Online Supplement to “A Minimal Switching Procedure for Constrained Ranking and Selection under Independent or Common Random Numbers” by Healey, Andradóttir, and Kim

Proof of Theorem 1: We refer to Batur and Kim (2010) and Hong and Nelson (2005) for discussion of when $\mathcal{F}_B^I$ and $\mathcal{MSS}$ satisfy Assumptions 3 and 4, respectively. Then, as $\mathcal{CMS}$ satisfies Assumptions 2, 3, and 4, we apply Lemma 1 and the fact that $\beta_1 = \beta_2/s$, and obtain that $\mathcal{CMS}$ guarantees

$$\Pr\{CS\} \geq (1 - \beta_2)^j + (1 - \beta_2) + (1 - \beta_2)^{k-j-1} - 2 \quad (5)$$

when the number, j , of unacceptable systems is less than k and $\Pr\{CS\} \geq (1 - \beta_2)^k$ when $j = k$. Remark 4.3 of Healey et al. (2012) shows that the lowest $\Pr\{CS\}$ occurs when $j < k$, so any β_2 that guarantees $\Pr\{CS\} \geq 1 - \alpha$ for $j < k$ also guarantees $\Pr\{CS\}$ when $j = k$.

Since $j = (k - 1)/2$ minimizes the right-hand side of (5), the definition of β_2 yields

$$\Pr\{CS\} \geq 2(1 - \beta_2)^{(k-1)/2} - \beta_2 - 1 = 1 - \alpha,$$

which concludes the proof. $\square$

Additional Tables and Figures for Section 5.2.1: See Tables A.1 and A.2 and Figures A.1, A.2, and A.3.

Table A.1: Average total cost of switches and samples for procedures with k independent systems, $s = 3$ constraints, and b feasible systems when switches and samples are equally costly.

$k = 15, b = 8$	DM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{HAK}$	3001	23741	26994	29535	1380	9808	13375	13571
$\mathcal{HAK}+$	4530	34578	29188	47467	2264	16321	14156	25053
$\mathcal{MD}_R$	3974	13213	29178	41854	1782	7191	14148	20143
$\mathcal{CMS}$	3519	12558	32158	39599	1850	6090	19460	22702
$k = 101, b = 51$								
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{HAK}$	26934	225122	225756	266992	4035	25758	32145	33575
$\mathcal{HAK}+$	41321	303608	243039	428341	6095	42181	33860	67040
$\mathcal{MD}_R$	36428	124535	242219	378446	4854	19431	33159	49367
$\mathcal{CMS}$	30650	111834	284169	359741	5075	14652	43422	51862

Table A.2: Average total cost of switches and samples for procedures with k independent systems, $s = 3$ constraints, and b feasible systems when switches are ten times as costly as samples.

$k = 15, b = 8$	DM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{HAK}$	5893	24002	137886	55918	3313	10069	69282	30788
$\mathcal{HAK}+$	23627	188889	159247	259783	11165	88476	76572	136504
$\mathcal{MD}_R$	19639	35033	159151	219355	8183	21326	76521	105845
$\mathcal{CMS}$	3780	12819	32419	39860	2102	6349	19714	22963
$k = 101, b = 51$								
$\mathcal{HAK}$	47558	226931	1140967	457853	8149	27567	162772	70429
$\mathcal{HAK}+$	218623	1661206	1328076	2347234	24882	223356	177588	360081
$\mathcal{MD}_R$	182742	349602	1320500	1978203	17519	69488	173585	254876
$\mathcal{CMS}$	32454	113638	285977	361550	6245	16175	44879	53670

(a) DM-H/L

(b) DM-H/H

(c) MIM-H/L

(d) MIM-H/H

Figure A.1: Average total cost for procedures under DM (top) and MIM (bottom) with H/L and H/H variance configurations as a function of relative cost between switches and samples when there are $k = 101$ independent systems with $s = 3$ constraints and $b = 51$ feasible systems.

(a) DM-L/L

(b) DM-L/H

(c) MIM-L/L

(d) MIM-L/H

Figure A.2: Average total cost for procedures under DM (top) and MIM (bottom) with L/L and L/H variance configurations as a function of relative cost between switches and samples when there are $k = 15$ independent systems with $s = 3$ constraints and $b = 8$ feasible systems.

(a) DM-H/L

(b) DM-H/H

(c) MIM-H/L

(d) MIM-H/H

Figure A.3: Average total cost for procedures under DM (top) and MIM (bottom) with H/L and H/H variance configurations as a function of relative cost between switches and samples when there are $k = 15$ independent systems with $s = 3$ constraints and $b = 8$ feasible systems.

Additional Tables for Section 5.2.2: See Tables A.3 and A.4.

