# Response to reviewers of ”Gust load alleviation: a sub/transonic wind tunnel experimental validation of a 2D aeroelastic airfoil”

## I. General comments

First of all, the authors both thank the Associate Editor and the three Reviewers for the time spent to read, evaluate and rate the paper initially entitled "Gust load alleviation: a sub/transonic wind tunnel experimental validation of a 2D aeroelastic airfoil" (now entitled "A new frequency domain subspace algorithm with restricting LMI regions poles location and its application on a wing tunnel test."). They are very grateful for the positive feedbacks, valuable comments and criticisms as well. They also hope that the proposed revised version is improved in quality and readability and that the effort push in the re-writing really improves the paper quality. Before detailing the corrections made, authors want to stress that, accordingly to the main comments identified, the paper has been modified in the sense that both its structure and title have been re-thought to address the main comments of the Editor and Reviewers (detailed hereafter)

## II. Responses to Reviewer #1

**I believe the present manuscript is acceptable for publication in the International Journal of Control. It features an overall very good technical quality, showing rigour in the theoretical derivations. I have also noted the important effort the authors made to comment in an ex- haustive manner their assumptions, claims and results. From a technical point of view, I have appreciated a lot the linear matrix inequalities they introduced in subspace identification. I think many different applications may find interest in restricting pole location when using a subspace algorithm. Finally, the use of real measurements acquired in wind tunnel testing conditions is an additional strong asset of the paper.**

We would like to thank Reviewer #1 for his very positive comments. We appreciate that reviewer' believes that the paper is well organised. Finally, we believe that he clearly summarizes the results of the paper. In the following, we respond the specific remarks pointed, and try to additional informations as well as in the paper.

### A. Remark #1

**there are multiple conflicts between US and UK spellings, e.g., dynamical (US), utilized (US), characterized (US), behaviour (UK), colour (UK).**

This problem has been fixed.

### B. Remark #2

**I have seen a number of typos, likely mean (should be by means), an other (should be another), made cleared (should be made clear), authors knowledge (should be authors' knowledge), there exist (should be there exists), amount other (should be among others), might leads (should be might lead), we won't (should be we will not), etc.**

Many typographical and grammatical errors have been fixed.

### C. Remark #3

**the authors use a lot the present perfect in the manuscript (has been, have been, etc.), while the simple present and simple past are most often recommended.** We tried to correct that.

### D. Remark #4

**page 2, Remark 3: despite the scope of the paper is restricted to linear systems, I think that people may find interest in the use of linear matrix inequalities in the context of non- linear system identification. For instance, the authors' developments look to me directly integrable in the nonlinear subspace algorithms proposed in Refs. [1,2,3] below. Perhaps the authors may want to comment on this.**

Others extensions can be investigated as in the non-linear context [1], [2] for instance. Generally speaking, the condition required in order to involve LMI poles location is to be able to write the problem under the following form $\varphi\hat{A}\Psi = b\Psi$.

These references have been added with a short comment. Of course these considerations must be taken with precautions since it would be necessary to really involve the LMI constraints in the non-linear context to evaluate the feasibility. It seems straightforward for [2], [3], and for [1] a solution would be to multiply equation (96) in [1] $\hat{A}R = x$ on the right by $R^{-1}$ since $R$ is supposed to be right invertible and on the left by $R$ to obtain finally $R\hat{A} = RxR^{-1}$ which is equivalent to $\varphi\hat{A} = b$ with $\varphi = R$ and $b = RxR^{-1}$.

### E. Remark #5

**I think the authors should cite Ref. [4] below when speaking about the a priori knowledge of the noise covariance to ensure consistency in subspace identification.**

This reference has been added.

## III. Responses to Reviewer #2

### A. Remark #1

**The paper discusses two methods for model identification/reduction in the frequency domain. The methods are applied to several benchmark problems and an experimental wind tunnel setup. I believe that there is a major interest in the reported work, but I find it hard to support the publication of the paper in its current form. The paper does not only contain a discussion of (already existing) methods (as the title suggests), but proposes an extension of the frequency domain subspace method with LMI constraints. I think that it will merit the paper by moving the focus on the novel contributions of the author (now Section 3). Furthermore, mentioning the wind tunnel setup in the title should be avoided: the merit of the paper is not limited to this specific setup. I would support the publication of this paper if the presentation can be majorly improved in the following way: try to focus on the novel contributions of the authors, and provide a briefer qualitative/quantitative discussion of the methods, perhaps in a shorter publication format. Moreover, in its current form, the paper gives the impression to be written in a rush: there are many typographical and grammatical errors. I am afraid that improving the paper will require a complete re-writing, but I believe that the paper is a good starting point for an improved paper. The authors should do a careful reading of the paper, as I have spotted dozens of typographical and grammatical errors.**

