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A numerical model of fibrous filters containing deposit.

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Abstract

This work is aimed at developing numerical methods to model the effects that particle deposit collected by fibrous filters has upon the flow field within the filter and hence upon further deposition. A numerical model of the flow field has been developed using the Boundary Element Method. The model contains a two dimensional single fibre with the boundary conditions applied on the computational domain accounting for the neighbouring fibres. A layer of porous material is assumed to cover the fibre modelling deposited particulate. The width of the porous layer, which varies along the fibre surface, is defined by particle behaviour determined in earlier work, Dunnett and Clement ¹. Once the flow field is known the motion of the particles in the flow is modelled. Small particles are considered where diffusion is the main mechanism by which they are captured by the fibres. Various situations are considered and the flow and particle behaviour investigated. In particular the effect of existing porous deposit on the fibres upon further deposition is studied.

Keywords: Boundary Element Method, fibrous filters, porous deposit

Introduction

Filtration by fibrous filters has numerous applications, protecting employees from airborne particulate in the form of face masks or larger scale filtration units, improving air quality in transport etc. Such filters generally consist of many threadlike fibres oriented

more or less normal to the fluid flow which passes through them. Some of the particles which are carried by the fluid flow are removed by collision and attachment to the fibres. Neglecting the influence of electrical effects the main mechanisms by which particles are removed are interception, impaction, diffusion and gravitational settling. The dominant mechanism for any application will depend upon the size of the particles. The collected particles accumulate forming complex structures which influence the fluid flow and further deposition. Eventually the filter will clog leading to regeneration or replacement. In order to understand and predict the performance of a filter it is necessary to have a good understanding of the fluid flow within the filter. Much work has been undertaken on this problem, in particular for clean filters, see for example the early works of Kuwabara² and Happel³ where 'cell' models were developed to describe the fluid flow. In these models the equations of the fluid flow are solved in a two dimensional 'cell', surrounding the single fibre, where the boundary conditions take into account neighbouring fibres. These have been widely adopted over the years. Other techniques have also been applied to solving the fluid flow, such as the control volume method, see Rao and Faghri⁴ and Dhaniyala and Liu⁵ and the Boundary Element Method, see Hildyard et al⁶, Ingham et al⁷. Once the flow is determined the transport of particles can be modelled and their possible removal by the fibres investigated. As for the fluid flow, extensive research has been undertaken into particle removal for clean fibres, see for example Fardi and Lui⁸, Ingham et al⁹, Pich¹⁰, Stechkina and Fuchs¹¹, Zhu et al¹², Asgharian and Chen¹³. Due to this research the performance of a clean fibre is reasonably well understood. However the case of a loaded filter has received less attention. Experimental work in this area includes Japuntich et al¹⁴ and Thomas et al¹⁵ where the penetration and pressure drop of various filters were considered under loading. More recently a group of researchers at Karlsruhe University have undertaken a study of the structure of deposit structures on single fibres and the effect the deposition has upon efficiency, see Hoferer et al¹⁶, Schollmeier et al^{17,18}. Modelling work in this area includes Stenhouse and Trottier¹⁹ and Stenhouse et al²⁰ where semi-empirical models of fibre efficiency as a function of deposit were developed. These models are restricted to specific cases and based on limited experimental data. The dendritic structures which form on the fibre surface were modelled by Payatakes and colleagues, Payatakes et al²¹, Payatakes and Gradon²² however these models are limited

due to the neglect of the effects of the dendrite upon the flow field. Other studies which attempt to take into account the change in the flow field due to the particle deposit are restricted due to their considerable use of computing time and resources, see Jung and Tien²³, Biggs et al²⁴, Karadimos and Ocone²⁵ and Przekop et al²⁶. More recently Kirsh²⁷ developed a mathematical model of Stokes flow in a model filter consisting of parallel cylinders with porous permeable shells. Although this work models the deposit as a porous layer, the same approach as adopted in this work, it is restricted in its use. This is because the model only considers layers of a constant width around the fibre which is not generally a realistic deposit. Also the model involves cumbersome mathematical expressions alongside numerical coefficients which would make any detailed study of particle motion time consuming.

Other past work which is relevant to the present study is the modelling of flow systems comprising of a coupled free/porous flow. Such flows have many practical applications, for example, seepage from streams bounded by porous banks, gas-cooled nuclear reactors where the nuclear fuel is separated from the coolant by a porous media, as well as filtration. As the two distinct flow regions, porous and free flow, are described by different equations and the coupling at the interface is not straightforward this subject has received considerable attention. In general there have been two main approaches to solving for the flow characteristics, a single domain, and a two domain, approach. In the single domain approach the porous layer is assumed to be a pseudo-fluid and the combined free flow and porous regimes are considered as a single flow domain. This results in a single momentum equation, a modified Navier Stokes equation with an additional Darcy term. The transition from the free to porous flow regime, or vice versa, is achieved through spatial variation of properties such as the permeability. This approach has been adopted by many authors, see Beckerman et al²⁸ and Bhattacharyya et al²⁹ for example, as, although the governing equations are more complex than for the two domain approach, no explicit boundary conditions are necessary on the interface between the free and porous flow. In the two domain approach the free flow region and the porous medium are considered as two distinct domains with distinct flow characteristics. The fluid flow in the free flow region is modelled by the Navier-Stokes, or Stokes, equation,

depending on the flow regime studied, and in the porous region by the Darcy equation, equation (1).

$$\nabla p = -\frac{\mu}{k} \underline{U} \quad (1)$$

Where k is the permeability of the porous region. These are then solved with appropriate boundary conditions applied on the interface. When a viscous fluid flows past a porous medium the effect of the viscous shear can penetrate beneath the surface and form a sort of boundary layer in the porous region whose size is of the order of $\sqrt{k}$. One principal difficulty in formulating the correct boundary conditions is that the Darcy equation is not compatible with the existence of such a region as it has no shear stress term associated with it. One way to overcome this is to use the Brinkman equation, equation (2), instead of the Darcy equation, see Haber and Mauri³⁰.

$$\nabla p = \tilde{\mu} \nabla^2 \underline{U} - \frac{\mu}{k} \underline{U} \quad (2)$$