Table A.3: Observed PCS for procedures with $k = 101$ systems, $s = 3$ constraints, $b = 51$ feasible systems, and induced correlation (ρ_x). PCS below $1 - \alpha = 0.95$ marked in bold.

	DM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{HAK}$	0.991	1.000	0.984	0.990	1.000	1.000	0.999	0.999
$\mathcal{HAK}(0)$	0.991	1.000	0.984	0.990	1.000	1.000	0.999	0.999
$\mathcal{HAK}(0.1)$	0.991	1.000	0.979	0.987	0.999	1.000	0.999	0.999
$\mathcal{HAK}(0.25)$	0.993	1.000	0.984	0.993	1.000	1.000	1.000	1.000
$\mathcal{HAK}(0.5)$	0.996	1.000	0.981	0.996	1.000	1.000	0.999	0.999
$\mathcal{HAK}(0.75)$	0.997	1.000	0.978	0.997	0.998	1.000	0.999	0.999
$\mathcal{HAK}(0.9)$	0.999	1.000	0.920	0.998	0.993	1.000	0.995	0.998
$\mathcal{CMS}$	0.999	0.999	1.000	0.999	1.000	0.999	1.000	0.999
$\mathcal{CMS}(0)$	0.999	0.999	1.000	0.999	1.000	1.000	1.000	0.999
$\mathcal{CMS}(0.1)$	0.998	0.999	0.998	0.997	0.999	1.000	1.000	1.000
$\mathcal{CMS}(0.25)$	0.998	0.998	0.998	0.997	1.000	1.000	1.000	0.999
$\mathcal{CMS}(0.5)$	0.991	0.991	0.989	0.990	0.999	0.999	0.999	0.999
$\mathcal{CMS}(0.75)$	0.948	0.934	0.925	0.922	0.997	0.995	0.994	0.995
$\mathcal{CMS}(0.9)$	0.865	0.850	0.690	0.676	0.988	0.985	0.949	0.943

Table A.4: Average number of required samples for procedures with $k = 101$ systems, $s = 3$ constraints, $b = 51$ feasible systems, and induced correlation (ρ_x).

	DM ($\sigma_{x_i}^2/\sigma_{y_{it}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{it}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{HAK}$	24643	224921	124066	245786	3577	25557	17631	29480
$\mathcal{HAK}(0)$	24682	225221	124089	246181	3580	25623	17633	29521
$\mathcal{HAK}(0.1)$	24126	225221	112539	240191	3484	25623	16013	28487
$\mathcal{HAK}(0.25)$	23430	225221	95674	233297	3377	25623	13568	27232
$\mathcal{HAK}(0.5)$	22758	225221	67137	226588	3272	25623	9534	25941
$\mathcal{HAK}(0.75)$	22618	225221	38554	225240	3248	25623	5557	25633
$\mathcal{HAK}(0.9)$	22616	225221	24306	225221	3248	25623	3553	25623
$\mathcal{CMS}$	30449	111633	283968	359540	4945	14483	43260	51661
$\mathcal{CMS}(0)$	30587	111620	284692	360551	4976	14584	43375	51837
$\mathcal{CMS}(0.1)$	28240	110065	258925	334157	4638	14418	38608	47043
$\mathcal{CMS}(0.25)$	24747	107584	217272	292433	4190	14175	31701	40400
$\mathcal{CMS}(0.5)$	18690	102880	147624	222455	3469	13781	20904	29339
$\mathcal{CMS}(0.75)$	13398	99333	78837	155095	2857	13378	10497	18979
$\mathcal{CMS}(0.9)$	11259	101862	40895	128976	2664	13168	5003	14607

Additional Tables and Figure for Section 5.2.3: See Tables A.5, A.6 and A.7 and Figure A.4.

Table A.5: Average number of required samples for $\mathcal{MD}_R$ and $\mathcal{CMS} + TS_4$ with $k = 15$ systems, $s = 3$ constraints, $b = 8$ feasible systems, and induced correlation (ρ_x).