Thank you for comment and synthesis. The paper has been modified in the sense proposed by the Reviewer. The title has been modified to point out the frequency domain subspace method with LMI constraints. The description of the application in the title has been lightened. In the abstract and in Section 1.2 Outlines the extension of the frequency domain subspace method is more deeply described. The parts which describe Loewner approach has been reduced to improve the visibility of the main contribution of this paper which concerns the LMI-based frequency domain subspace. More generally speaking, authors believe that it is not necessarily relevant to reduce the application part and to avoid wind tunnel test in the title since this publication has been done in the context of a special issue dedicated to applications and more specifically to "Identification and Control of Nonlinear Electro-Mechanical Systems" where applications play an important role. This point has been confirmed to the authors by guest editors. That is why authors share the point of view on the necessity to point out more specifically the LMI based method applied to subspace, and modifications have been done in this sense in the title, abstract and introduction, but due to the specific topic of this special issue, the applicative parts should remain important in the article. Many typographical errors have been fixed.

### B. Remark #2

**title: perhaps mentioning the wind tunnel setup is not an advantage for the 'visibility' of the paper; I believe that the focus of this paper should be on the novel subspace-LMI method of the authors.**

The title has been modified to point out the novel subspace-LMI method, but a reference to the wind tunnel test is preserved to be aligned with the special issue topic.

### C. Remark #3

**page 1, lines 44-56 (also Remark 2 on page 2, lines 36-46): A remark regarding the distinction between time domain and frequency domain system identification on page 1: the bottom paragraph (lines 44-56) seems to suggest that time domain identification always requires an impulse response experiment. However, this is obviously not the case. Persistency of excitation is also important for FD identification. Moreover, theoretically there is an equivalence between TD and FD identification, however for some applications or input signals, one can favor one over the other.**

Authors agree with the reviewer. This remark (Remark 3) was not correctly presented and has been deeply modified to explain more clearly the choice of frequency domain representation.

## IV. Responses to Reviewer #3

### A. Remark #1

**In this paper....(introduction of a stability constraint)**

We would like to thank Reviewer #2 for his comments.

### B. Remark #2

**Improving...references.**

We believe that the modifications performed in the paper increases its overall clarity. Besides 8 references have been added to improve the global quality of the revised version.

### C. Remark #3

**Explaining....only**

Authors wish to clarify the presentation to point out the complementary of these two approaches. Results clearly show that Loewner approach is able to provide state-space realisations from frequency domain data in a quite fast manner and with a very high (and controlled) accuracy level. When the optimal order is chosen one then have the guarantee to obtain an exact interpolation between data and for example, in LAH benchmark, the residual error is very weak (close to $10^{-13}$) when the orders of identified model and reference model are equals. However, in the noisy context, this approach can lead to irrelevant dynamics (as classical subspace approaches). That is why, to tackle this problem a modified subspace approach is proposed to obtain a model which is used as a synthesis model and to validate closed loop results with time domain data. Finally, to validate the frequency domain responses of the closed-loop an accurate model $\hat{H}$ is obtained involving the Loewner approach.

### D. Remark #4

**In Sub-Section I.I...paper"**

See detailed responses here after.

### E. Remark #5

**First...models"?**

This remark was mentioned since the Loewner framework was initially settled for model reduction purpose. However, in this case, the internal variable is obviously unknown, as in the black box model identification.

### F. Remark #6

**Second...for me**

- By model approximation method (as in the Loewner approach) authors mean a method which is able to determine a state-space representation from input/output data which correspond generally speaking to the resolution of ODE (or even irrational equations [4]).
- By model identification method (as subspace approach) the objective is the same one as previously but in an experimental context since inputs/output data correspond to noisy experimental data.
- By model reduction authors mean that method which are able to provide a reduced order model from a full order model.