In (2) $\tilde{\mu}$ is an effective viscosity which is identical to μ for high porosity media. The higher order of equation (2), compared with equation (1), allows for continuity of both velocity and stress on the boundary. However the complexity of the mathematical fluid dynamics problem has been increased. Due to the formation of the boundary layer equation (2) is only applicable to high permeability materials. An alternative method is to solve equation (1) in the porous region with additional assumptions on the interface boundary, Vainshtein et al³¹. One of these is a linear relationship between the slip velocity in the free flow and its derivative normal to the surface, Beavers and Joseph³².

$$\frac{\partial u_f}{\partial y} = \frac{\alpha_{BJ}}{\sqrt{k}} (u_f - u_m) \quad (3)$$

Where u_f is the velocity of the freely flowing fluid calculated at the interface and u_m is the seepage velocity measured inside the porous region, The constant α_{BJ} is dependent upon the structure of the porous medium. This relationship was determined from experimental work measuring the mass efflux of a Poiseuille flow over a naturally permeable block, which confirmed the formation of the boundary layer formed close to the interface. The validity of the condition has been widely supported, see Nield and Bejan³³.

The numerical solution of coupled free/porous flow problems has generally been performed using finite element methods, for example Salinger et al³⁴, Gartling et al³⁵ and Costa et al³⁶. However the Boundary Element Method (BEM) is an ideal method to apply to the problem as it only requires the boundaries of the 2 regions to be divided into elements in order to solve throughout the flow domain.

Hence, in this work a mathematical model is developed, using the Boundary Element Method, which models the deposit collected on a fibre as a porous layer. In this initial study of the effects of the porous deposit upon the flow behaviour within a filter the flow behaviour within the porous media is described by Darcy's law, equation (1). This is not believed to be restrictive for the situations considered as for the low permeabilities of the porous media considered the size of the boundary layer within the media is negligible. Using the method adopted the width of the layer can vary around the fibre, modelling real life situations. Considering particles for which diffusion is the main mechanism of deposit, the rate at which they deposit upon a clean fibre is determined. It is then possible to determine the shape formed by the surface of deposited particles, this is treated as porous and the flow field and particle behaviour recalculated. In this way the deposit structure formed as loading increases can be determined. This has been investigated for various values of the relevant parameters.

Formulation

In this work creeping flow through an array of fibres containing porous deposit is modelled. The fibres are modelled as infinitely long cylinders of the same size, parallel to each other and with their axis perpendicular to the flow. Hence the flow is two dimensional and the motion takes place in the plane perpendicular to the cylinder axis. Using a similar approach to that of Kuwabara² the cylinders are assumed to be staggered and a region representative of the filter as a whole considered. This region consists of a single fibre (cylinder), radius r_0 , enclosed by a coaxial cylinder of radius h_1 . Boundary conditions are imposed on the outer cylinder to account for neighbouring fibres. The fibre is assumed to have a layer of porous deposit on its surface. Hence, due to symmetry, the domain considered is that shown in Figure 1. As the width of the deposit is not necessarily constant around the fibre surface the flow field is not symmetrical front to

back. The distance h_1 is defined by the condition that the packing fraction of the fibre within this cylinder is identical to that of the fibres within the filter as a whole. This results in the condition $h_1 = \frac{r_0}{\sqrt{c}}$, where the packing fraction of the filter, c , is defined as the fraction of the perceived volume of the filter that is actually occupied by fibres. There are two regions to the flow field, region I outside the porous layer where the equations of motion, assuming that viscosity is dominant, are given by:

$$\begin{aligned}\nabla p &= \mu \nabla^2 \underline{U} \\ \nabla \cdot \underline{U} &= 0\end{aligned}\tag{4}$$

where p is the fluid pressure and μ the fluid viscosity. The assumption that viscosity is dominant implies that the Reynolds number of the flow, $Re = \frac{2r_0\rho U_0}{\mu}$, is small, where ρ is the density of the air.

The flow inside the porous layer, region II, is described by Darcy's equation, equation (1).

Introducing the stream function ψ the components of fluid velocity u_r and u_θ in the r and θ directions respectively are given by:

$$u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta} \quad \text{and} \quad u_\theta = -\frac{\partial \psi}{\partial r}\tag{5}$$

In terms of ψ equations (4) and (1) become:

$$\nabla^4 \psi_f = 0\tag{6}$$

and

$$\nabla^2 \psi_p = 0\tag{7}$$

respectively. The subscripts f and p refer to the regions in which the equations apply, fluid, region I, and porous, region II.

Considering region I, where ψ_f satisfies equation (6), this can be written in its coupled equivalent form,

$$\begin{aligned}\nabla^2 \psi_f &= \omega_f \\ \nabla^2 \omega_f &= 0\end{aligned}\tag{8}$$

Where ω_f is the fluid vorticity which is a measure of the tendency of the fluid to rotate or to cause rotation of suspended bodies during flow.

In order to solve equations (8) at any point $\underline{p}$ in the two dimensional domain S_f bounded by the contour C_f the equivalent pair of coupled integral equations

$$\begin{aligned}\eta(\underline{p})\psi_f(\underline{p}) &= \int_{C_f} \left\{ \psi_f(\underline{q})G'_1(\underline{p},\underline{q}) - \psi'_f(\underline{q})G_1(\underline{p},\underline{q}) + \omega_f(\underline{q})G'_2(\underline{p},\underline{q}) - \omega'_f(\underline{q})G_2(\underline{p},\underline{q}) \right\} d\mathbf{q} \\ \eta(\underline{p})\omega_f(\underline{p}) &= \int_{C_f} \left\{ \omega_f(\underline{q})G'_1(\underline{p},\underline{q}) - \omega'_f(\underline{q})G_1(\underline{p},\underline{q}) \right\} d\mathbf{q}\end{aligned}\tag{9}$$

may be used. Where

(i) $\underline{p} \in S_f \cup C_f$, $\underline{q} \in C_f$

(ii) A prime denotes differentiation with respect to the outward normal to S_f at $\underline{q} \in C_f$.

(iii) $d\mathbf{q}$ is the differential increment of C_f at $\underline{q} \in C_f$.