	DM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{MD}_R$	2233	10788	14736	22132	1071	5620	7217	10621
$\mathcal{MD}_R(0)$	2239	10830	14774	22197	1074	5635	7234	10666
$\mathcal{MD}_R(0.1)$	2096	10708	13410	20871	1012	5581	6554	10013
$\mathcal{MD}_R(0.25)$	1894	10520	11263	18860	921	5500	5507	9128
$\mathcal{MD}_R(0.5)$	1584	10212	7794	15701	791	5371	3792	7734
$\mathcal{MD}_R(0.75)$	1280	9904	4325	12677	690	5267	2082	6437
$\mathcal{MD}_R(0.9)$	1122	9741	2231	10830	656	5235	1074	5634
$\mathcal{CMS}$	3490	12529	32129	39570	1822	6061	19432	22673
$\mathcal{CMS}(0) + TS_4$	3502	12593	32335	39562	1846	6077	19530	22727
$\mathcal{CMS}(0.1) + TS_4$	3195	12413	29072	36287	1674	5996	17509	20693
$\mathcal{CMS}(0.25) + TS_4$	2761	12121	24037	31280	1447	5876	14226	17413
$\mathcal{CMS}(0.5) + TS_4$	2083	11570	16335	23611	1083	5673	9428	12533
$\mathcal{CMS}(0.75) + TS_4$	1495	10908	9061	16618	778	5467	5151	8471
$\mathcal{CMS}(0.9) + TS_4$	1215	10073	4808	14819	675	5291	2826	7402

Table A.6: Average number of required switches for $\mathcal{MD}_R$ and $\mathcal{CMS} + TS_4$ with $k = 15$ systems, $s = 3$ constraints, $b = 8$ feasible systems, and induced correlation (ρ_x).

	DM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{MD}_R$	1741	2425	14442	19722	711	1571	6930	9522
$\mathcal{MD}_R(0)$	1747	2442	14480	19792	714	1576	6948	9570
$\mathcal{MD}_R(0.1)$	1589	2173	13112	18267	642	1415	6267	8798
$\mathcal{MD}_R(0.25)$	1349	1769	10957	15877	532	1164	5219	7727
$\mathcal{MD}_R(0.5)$	929	1094	7454	11590	356	722	3499	5780
$\mathcal{MD}_R(0.75)$	404	424	3928	6310	165	280	1772	3489
$\mathcal{MD}_R(0.9)$	63	64	1741	2440	39	49	714	1575
$\mathcal{CMS}$	29	29	29	29	28	29	28	29
$\mathcal{CMS}(0) + TS_4$	29	29	29	29	28	29	28	29
$\mathcal{CMS}(0.1) + TS_4$	29	29	29	29	28	29	28	29
$\mathcal{CMS}(0.25) + TS_4$	29	29	29	29	27	28	28	29
$\mathcal{CMS}(0.5) + TS_4$	29	29	29	29	26	27	28	29
$\mathcal{CMS}(0.75) + TS_4$	28	28	29	29	23	24	28	29
$\mathcal{CMS}(0.9) + TS_4$	24	24	29	29	21	22	28	29

Table A.7: Average total cost of switches and samples for procedures with $k = 15$ systems, $s = 3$ constraints, $b = 8$ feasible systems, and induced correlation (ρ_x) when switches are ten times as costly as samples.

$k = 15, b = 8$	DM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)				MIM ($\sigma_{x_i}^2/\sigma_{y_{i\ell}}^2$)			
	L/L	L/H	H/L	H/H	L/L	L/H	H/L	H/H
$\mathcal{MD}_R$	19639	35033	159151	219355	8183	21326	76521	105845
$\mathcal{MD}_R(0)$	19708	35252	159578	220114	8217	21390	76719	106361
$\mathcal{MD}_R(0.1)$	17991	32442	144532	203543	7432	19729	69228	97995
$\mathcal{MD}_R(0.25)$	15384	28210	120831	177634	6246	17137	57700	86395
$\mathcal{MD}_R(0.5)$	10871	21157	82336	131603	4348	12596	38787	65534
$\mathcal{MD}_R(0.75)$	5322	14148	43609	75777	2344	8064	19802	41327
$\mathcal{MD}_R(0.9)$	1754	10378	19642	35230	1044	5726	8213	21381
$\mathcal{CMS}$	3780	12819	32419	39860	2100	6349	19714	22963
$\mathcal{CMS}(0) + TS_4$	3791	12883	32625	39852	2123	6365	19812	23017
$\mathcal{CMS}(0.1) + TS_4$	3485	12703	29362	36577	1950	6283	17791	20983
$\mathcal{CMS}(0.25) + TS_4$	3051	12411	24327	31570	1720	6160	14508	17703
$\mathcal{CMS}(0.5) + TS_4$	2372	11859	16625	23901	1343	5943	9710	12823
$\mathcal{CMS}(0.75) + TS_4$	1779	11192	9351	16908	1010	5709	5433	8761
$\mathcal{CMS}(0.9) + TS_4$	1459	10316	5098	15109	889	5515	3106	7690

(a) DM-H/L

(b) DM-H/H

(c) MIM-H/L

(d) MIM-H/H

Figure A.4: Average total cost for procedures under DM (top) and MIM (bottom) with the H/L and H/H variance configurations as a function of relative cost between switches and samples when there are $k = 15$ systems with $s = 3$ constraints, $b = 8$ feasible systems, and induced correlation $\rho_x = 0.25$.