However the most important point to keep in mind is that methods for model approximation can be used in model identification and reduction context. And methods for model identification can be used for model approximation and and model reduction. Of course methods for model reduction cannot be used for model approximation and model identification since there suppose to have a full order model as starting point. Results can be different according to methods. Typically an important topic in the identification method is the behaviour of the algorithm in presence of noise. It is not at all the case for model approximation methods [5] where a more central topic is the solution optimality and the exact interpolation guarantee.

Indeed, as written above, the Loewner framework is well known in the so-called model approximation from data or data-driven model approximation.

### G. Remark #7

**Third...ill-defined**

By close to the identification field, authors mean that naming them approximation from data or identification is more matter of viewpoint and background. From the authors point of view, they are quite close to each other. The main difference is that the model approximation community does not classically consider the noise information in the data (like mean, variance, and generally speaking, stochastic effects), while the identification community clearly take this consideration in a very mathematically consistent way.

### H. Remark #8

**Fourth....data-sets**

Authors agree with reviewer that is why this specific remark has been suppressed and the part which talks about the choice of frequency domain data has been widely modified.

### I. *Remark #9*

**Please...at all**

This section has been modified to take into account the different remarks.

### J. *Remark #10*

**Why...forms**

Note that the consideration of the descriptor form is actually not restrictive and reviewer can notice that this form is used in the Loewner part (equation 13). This form does not intend to treat non invertible $E$ matrices only, but is rather more general formulation and is also a way to handle non zero $D$ term (direct feed-through) within the Loewner approach. Indeed, it is well known that $D$ term can be recast as a singular $E$ term. However, authors stress that in the presented application, we always obtain an invertible $E$ matrix. The descriptor $E$ has been suppressed in the initial formulation but a remark has been added to explain that Loewner approach can use the descriptor $E$.

### K. *Remark #11*

**Please...exclusively**

Authors share this point of view. That is why this remark has been deeply modified.

### L. *Remark #12*

**Please...techniques**

Reviewer' right, it is not that standard. We aimed at pointing the main limitation of our approach, with is limited to linear forms. The term standard has been removed and a reference has been added [6].

### M. *Remark #13*

**In sub-Section 1.1...well**

No assumption is done on the noise which affects the data for several reasons:

- The Loewner approach is not developed to take into account a noisy context. That is why no theoretical result exists with this approach to prove for example the consistency in presence of noise is provided. Of course it is possible to apply a specific filtering on data to improve results with Loewner approach;
- The frequency domain subspace approach is able to be consistent in presence of noise. But assumptions to obtain this result are difficult to validate on the WTT. Typically these assumptions are based on the knowledge of the noise relative variance for each frequency sample;
- It seems very difficult to validate the noise assumptions to prove the consistency of the subspace algorithm;
- The noise influence is rather limited, even if, this noise leads to obtain unstable models with Loewner and subspace approaches.
- In fact the objective is to obtain relevant identified models (not only stable but also skipped of irrelevant dynamics) without any assumption on the noise (assumptions impossible to validate). Of course it is not possible to prove the consistency of the algorithm but the objective here is not theoretical but rather practical: to obtain a relevant identified model for the control with a straightforward methodology represented here by the frequency domain subspace approach with LMI poles location. And the main advantage is to obtain a relevant identified model without noise assumption but with the a priori knowledge of the real system.

### N. *Remark #14*

**In Problem I...Minimal?**

- By low order, one means an order which made possible the control design, e.g. $r \ll 100$
- By minimal, one means that the McMillian degree of the realisation is indeed minimal.

### O. *Remark #15*

**With...identification?**

Indeed, it is possible to extend this with noisy data. However, as pointed above, no exploitation of the noise property is done so far. The Loewner framework is an interpolatory-oriented framework and ensured interpolation condition. This property should then be taken carefully into account when noisy data are to be treated. In theory one can use with no limitation, however, to the authors point of view, in practice, pre-filtering should be performed.

### *P. Remark #16*

**In this...approach**

Indeed, there is a link. The pulsation $\omega_i$ are the so-called driving frequencies and are represented by $\mathbf{\Lambda}$ and $\mathbf{M}$. A remark has been added after (5).