(iv) $\eta(\underline{p})$ is defined by :

$$\eta(\underline{p}) = \left\{ \begin{array}{ll} 0 & \text{if } \underline{p} \notin S_f \cup C_f \\ 2\pi & \text{if } \underline{p} \in S_f \\ \text{Internal angle included between the tangents to } C_f \text{ on either side of } \underline{p} \text{ if } \underline{p} \in C_f \end{array} \right\}$$

$$(v) \ G_1(\underline{p},\underline{q}) = \ln|\underline{p}-\underline{q}|, \quad G_2(\underline{p},\underline{q}) = \frac{1}{4}|\underline{p}-\underline{q}|^2 \left\{ \ln|\underline{p}-\underline{q}| - 1 \right\}$$

In this problem the contour C_f is the boundary ABCD shown in figure 1 and S_f is the region enclosed by the boundary.

Using the numerical approach described in Kelmanson³⁷ the contour C_f is divided into N_f straight line segments $C_{f_1} \dots C_{f_{N_f}}$, such that

$$C_f = \bigcup_{j=1}^{N_f} C_{f_j}\tag{10}$$

On each of the C_{f_j} it is assumed that $\psi_f, \psi'_f, \omega_f$ and ω'_f have the piecewise constant value of $\psi_{f_j}, \psi'_{f_j}, \omega_{f_j}$ and ω'_{f_j} respectively. Where they take their values at the midpoint of the segment. Equations (9) are then approximated by:

$$\eta(\underline{p})\psi_f(\underline{p}) = \sum_{j=1}^{N_f} \left\{ \psi_{f_j} \int_{C_{f_j}} G'_1(\underline{p}, \underline{q}) d\mathbf{q} - \psi'_{f_j} \int_{C_{f_j}} G_1(\underline{p}, \underline{q}) d\mathbf{q} + \omega_{f_j} \int_{C_{f_j}} G'_2(\underline{p}, \underline{q}) d\mathbf{q} - \omega'_{f_j} \int_{C_{f_j}} G_2(\underline{p}, \underline{q}) d\mathbf{q} \right\} \quad (11)$$

$$\eta(\underline{p})\omega_f(\underline{p}) = \sum_{j=1}^{N_f} \left\{ \omega_{f_j} \int_{C_{f_j}} G'_1(\underline{p}, \underline{q}) d\mathbf{q} - \omega'_{f_j} \int_{C_{f_j}} G_1(\underline{p}, \underline{q}) d\mathbf{q} \right\}$$

Applying equations (11) at the midpoint $\underline{p}=\underline{q}_i, i=1 \dots N_f$, of each interval they reduce to the coupled system of equations:

$$\sum_{j=1}^{N_f} \{A_{ij}\psi_{f_j} + B_{ij}\psi'_{f_j} + C_{ij}\omega_{f_j} + D_{ij}\omega'_{f_j}\} = 0 \quad i = 1 \dots N_f \quad (12)$$

$$\sum_{j=1}^{N_f} \{A_{ij}\omega_{f_j} + B_{ij}\omega'_{f_j}\} = 0$$

Where

$$A_{ij} = \int_{C_{f_j}} G'_1(\underline{q}_i, \underline{q}) d\mathbf{q} - \eta_j \delta_{ij}$$

$$B_{ij} = - \int_{C_{f_j}} G_1(\underline{q}_i, \underline{q}) d\mathbf{q}$$

$$C_{ij} = \int_{C_{f_j}} G'_2(\underline{q}_i, \underline{q}) d\mathbf{q}$$

$$D_{ij} = - \int_{C_{f_j}} G_2(\underline{q}_i, \underline{q}) d\mathbf{q} \quad (13)$$

δ_{ij} is the Kronecker delta, $\eta_i = \eta(\underline{q}_i)$.

Analytic expressions are available for the integrals given in equation (13), see Kelmanson³⁷. Hence if any two of $\psi_f, \psi'_f, \omega_f$ and ω'_f are known at each point on the boundary of region I then equations (12) can be used to determine the unknowns on the

boundary. Equations (11) can then be applied to determine ψ_f and ω_f at any point in the region. However the boundary conditions do not give enough information to make that possible. From the physical problem it is known that DA and BC are symmetry boundaries hence in terms of ψ_f and ω_f the conditions are:

$$i) \quad \psi_f=0, \omega_f=0 \text{ on DA and BC} \quad (14a)$$

The outer boundary CD is the boundary of the cell where the conditions imposed account for the presence of the surrounding fibres. For clean fibres, the two principle single fibre theories, Happel³ and Kuwabara², differ in the conditions imposed. The former requires the tangential stress to vanish on the boundary and the latter requires the vorticity to vanish. In this work the condition imposed by Kuwabara² is used as it has been found experimentally to accurately describe the flow field, Brown³⁸, and numerically to be valid for small Reynolds number flows, Liu and Wang³⁹. The condition is valid for arrays of cylinders parallel to each other and directed normal to the flow. For other situations different boundary conditions would need to be imposed. It is also assumed that the flow is moving with the mean velocity U_0 on CD, see Fig. 1, hence

$$ii) \quad \psi_f=U_0 r \sin\theta, \omega_f=0 \text{ on CD} \quad (14b)$$

The inner boundary AB is the interface boundary between the porous region and the fluid region and is considered later.

Considering region II. In this case ψ_p satisfies Laplace's equation, equation (7). In order to solve this equation at any point $\underline{p}$ in the two dimensional domain S_p bounded by the contour C_p , Green integral formula (15) may be used.

$$\eta(\underline{p})\psi_p(\underline{p}) = \int_{C_p} \{ \psi_p(\underline{q})G'_3(\underline{p}, \underline{q}) - \psi'_p(\underline{q})G_3(\underline{p}, \underline{q}) \} d\mathbf{q} \quad (15)$$

Where $G_3 = \ln|\underline{p} - \underline{q}|$. In order to solve equation (15), as before the contour C_p is divided

into a number N_p of subintervals $C_{p_1} \dots C_{p_{N_p}}$ such that $C_p = \bigcup_{j=1}^{N_p} C_{p_j}$. On each of the

C_{p_j} it is assumed that ψ_p and ψ'_p have the piecewise constant value of ψ_{p_j} and ψ'_{p_j} respectively, taken at the midpoints of the segments. Taking $\underline{p}$ at the midpoint $\underline{q}_i$, $i=1 \dots N_p$, of each interval gives:

$$\sum_{j=1}^{N_p} \{F_{ij}\psi_{p_j} + H_{ij}\psi'_{p_j}\} = 0 \quad i=1----N_p \quad (16)$$

