43	600	0.05	0.05	0.9016	11	43	600	0.05	0.05	0.9016
12	45	400	0.05	0.05	0.9827	12	45	400	0.05	0.05	0.9827
13	40	800	0.05	0.05	0.7831	13	40	800	0.05	0.05	0.7831
14	44	800	0.05	0.05	0.8164	14	44	800	0.05	0.05	0.8164
15	40	400	0.05	0.04	0.9909	15	40	400	0.05	0.04	0.9909
16	40	600	0.05	0.05	0.8806	16	45	400	0.05	0.04	0.9967
17	41	400	0.05	0.04	0.9924						
18	42	400	0.05	0.04	0.9937						
19	43	400	0.05	0.04	0.9948						
20	44	400	0.05	0.04	0.9958						
21	42	600	0.05	0.04	0.9477						
22	45	400	0.05	0.04	0.9967						

Table 10. Performance of SMED and our method with positive and negative DRPCS and $\gamma=3$ (Same as Table 11).

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	44	600	0.15	0.04	0.8406	2	44	600	0.15	0.04	0.8406
3	45	400	0.15	0.05	0.7268	3	45	400	0.15	0.05	0.7268
4	40	400	0.15	0.03	0.8199	4	44	1200	0.1	0.03	0.8406
5	44	1200	0.1	0.03	0.8406	5	45	400	0.15	0.03	0.8602
6	44	800	0.1	0.03	0.7554	6	44	600	0.1	0.03	0.8528
7	40	400	0.15	0.04	0.7328	7	40	400	0.1	0.03	0.9381
8	44	600	0.15	0.03	0.7294	8	43	400	0.1	0.03	0.9521
9	45	400	0.05	0.05	0.9827	9	45	400	0.1	0.03	0.9597
10	44	400	0.05	0.05	0.9805	10	44	400	0.1	0.03	0.9560
11	42	400	0.05	0.05	0.9754	11	42	400	0.1	0.03	0.9477
12	43	400	0.05	0.05	0.9780	12	41	400	0.1	0.03	0.9431
13	40	400	0.05	0.05	0.9692	13	40	400	0.05	0.03	0.9999
14	44	600	0.05	0.05	0.9078						
15	41	400	0.05	0.05	0.9725						
16	40	800	0.05	0.05	0.7831						
17	40	600	0.05	0.05	0.8806						
18	41	400	0.05	0.04	0.9924						
19	40	400	0.05	0.04	0.9909						
20	42	400	0.05	0.04	0.9937						
21	43	400	0.05	0.04	0.9948						
22	44	400	0.05	0.04	0.9958						
23	45	400	0.05	0.04	0.9967						

Table 11. Performance of SMED and our method with positive DRPCS and $\gamma=3$ (Same as Table 12).

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	44	2000	0.05	0.03	0.7295	2	44	2000	0.05	0.03	0.7295
3	44	1800	0.05	0.04	0.8406	3	45	400	0.15	0.05	0.7268
4	45	400	0.15	0.05	0.7268	4	45	400	0.15	0.03	0.8602
5	44	1400	0.05	0.05	0.8098	5	44	1200	0.1	0.03	0.8406
6	44	1200	0.1	0.03	0.8406	6	44	1400	0.05	0.05	0.8098
7	44	1400	0.05	0.04	0.7261	7	45	400	0.1	0.03	0.9597
8	45	400	0.15	0.03	0.8602	8	42	400	0.1	0.03	0.9477
9	45	400	0.1	0.03	0.9597	9	44	400	0.1	0.03	0.9560
10	43	400	0.1	0.03	0.9521	10	40	400	0.1	0.03	0.9381
11	40	400	0.1	0.03	0.9381	11	44	600	0.1	0.03	0.8528
12	44	400	0.1	0.03	0.9560	12	43	400	0.1	0.03	0.9521
13	42	400	0.1	0.03	0.9477	13	41	400	0.1	0.03	0.9431
14	44	600	0.1	0.03	0.8528	14	40	400	0.05	0.03	0.9999
15	41	400	0.1	0.03	0.9431						
16	43	400	0.05	0.03	0.9998						

c. Simulations for $\gamma = 5$

Table 12. Performance of SMED and our method with negative DRPCS and $\gamma=5$.

Experiment Number	SMED					Experiment Number	Our Method				
	Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density		Laser Power (W)	Laser Velocity (mm/s)	Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	45	400	0.15	0.05	0.7268	2	45	400	0.15	0.05	0.7268
3	41	800	0.15	0.03	0.9496	3	41	800	0.15	0.03	0.9496
4	44	600	0.15	0.03	0.7294	4	44	600	0.15	0.03	0.7294
5	42	800	0.15	0.03	0.9067	5	42	800	0.15	0.03	0.9067
6	43	800	0.15	0.03	0.8708	6	43	800	0.15	0.03	0.8708
7	40	600	0.1	0.03	0.8199	7	40	400	0.1	0.03	0.9381
8	41	600	0.15	0.04	0.9496	8	40	800	0.1	0.03	0.7328
9	42	600	0.15	0.04	0.9067	9	41	400	0.1	0.03	0.9431
10	43	600	0.15	0.04	0.8708	10	42	400	0.1	0.03	0.9477
11	40	400	0.15	0.04	0.7328	11	43	400	0.1	0.03	0.9521
12	45	400	0.15	0.04	0.7621	12	44	400	0.1	0.03	0.9560
13	40	1000	0.1	0.03	0.7638	13	45	400	0.1	0.03	0.9597
14	44	600	0.15	0.04	0.8406	14	40	400	0.05	0.03	0.9999
15	40	400	0.1	0.04	0.8602						
16	40	800	0.1	0.04	0.8155						
17	42	400	0.1	0.04	0.8757						
18	45	400	0.1	0.04	0.8962						
19	43	400	0.1	0.04	0.8829						
20	44	400	0.1	0.04	0.8898						
21	40	600	0.1	0.04	0.7328						
22	44	600	0.1	0.04	0.7554						
23	40	400	0.05	0.05	0.9692						
24	41	400	0.1	0.04	0.8682						
25	41	400	0.05	0.05	0.9725						
26	42	400	0.05	0.05	0.9754						
27	43	400	0.05	0.05	0.9780						
28	44	400	0.05	0.05	0.9805						
29	42	600	0.05	0.05	0.8950						
30	45	400	0.05	0.05	0.9827						
31	40	800	0.05	0.05	0.7831						
32	44	800	0.05	0.05	0.8164						
33	40	600	0.05	0.05	0.8806						
38	44	600	0.05	0.05	0.9078						
35	41	600	0.05	0.05	0.8880						
36	43	600	0.05	0.05	0.9016						
37	40	400	0.05	0.04	0.9909						
38	45	400	0.05	0.04	0.9967						