### *Q. Remark #17*

**The...efficient**

A specific comment has been added to address these different approaches. To sum up the main advantage of the approach in this article is to propose a more general formulation which is able not only to take into account stability but more generally any regions of $s$-plane/$z$-plane as long as they can be described by an LMI. This general formulation (not limited to the discrete Lyapunov function as illustrated in [7]) allows enlarging the application field. For example it is well known that identification of ground aircraft, named Ground Vibration Test, that flexible modes have a minimal damping of 5% for instance. This kind of constraint can be directly used by our approach.

### *R. Remark #18*

**In Sub-Section 4.2...observations**

Authors are surprised by this comment. The conclusion given in the article is the following one: "As first conclusion, the Loewner approach is particularly dedicated to provide a reduced order LTI models from frequency data, which are here, generated by a full order model in the noise-free context. In fact the Loewner approach is an interpolation based method which is exact when the order is not imposed. This property allows to obtain residual error which is, in general case, weaker than with the subspace approach. More generally, Loewner approach is an efficient method to interpolate frequency domain responses obtained by a numerical black box tool."

It is clear that the computational time is not presented as the only difference between the 2 methods. But it is also clear too that the computation time obtained between these 2 methods can be very important. An explication is given in the article and based on the minimum value for $q$ to obtain a relevant model. And a large value $q$ leads to large size-scale matrices. Nevertheless, this difference in favour of Loewner approach should be balanced by the fact that results are obtained on these benchmarks as indicated by the reviewer. Here the specificity of these models is to contain a large number of flexible modes as in the case of an aeroelastic model. But of course other kind of models could lead to different results in term of computation time. That is why a sentence has been added to moderate the remark concerning the velocity.

### *S. Remark #19*

**In Sub-Section 4.2...account**

First of all, in the noise free context and when data are generated by an LTI model $H(s)$ the subspace approach is said correct, which means that the $\mathcal{H}_\infty$ error between the identified model $\hat{H}$ and the true model $H$ is null. In other words $\hat{H}(s) = H(s)$ and consequently if $H(s)$ is stable then $\hat{H}(s)$ is stable too. Of course a condition for this is to choose an order for the identified model which is equal to the order of $H(s)$ as for LAH benchmark. And this "optimal" order is easily detected by the SVD of the extended observability matrix. For the Loewner approach we have a similar result since the optimal order can be obtained automatically and in this case the interpolation is exact. When the identified model order is inferior to the full order model it is possible to think that unstable models can be obtained but in our applications it is not the case. This result is directly linked to the good property of the subspace and Loewner approaches. A remark has been added to indicate that no LMI constraint has been added.

### *T. Remark #20*

**In Sub-Section 4.3...paper**

Authors agree with this comment. But from a practical point of view, since authors have used intensively the two approaches, still the Loewner approach remains a very efficient one to obtain models to validate closed loop frequency domain response. Firstly because the residual error is in general case very weak, even in the noisy context. And secondly the calculation time in our applications were particularly limited in comparison with the subspace approach with or without LMI. That is why the Loewner approach has been involved to obtain an LTI model of high order to evaluate the load alleviation. A conclusion in section 4.3 has been added to represent more precisely the authors' feeling.

### *U. Remark #21*

**I...identification**

It is a point of view. Results with benchmarks are obtained in the noise-free context in the first part. And experimental data are not strongly noisy as illustrated in figure 13 right. That is why model approximation methods are appropriate in our case.

### V. Remark #22

**What..."fiction"**

Indeed, its fictitious, not fiction... One means from a process which does not exists, but which generates exactly these data. This notation allows to

### W. Remark #23

**In section 4...section 5**

Some additional informations are given in the article but these informations are not at all essential. Of course it is possible to obtain slightly different results according to sample period or the data length but it is not the problem addressed here. That is why to lighten the presentation and to make more visible the results, details are omitted. No Monte Carlo simulation have been performed. Sample period and bandwidth information have been added. For section 5 the sample period and the number of frequency are already given. The bandwidth has been added with the value of $q$.

### X. Remark #24

**In Sub-Section...or not**

The value of $\epsilon_m$ has been added. $\epsilon_a$ is tuned according to the benchmark to act on the weak gain frequency domain response only. The data are very moderately noisy.

## V. Conclusion

We hope that our answers have properly addressed all the elements raised by the Associate Editor and the Reviewers. Once again, we are grateful for the time spent evaluating the article and we hope that this revised version sticks to the acceptance standards of the International Journal of Control.

## REFERENCES


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