Where

$$F_{ij} = \int_{C_{p_j}} G'_3(\underline{q}_i, \underline{q}) d\mathbf{q} - \eta_j \delta_{ij} \quad (17)$$

$$H_{ij} = - \int_{C_{p_j}} G_3(\underline{q}_i, \underline{q}) d\mathbf{q}$$

Hence if ψ_p or ψ'_p are known at all points on the boundary, equation (16) can be used to determine the unknowns on the boundary. Equation (15) can then be applied to determine ψ_p at any point in the domain. From the physical problem it is known that AE and FB are symmetry boundaries and that EF is the fibre surface. The boundary AB is the interface boundary. This gives the conditions:

$$\psi_p = 0 \text{ on AE, EF and FB} \quad (18)$$

Enforcing the streamfunction to be constant on the fibre surface EF ensures there is no flow across the surface however due to the order of equation (7) describing the fluid flow within the porous region, it is not possible to enforce the no slip condition there .

Therefore, there will be inaccuracies in the flow very close to the fibre surface. However it is not expected that these inaccuracies will significantly affect the flow outside the deposit.

Equations (12) and (16) give $2N_f + N_p$ equations in the variables ψ_f , ψ'_f , ω_f , ω'_f , ψ_p and ψ'_p . Using the boundary information given in (14) and (18) there are $2N_f + N_p + 3N_1$ unknowns, where N_1 is the number of elements on the interface boundary AB. These are:

ψ'_f and ω'_f on DA, BC and CD.

ψ'_p on AE, EF and FB.

ψ_f , ψ'_f , ω_f , ω'_f , ψ_p and ψ'_p on AB.

Hence in order to solve the problem more information is needed on AB.

Physically it is known that:

- i) The normal velocity must be continuous across the boundary
- ii) The pressure gradient along the boundary is continuous

The third condition is on the tangential velocity at the boundary. Experiments have shown, Beavers and Joseph³², that the velocity of the freely flowing fluid at the interface is greater than the velocity of the fluid in the porous medium. In that work a slip wall interface condition was proposed, equation (3), which has been widely used and validated. A generalisation of that condition, see Nield and Bejan³³, has been assumed here. Hence the conditions to be applied on AB are:

$$\text{i)} \quad \frac{\partial \psi_f}{\partial T} = \frac{\partial \psi_p}{\partial T} \quad (19)$$

$$\text{ii)} \quad \frac{\partial p}{\partial T} \text{ is continuous} \quad (20)$$

$$\text{iii)} \quad \left(\frac{\partial u_T}{\partial N} + \frac{\partial u_N}{\partial T} \right)_f = \frac{\alpha_{BJ}}{\sqrt{k}} \left((u_T)_f - (u_T)_p \right) \quad (21)$$

Where T is the tangent to the boundary, N is the normal to the boundary and α_{BJ} is a dimensionless slip parameter which depends on the material parameters that characterize the structure of the permeable material.

Considering condition (ii), equation (2) gives $\frac{\partial p}{\partial T} = \mu \nabla^2 u_T$ on the fluid side of the boundary and equation (3) gives $\frac{\partial p}{\partial T} = -\frac{\mu}{k} u_T$ on the porous side, where u_T is the fluid velocity in the tangential direction. This results in the condition

$$\frac{\psi'_p}{k} = \omega'_f \quad (22)$$

on AB.

Equation (21) results in the condition:

$$\psi'_p = -\psi'_f + \frac{\sqrt{k}}{\alpha_{BJ}} \left(-\frac{\partial^2 \psi}{\partial N^2} + \frac{\partial^2 \psi}{\partial T^2} \right)_f \quad (23)$$

on AB. In order to apply this condition it is necessary to express the terms inside the brackets in terms of ψ_f , ψ'_f , ω_f and ω'_f .

Earlier work, Dunnett and Clement¹, found that for a certain range of the parameters κ and Pe, the surface formed by the early stages of deposit can be approximated by

$$r = r_0 \left[1 + \lambda (1 - a\beta^p)^b \right] \quad (24)$$

where r_0 is the radius of the clean fibre, λ is the thickness of the deposit on the fibre front divided by r_0 and a , p and b are fitted constants. The angle β is measured from the front of the fibre and is given by $\beta = \pi - \theta$. The parameter κ is the ratio of particle and fibre diameter and Pe is the Peclet number given by $Pe = \frac{2U_0 r_0}{D}$ with U_0 the mean flow velocity and D the coefficient of diffusion of the particles. Hence equation (24) has been taken here to define the boundary AB.

Writing $\frac{\partial^2 \psi}{\partial N^2}$ in equation (23) in terms of $\nabla^2 \psi$ gives

$$\psi'_p = -\psi'_f + \frac{\sqrt{k}}{\alpha_{BJ}} \left(-F(\beta) \nabla^2 \psi + 2 \frac{\partial^2 \psi}{\partial T^2} - G(\beta) \psi' + H(\beta) \frac{\partial \psi}{\partial T} \right)_f \quad (25)$$

Where F, G and H are known functions of β . Using (8) and (19) this gives

$$\psi'_p = -\psi'_f + \frac{\sqrt{k}}{\alpha_{BJ}} \left(-F(\beta) \omega_f + 2 \frac{\partial^2 \psi_p}{\partial T^2} - G(\beta) \psi'_f + H(\beta) \frac{\partial \psi_p}{\partial T} \right) \quad (26)$$

In order to apply this equation the partial derivatives wrt T are approximated using finite differences so that the right hand side of expression (26) only contains ψ_p , ψ'_f , ω_f on the surface.