Table 13. Performance of SMED and our method with negative DRPCS and $\gamma=5$.

Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	SMED Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Our Method Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	45	400	0.15	0.03	0.8602	2	45	400	0.15	0.03	0.8602
3	45	400	0.15	0.04	0.7621	3	45	400	0.15	0.04	0.7621
4	40	400	0.15	0.03	0.8199	4	40	400	0.15	0.03	0.8199
5	44	1200	0.1	0.03	0.8406	5	44	1200	0.1	0.03	0.8406
6	44	1000	0.1	0.03	0.7295	6	44	1000	0.1	0.03	0.7295
7	44	600	0.15	0.03	0.7294	7	44	600	0.15	0.03	0.7294
8	44	400	0.15	0.03	0.8528	8	44	400	0.15	0.03	0.8528
9	45	400	0.15	0.05	0.7268	9	45	400	0.15	0.05	0.7268
10	41	1200	0.1	0.03	0.9496	10	41	1200	0.1	0.03	0.9496
11	42	1200	0.1	0.03	0.9067	11	42	1200	0.1	0.03	0.9067
12	43	1200	0.1	0.03	0.8708	12	43	1200	0.1	0.03	0.8708
13	40	1000	0.05	0.03	0.8806	13	40	1000	0.05	0.03	0.8806
14	44	1000	0.05	0.03	0.9078	14	44	1000	0.05	0.03	0.9078
15	43	1000	0.05	0.03	0.9016	15	43	1000	0.05	0.03	0.9016
16	44	800	0.05	0.03	0.9560	16	44	800	0.05	0.03	0.9560
17	43	800	0.05	0.03	0.9521	17	43	800	0.05	0.03	0.9521
18	42	800	0.05	0.03	0.9477	18	42	800	0.05	0.03	0.9477
19	40	800	0.05	0.03	0.9381	19	40	800	0.05	0.03	0.9381
20	41	800	0.05	0.03	0.9431	20	41	800	0.05	0.03	0.9431
21	43	600	0.05	0.03	0.9877	21	43	600	0.05	0.03	0.9877
22	44	600	0.05	0.03	0.9894	22	44	600	0.05	0.03	0.9894
23	42	600	0.05	0.03	0.9859	23	42	600	0.05	0.03	0.9859
24	41	600	0.05	0.03	0.9838	24	41	600	0.05	0.03	0.9838
25	40	600	0.05	0.03	0.9815	25	40	600	0.05	0.03	0.9815
26	42	400	0.05	0.03	0.9999	26	42	400	0.05	0.03	0.9999

Table 14. Performance of SMED and our method with positive DRPCS and $\gamma = 5$.

Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	SMED Hatch Distance (mm)	Laser Thickness (mm)	Relative Density	Experiment Number	Laser Power (W)	Laser Velocity (mm/s)	Our Method Hatch Distance (mm)	Laser Thickness (mm)	Relative Density
1	44	800	0.15	0.03	0.8406	1	44	800	0.15	0.03	0.8406
2	44	2000	0.05	0.03	0.7295	2	44	2000	0.05	0.03	0.7295
3	44	1800	0.05	0.04	0.8406	3	45	400	0.15	0.05	0.7268
4	45	400	0.15	0.05	0.7268	4	45	400	0.15	0.03	0.8602
5	44	1400	0.05	0.05	0.8098	5	40	400	0.15	0.03	0.8199
6	44	1200	0.1	0.03	0.8406	6	44	1200	0.1	0.03	0.8406
7	44	1600	0.05	0.04	0.7481	7	45	400	0.1	0.03	0.9597
8	45	400	0.15	0.03	0.8602	8	44	400	0.1	0.03	0.9560
9	40	400	0.15	0.03	0.8199	9	42	400	0.1	0.03	0.9477
10	45	400	0.1	0.03	0.9597	10	43	400	0.1	0.03	0.9521
11	44	400	0.1	0.03	0.9560	11	40	400	0.1	0.03	0.9381
12	42	400	0.1	0.03	0.9477	12	41	400	0.1	0.03	0.9431
13	43	400	0.1	0.03	0.9521	13	44	600	0.1	0.03	0.8528
14	40	400	0.1	0.03	0.9381	14	42	400	0.05	0.03	0.9999
15	41	400	0.1	0.03	0.9431						
16	44	600	0.1	0.03	0.8528						
17	44	400	0.05	0.03	0.9997						