The conditions given above on AB are used to reduce the unknowns there to ψ_p , ψ'_f and ω_f by using equation (19) to express ψ_f in terms of ψ_p , equation (22) to express ω'_f in terms of ψ'_p and equation (26) to express ψ'_p in terms of ψ'_f , ω_f and ψ_p

Considering equation (16), this can be written as:

$$\sum_{AB} \left\{ F_{ij} \psi_{p_j} + H_{ij} \psi'_{p_j} \right\} + \sum_{BF EA} \left\{ F_{ij} \psi_{p_j} + H_{ij} \psi'_{p_j} \right\} = 0 \quad i=1 \dots N_p \quad (27)$$

Applying the boundary conditions gives:

$$\sum_{AB} \left\{ F_{ij} \psi_{p_j} + H_{ij} \left(-\psi'_{f_j} + \frac{\sqrt{k}}{\alpha_{BJ}} \left(-F(\beta) \omega_{f_j} + 2 \frac{\partial^2 \psi_{p_j}}{\partial T^2} - G(\beta) \psi'_{f_j} + H(\beta) \frac{\partial \psi_{p_j}}{\partial T} \right) \right) \right\}_{i=1 \dots N_p} \\ + \sum_{BFEA} H_{ij} \psi'_{p_j} = - \sum_{BFEA} F_{ij} \psi_{p_j} \quad (28)$$

The partial derivatives wrt T are expressed in terms of ψ_{p_j} and neighbouring values.

Considering equation (12), taking into account the boundary information, these can be written as:

$$\sum_{AB} \{ A_{ij} \psi_{f_j} + B_{ij} \psi'_{f_j} + C_{ij} \omega_{f_j} + D_{ij} \omega'_{f_j} \} + \sum_{BCDA} \{ B_{ij} \psi'_{f_j} + D_{ij} \omega'_{f_j} \} = - \sum_{BCDA} \{ A_{ij} \psi_{f_j} + C_{ij} \omega_{f_j} \} \\ i = 1 \dots N_f \quad (29)$$

$$\sum_{AB} \{ A_{ij} \omega_{f_j} + B_{ij} \omega'_{f_j} \} + \sum_{BCDA} \{ B_{ij} \omega'_{f_j} \} = - \sum_{BCDA} \{ A_{ij} \omega_{f_j} \}$$

Using (21) and (26) these become:

$$\sum_{AB} \left\{ A_{ij} \psi_{f_j} + B_{ij} \psi'_{f_j} + C_{ij} \omega_{f_j} + \frac{D_{ij}}{k} \left(-\psi'_{f_j} + \frac{\sqrt{k}}{\alpha_{BJ}} \left(-F(\beta) \omega_{f_j} + 2 \frac{\partial^2 \psi_{p_j}}{\partial T^2} - G(\beta) \psi'_{f_j} + H(\beta) \frac{\partial \psi_{p_j}}{\partial T} \right) \right) \right\} \\ + \sum_{BCDA} \{ B_{ij} \psi'_{f_j} + D_{ij} \omega'_{f_j} \} = - \sum_{BCDA} \{ A_{ij} \psi_{f_j} + C_{ij} \omega_{f_j} \} \quad i = 1 \dots N_f \quad (30)$$

and

$$\sum_{AB} \left\{ A_{ij} \omega_{f_j} + \frac{B_{ij}}{k} \left(-\psi'_{f_j} + \frac{\sqrt{k}}{\alpha_{BJ}} \left(-F(\beta) \omega_{f_j} + 2 \frac{\partial^2 \psi_{p_j}}{\partial T^2} - G(\beta) \psi'_{f_j} + H(\beta) \frac{\partial \psi_{p_j}}{\partial T} \right) \right) \right\} + \\ \sum_{BCDA} \{ B_{ij} \omega'_{f_j} \} = - \sum_{BCDA} \{ A_{ij} \omega_{f_j} \} \quad i = 1 \dots N_f \quad (31)$$

Approximating the partial derivatives using finite differences and using equation (19),

equations (30) and (31) are written in terms of the unknowns ψ_p , ψ'_f and ω_f on AB,

ψ'_f and ω'_f on BCDA and the knowns ψ_f and ω_f on BCDA . Hence equations (28), (30)

and (31) give $2N_f + N_p$ equations in the $2N_f + N_p$ unknowns:

ψ_p , ψ'_f and ω_f on AB

ψ'_p on BFEA

ψ'_f and ω'_f on BCDA

These were solved and then the complete boundary information on AB obtained, using equation (19) to obtain ψ_f , equation (26) to obtain ψ'_p , and equation (22) to obtain ω'_f .

Before making any calculations all quantities were non-dimensionalised, distances with respect to fibre radius, r_0 , and velocities with respect to the mean flow velocity, U_0 .

Once all the boundary information for the two regions I and II is available equations (11) can be used to determine ψ_f and ω_f anywhere in region I, outside the porous layer and equation (15) can be used to determine ψ_p anywhere within the porous layer, region II. Equations (5) gives the flow velocity in regions I and II. Once the flow velocities are known the motion of particles within the flow can be determined.

Particle motion

In this work small particles are considered so that the major mechanism by which they are captured is diffusional deposition, in which the combined effect of the air motion and the Brownian motion of the particle brings it into contact with a fibre. In this case, for steady state conditions and neglecting the influence of electric effects, the particle concentration, n , is described by equation (24)

$$u_r \frac{\partial n}{\partial r} + \frac{u_\theta}{r} \frac{\partial n}{\partial \theta} = D \left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} + \frac{1}{r^2} \frac{\partial^2 n}{\partial \theta^2} \right) \quad (32)$$

Where D is the coefficient of diffusion of the particles and u_r and u_θ are the components of fluid velocity in the r and θ directions respectively in either region I, clear fluid, or region II, porous region. Non-dimensionalising all quantities equation (24) becomes:

$$u_r \frac{\partial n}{\partial r} + \frac{u_\theta}{r} \frac{\partial n}{\partial \theta} = \frac{2}{Pe} \left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} + \frac{1}{r^2} \frac{\partial^2 n}{\partial \theta^2} \right) \quad (33)$$

Where all quantities in the equation are non-dimensional and Pe is the Peclet number, which is a measure of the relative magnitude of the diffusional motion of the particles and the convective motion of the air past the fibre. The particle concentration has been non-dimensionalised with respect to n_0 , the particle concentration at the cell boundary. For

fibrous filtration the third term on the right hand side of equation (25) is much smaller than the other terms and hence has been neglected. The problem of determining the particle concentration hence reduces to solving

$$u_r \frac{\partial n}{\partial r} + \frac{u_\theta}{r} \frac{\partial n}{\partial \theta} = \frac{2}{Pe} \left(\frac{\partial^2 n}{\partial r^2} + \frac{1}{r} \frac{\partial n}{\partial r} \right) \quad (34)$$

subject to the appropriate boundary conditions. The boundary condition for the non-dimensional particle concentration n , on the cell boundary is $n=1$. For this to be true it is

necessary that $Pe \gg c^{3/2}$ as particles diffuse to the fibre from a layer adjacent to its surface, and in order for the condition at the cell boundary to be true it is necessary that the non dimensional thickness of this layer, δ , is a lot smaller than the non-dimensional cell radius, h_1 . It is known that $\delta = O\left(Pe^{-1/3}\right)$, see Stechkina and Fuchs¹⁰ and $h_1 = c^{-1/2}$

and hence the condition that $\delta \ll h_1$ leads to the restriction on Pe stated above. The condition $n=0$ at the interception radius must also be satisfied as it is assumed that particles adhere to the porous surface formed by the previously deposited particles and are not able to travel through it. The interception radius is given by $r_s + \kappa$, where r_s is the non-dimensional distance from the fibre centre to the surface formed by the fibre + deposit

and $\kappa = \frac{r_p}{r_0}$ where r_p is the particle radius. In order to solve equation (26) it is necessary

to apply a third boundary condition. This was done in a similar way to Stechkina and Fuchs¹⁰. Along the line $\theta=\pi$, $u_\theta=0$, and hence equation (26) reduces to an ordinary differential equation:

$$\frac{2}{Pe} \frac{d^2 n}{dr^2} + \frac{dn}{dr} \left(\frac{2}{r \cdot Pe} - u_r \right) = 0 \quad (35)$$

This was solved numerically by approximating the derivatives using finite differences and determining u_r using the BEM method described earlier. This resulted in values of n at finite points along the line $\theta=\pi$. With this information equation (26) was then solved numerically using finite differences to give values of n at finite points in the domain of interest, the region outside the surface formed by previously deposited particles. It is assumed that particles are attached to the porous boundary if they impact with it.

The rate at which particles are deposited on a surface by diffusion is proportional to the concentration gradient $\frac{\partial n}{\partial N}$ on the surface, where N is the normal to the surface. Hence in this work $\frac{\partial n}{\partial N}$ on the deposit surface is investigated for various situations. Also once $\frac{\partial n}{\partial N}$ is known the new surface formed by depositing particles can be determined for a certain time interval. This new surface then defines the boundary of the porous layer and the flow field can be recalculated using the process described in the previous section. In this way the shape of the surface can be investigated as deposition increases.

Results

Considering initially the flowfield, in order to solve the equations of motion for the two regions it is necessary to impose a value for α_{BJ} , the empirical constant in the boundary condition (21). In the original work of Beavers and Joseph³² it was stated that this quantity is independent of the fluid viscosity but is dependent upon the material parameters that characterise the structure of the porous media. In their work various porous media were investigated and α_{BJ} was found to range from 0.1 to 4 with the value increasing with an increase in permeability. This behaviour was confirmed in a later numerical study by Sahraoui and Kaviany⁴⁰ where α_{BJ} was also found to depend on the extent of the plain medium, the flow direction and the Reynolds number based on the size of the particles making up the porous material. They investigated the flow past arrays of circular cylinders and found that for a square array α_{BJ} increases from 1.2 to 4 as the porosity of the array increases. They also found that α_{BJ} may be as low as 0.4 depending on the surface structure. Hence, although α_{BJ} is expected to lie in the range 1.2 to 4, the effect on the flow of varying α_{BJ} beyond this range in this present work has been investigated. As an example of the results obtained the velocity of the flow along the symmetry axis in front of the fibre is shown in Fig. 2a for $c=0.05$, $k'=0.001$ and 0.1 and $\alpha_{BJ}=0.4, 1, 2, 4$ and 10 . Where k' is the non-dimensional permeability of the porous layer, $k' = \frac{k}{r_0^2}$. In the results shown equation (24) has been used to define the boundary of the porous deposit with $\lambda = 0.5$. As can be seen, for the lowest value of k' shown the

differences in the axial velocity obtained for the different values of α_{BJ} considered is small. The biggest difference occurring between $\alpha_{BJ}=0.4$ and the larger values. This smaller value of α_{BJ} is really at the extreme of the values likely to occur. For the larger value of k' the differences in the values of u obtained for the various α_{BJ} are greater with once again the greatest difference being between $\alpha_{BJ}=0.4$ and the other values. The larger values of k' correspond to larger permeabilities of the porous media where it is expected, from past work, that α_{BJ} is at the upper end of the range 1.2 to 4. The streamlines of the flow are shown in figs 2b and c for $k'=0.001$ and 0.1 respectively for $\alpha_{BJ}=1$ and 10. In these figures, due to symmetry, only half of the fibre is shown and the porous region is enclosed within the dotted curve. Only the flow in the region close to the fibre + deposit is shown. No significant differences in the flow can be seen for $k'=0.001$ but there are some slight differences for $k'=0.1$. Hence the effects of α_{BJ} appear to only be significant for the higher values of k' corresponding to the larger permeabilities. Though for the values of α_{BJ} expected for the larger permeabilities, greater than one, the effects are not great. In order to validate the numerical model developed a comparison has been made with an available analytic solution. Considering the cell model described here with the fibre covered by a solid deposit of constant width λr_0 around the fibre surface, the problem then reduces to solving equation (6) in the region $r > r_0(1 + \lambda)$. Adopting the same approach as Kuwabara², a particular solution of (6) is

$$\psi_f = \left(Ar + \frac{B}{r} + Cr \ln\left(\frac{r}{r_0}\right) + Dr^3 \right) \sin \theta \quad (36)$$

where A,B,C and D are arbitrary constants which can be determined from the boundary conditions. In this case these conditions are:

$$i) \quad u_r = u_\theta = 0 \quad \text{on } r = r_0(1 + \lambda) \quad (37)$$

$$ii) \quad \psi_f = U_0 r \sin \theta, \quad \omega_f = 0 \quad \text{on } r = \frac{r_0}{\sqrt{c}} \quad (38)$$

Non-dimensionalising, as before, the constants are determined to be given by:

$$A = \frac{-4 - 4c + (1 + \lambda)^2 c(3c + 4) - 4(c + 2) \ln(1 + \lambda) + c^2 (1 + \lambda)^4 - 4c(1 + \lambda)^2 \ln(1 + \lambda)}{F} \quad (39a)$$

$$B = \frac{2(1+\lambda)^2(-c(1+\lambda)^2 - c^2(1+\lambda)^2 + 2c + 2)}{F} \quad (39b)$$

$$C = \frac{4(2 + c(1+\lambda)^2)}{F} \quad (39c)$$

$$D = -\frac{c(2 + c(1 + (1+\lambda)^2))}{F} \quad (39d)$$

$$F = -(1+\lambda)^2 2c \ln c + 3c(1+\lambda)^2 - 4 \ln c - 6 - 2c^3(1+\lambda)^4 + 5c^2(1+\lambda)^2 + c^2(1+\lambda)^4 - 4c(1+\lambda)^2 \ln(1+\lambda) - 8 \ln(1+\lambda) - 4c \ln(1+\lambda) - 2c \ln c - c \quad (39e)$$

In the numerical model the non-dimensional porous layer is taken to have a constant width, $\lambda=0.1$, and various values of k' are considered. As k' decreases the permeability of the layer decreases and hence the solution should tend to the analytical solution for a solid deposit described above. This is seen to occur in Figure 3 where the velocity along the symmetry axis in front of the fibre is shown. In the numerical results α_{BJ} has been taken to be 3. As can be seen as k' decreases u tends to the analytical expression, as expected, with good agreement seen for the smallest values of k' taken. In fact there is no discernable difference between the results for $k'=0.001$, 0.00001 and the analytical solution. Hence the model is seen to be predicting the correct behaviour.

Considering the effects of k' , the non-dimensional permeability upon the flow, Figures 4a and b show the flow streamlines for various values of k' for $c=0.05$, $\alpha_{BJ}=3$, $\lambda=0.5$.

Equation (24) has been used to define the boundary of the porous deposit. In fig. 4a the streamlines are shown for $k'=0.001$ and 0.01 and in fig. 4b for $k'=0.1$ and 0.2 . As can be seen for the smaller values of k' , low permeability of the porous layer, fig. 4a, the streamlines are shifted away from the deposit with very little flow passing through. At

$k'=0.001$ it was found that for $\Psi = \frac{\psi}{r_0 U_0} > 0.001$ all streamlines lie above the porous

layer. As k' increases more flow passes through the layer, as seen in fig. 4b with the porous layer offering less resistance to the flow.

In order to investigate the effects of the filter packing fraction, c , upon the flowfield results were obtained for various values of c for particular values of k' . As an example of the results obtained Figure 5 shows the streamlines of the flow for $k'=0.01$ with $\alpha_{BJ}=3$ and $\lambda=0.5$. In fig. 5a streamlines are shown for $c=0.005$ and 0.01 and in fig. 5b for $c=0.05$

and 0.1. As can be seen, as c increases the flow is moving faster near the fibre with more of the fluid passing through the porous deposit. This is to be expected as the larger the value of c the closer the fibres are to each other in the filter. Various values of k' were considered and the effect of varying c was found to be the same.

Taking the motion of the small particles into account equation (32) was solved for n the non-dimensional particle concentration. In earlier work, Dunnett and Clement¹, equation (24), defining the shape of the layer of deposit, was determined for small particles where diffusion is the main mechanism of capture. In order for that equation to be valid it is necessary that δ is significantly smaller than κ . This restricts the validity of equation (24) to $Pe \leq 1.4 \times 10^5$ and $\kappa \leq 0.05$.

It has been shown; see Nield and Bejan³³ that the permeability k , from Darcy's equation, can be related to the average diameter of a particle in the porous layer by:

$$k = \frac{\phi^3 r_p^2}{45(1 - \phi)^2} \quad (40)$$

Where ϕ is the porosity of the layer, which is the fraction of the porous media that is occupied by void space.

Hence

$$k' = \frac{\phi^3 \kappa^2}{45(1 - \phi)^2} \quad (41)$$

In this work, values of Pe and κ relating to real situations have been considered and the build up of deposit on the fibre considered. This has been achieved by, for particular values of Pe and κ :

- i) initially determining the flow field for a clean fibre and solving equation (32) for n . This enables $\frac{\partial n}{\partial N}$ to be determined on the fibre surface, where N is the normal to the surface.

- ii) The coefficients a, b and p which give the best fit to the equation

$$\frac{\partial n}{\partial N} = \left[\frac{\partial n}{\partial N} \right]_{\beta=0} (1 - a\beta^p)^b.$$

are then determined.

iii) As the rate at which particles deposit is proportional to $\frac{\partial n}{\partial N}$ the equation of the surface of the porous material on the fibre is written as

$$r = 1 + \alpha \frac{\partial n}{\partial N} = 1 + \lambda (1 - a\beta^p)^b$$

iv) The new flow field and particle concentration can then be obtained as described in this work and $\frac{\partial n}{\partial N}$ on the porous surface determined.

v) Step ii) is then repeated and more deposit added with the new surface of deposited porous material given by $r_{\text{new}} = r_{\text{old}} + \alpha \frac{\partial n}{\partial N} = r_{\text{old}} + \lambda (1 - a\beta^p)^b$

Steps iv) and v) are then repeated as deposit grows.

As an example, the variation of $\frac{\partial n}{\partial N}$ with β is shown, for $Pe = 1.1 \times 10^5$, $\kappa=0.02$ and $\phi=0.9$

in Figure 6a. In this figure the solid line represents the variation of $\frac{\partial n}{\partial N}$ on the boundary of the fibre for a clean filter. The amount of deposited material added before the flow field was recalculated was taken to be such that the non-dimensional width of the porous layer added was 0.2 at the front of the fibre. The variation of $\frac{\partial n}{\partial N}$ with β is shown in figure 6a for 1,2 and 3 layers of deposited material. As can be seen the effect of the deposit is to increase the value of $\frac{\partial n}{\partial N}$, and hence the amount of deposit, at the front of the fibre and decrease it at the back. This effect continues as the amount of deposited material increases, as seen by the curves for 2 and 3 layers of deposit. For the values of κ and ϕ taken k' is small, of order 10^{-4} , and hence very little flow is passing through the porous layer, see Fig. 4a, despite 90% of the porous media being occupied by void space. Increasing the percentage of void space in the porous layer further, Fig. 6b shows the variation of $\frac{\partial n}{\partial N}$ with β for $Pe = 1.1 \times 10^5$, $\kappa=0.02$, $\phi=0.95$ which results in a value of k' of the order 10^{-3} . In this case the effect of increasing deposition on the front of the fibre and decreasing it at the rear is more marked than in the case of $\phi=0.9$ as more flow moves through the porous layer and hence increasing the convection of particles to the front of

the fibre. For the case of $\phi=0.95$ results were only obtained up to 2 layers of deposit on the fibre as the shape of the curve of $\frac{\partial n}{\partial N}$ as a function of β after that could no longer be approximated well by equation (24). This indicates that interception of particles onto the porous boundary is becoming increasingly important. Other values of Pe and κ have been investigated and the same behaviour is found to occur, as an example the variation of $\frac{\partial n}{\partial N}$ with β for $Pe=1500$, $\kappa=0.05$, $\phi=0.95$ is shown in figure 6c. As before it can be seen that $\frac{\partial n}{\partial N}$ increases at the front of the fibre and decreases at the back as the amount of deposited material increases. For the value of κ and ϕ taken in this example k' is of the order 10^{-2} , as although the value of the porosity ϕ is the same as in the previous figure, the value of κ , the relative size of the deposited particles to the fibre, has increased. This results in an increase in the non-dimensional permeability.

An important characteristic used to evaluate the performance of a filter is the particle collection efficiency, η , which is the measure of the efficiency of a single representative fibre in the filter. This is defined to be the ratio of the number of particles that are actually removed by the fibre to the number entering the 'cell'. As shown by Dunnett and Clement¹ this can be written as

$$\eta = \frac{2I\sqrt{c}}{Pe} \quad (42)$$

where I is the integral

$$I = \int_{\theta=0}^{\theta=\pi} \left[\frac{\partial n}{\partial N} \right]_{r=r_1} ds$$

r_1 is the interception radius which will change as deposition of particles occurs.

Figure 7 shows the variation of η with ϕ when there is one layer of deposit on the fibre for $Pe=1500$, $\kappa=0.05$. As can be seen the efficiency increases as ϕ increases, leading to an increase in the permeability of the deposit. Other values of Pe and κ were considered and the same behaviour was seen to occur.

Conclusions

One of the major problems in understanding the performances of fibrous filters is in describing their behaviour after they have become loaded with deposited particles. In this paper a relatively straightforward mathematical model has been described in detail that considers the particulate deposit on the fibres to be a porous medium. The model enables the airflow around and through the porous medium to be obtained in an efficient manner. In determining the flow field there are a few parameters to be considered, the packing fraction of the filter, the permeability of the porous deposit and the size of the deposit on the fibres. The model developed allows the effects of the parameters upon the flow field and hence upon particle transport, to be investigated. As expected it is seen that as the packing fraction increases, so more of the filter space is occupied by fibres, more of the flow passes through the porous deposit. Hence increasing the chance of more particles being deposited. Also it is shown that as the permeability of the porous deposit increases more flow passes through the deposit.

Once the flow field is known the transport of particles within the airflow can be modelled. In this work a study has been made of the effect of particle deposit on fibrous filters on the airflow and particle transport within the filter and hence further particle deposition. The behaviour of small particles has been considered where the dominant mechanism by which particles are captured by the fibres is diffusion. Though other size particles could just as easily be considered, where other mechanisms such as impaction are important. It is seen that the effect of the deposit upon further deposition is to increase the rate at the front of the fibre and decrease it at the back. This effect of deposit has also been seen earlier for solid deposit, Dunnnett and Clement¹. Interestingly it is seen here that, for the small particles considered, where diffusion is the main mechanism of deposit, the permeability of the deposit formed is usually of the order that no significant flow passes through it. Hence, in those cases assuming a solid deposit may be a valid assumption. However this may not be the case when the interception of the particles onto the surface becomes significant.

The main advantage of the method described in recalculating the flow field following deposition is its simplicity. The Boundary Element Method only requires the boundary of the solution domain to be divided into segments and as, for the physical conditions

considered here, the new surface formed by the deposit can be easily described, this is a straightforward matter.

The future aim is to use the model of the flow field developed to investigate the pattern of particle deposition within filters for various size particles and study the effects upon efficiency. It is planned to consider different mechanisms of deposition, such as interception.

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Figure Captions.

Figure 1. Computational domain.

Figure 2. Flow field for $c=0.05$, $k'=0.001$ and 0.1 , $\lambda=0.5$ (a) u on symmetry axis in front of fibre for $\alpha_{BJ}=0.4, 1, 2, 4$ and 10 , (b) streamlines for $k'=0.001$, $\alpha_{BJ}=1$ and 10 , (c) streamlines $k'=0.1$ for $\alpha_{BJ}=1$ and 10 .

Figure 3. Velocity along the symmetry axis, in front of the fibre, as a function of the distance from the porous layer of constant width for $c=0.05$, $\lambda=0.1$ and various values of k' . Also shown is an analytical expression for a solid deposit.

Figure 4. Streamlines $\Psi=0, 0.1, 0.21, 0.31, 0.41, 0.51$ for $c=0.05$, $\alpha_{BJ}=3$, $\lambda=0.5$ for (a) $k'=0.001$ and 0.01 , (b) $k'=0.1$ and 0.2

Figure 5. Streamlines for $k'=0.01$, $\alpha_{BJ}=3$, $\lambda=0.5$, Ψ takes values $0, 0.1, 0.21, 0.31, 0.41, 0.51$ for (a) $c=0.005$ and 0.01 , (b) $c=0.05$ and 0.1 .

Figure 6. The variation of $\frac{\partial n}{\partial N}$ with β for $c=0.05$ (a) $Pe=1.1 \times 10^5$, $\kappa=0.02$, $\phi=0.9$, (b) $Pe=1.1 \times 10^5$, $\kappa=0.02$, $\phi=0.95$, (c) $Pe=1500$, $\kappa=0.05$, $\phi=0.95$.

Figure 7. The variation of η with ϕ for $Pe=1500$, $\kappa=0.05$

Figure. 1.

Figure 2a.

Figure 2b.

Figure 2c.

Figure 3

Figure 4a

Figure 4b

Figure 5a.

Figure 5b.

Figure 6a.

Figure 6b.

Figure 6c.

Figure 7.