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A Memetic Algorithm Based on Probability Learning for Solving the Multidimensional Knapsack Problem

Zuocheng Li, Lixin Tang *Senior Member, IEEE*, and Jiyin Liu

Abstract—The multidimensional knapsack problem (MKP) is a well-known combinatorial optimization problem with many real-life applications. In this paper, a memetic algorithm based on probability learning (MA/PL) is proposed to solve MKP. The main highlights of this paper are twofold: problem-dependent heuristics for MKP and a novel framework of MA/PL. For the problem-dependent heuristics, we first propose two kinds of logarithmic utility function (LUF) based on the special structure of MKP, in which the profit value and weight vector of each item are considered simultaneously. Then, LUFs are applied to effectively guide the repair operator for infeasible solutions and the local search operator. For the framework of MA/PL, we propose two problem-dependent probability distributions to extract the special knowledge of MKP, i.e., the marginal probability distribution (MPD) of each item and the joint probability distribution (JPD) of two conjoint items. Next, learning rules for MPD and JPD, which borrow ideas from competitive learning and binary Markov chain, are proposed. Thereafter, we generate MA/PL's offspring by integrating MPD and JPD, such that the univariate probability information of each item as well as the dependency of conjoint items can be sufficiently used. Results of experiments on 179 benchmark instances and a real-life case study demonstrate the effectiveness and practical values of the proposed MKP.

Index Terms—Combinatorial optimization, multidimensional knapsack problem, memetic algorithm, probability learning, competitive learning, Markov chain.

I. INTRODUCTION

THE MULTIDIMENSIONAL knapsack problem (MKP) is a generalized version of the conventional 0-1 knapsack problem. It is to select a subset of given items and put them into a multidimensional knapsack in order to maximize the total profit subject to the limited capacity of each dimension of the knapsack.

MKP is one of the most popular combinatorial optimization problems with rich practical applications, including cargo loading [1], cutting stock [2], portfolio selection [3], capital budgeting [4] and resource allocation in distributed systems [5].

Let $\mathcal{N}$ be the set of given items, $\mathcal{N} = \{1, \dots, N\}$, $\mathcal{M}$ the set of dimensions, $\mathcal{M} = \{1, \dots, M\}$, W_j the capacity of dimension $j \in \mathcal{M}$, p_i the profit of item $i \in \mathcal{N}$, and w_{ij} the weight of item i in dimension j . Then, MKP can be formulated as follows:

$$\text{Max} \quad f = \sum_{i \in \mathcal{N}} p_i x_i \quad (1)$$

$$\text{Subject to} \quad \sum_{i \in \mathcal{N}} w_{ij} x_i \leq W_j, \forall j \in \mathcal{M}, \quad (2)$$

$$x_i \in \{0, 1\}, \forall i \in \mathcal{N}, \quad (3)$$

where x_i defined in Constraints (3) is a 0-1 decision variable indicating whether item i is selected. The objective of the model (1) is to maximize the total profit. Constraints (2) ensure that the constraint of each dimension must be satisfied.

MKP is known to be NP-hard [6], and so the size of the problem that can be solved by exact algorithms [7]–[9] is limited. Even the latest exact algorithms (such as [9]) still have difficulties for instances with $N = 250$ and $M = 30$. In this case, metaheuristics can be suitable solution methods because they can obtain near optimal solutions within acceptable running time [10]. Memetic algorithm (MA) is a metaheuristic which uses a hybrid search model integrating evolutionary algorithms [11]–[12] and local search to achieve a good balance between the global exploration and the local exploitation. It has been proved that MAs are effective for many combinatorial optimization problems [13]–[16] and in particular for MKP [17]–[19]. This motivates our MA search framework for MKP.

Learning-based mechanisms have been widely advocated in enhancing the search performances of MAs [20]–[22]. For most learning-based MAs, certain kinds of probability models are adopted to extract problem-specific knowledge and generate

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Z. Li is with Key Laboratory of Data Analytics and Optimization for Smart Industry (Northeastern University), Ministry of China, Shenyang, 110819, China (e-mail: zuocheng_li@163.com).

L. Tang is with Institute of Industrial & Systems Engineering, Northeastern University, Shenyang, 110819, China (e-mail: lixintang@mail.neu.edu.cn).

J. Liu is with the School of Business and Economics, Loughborough University, Leicestershire LE11 3TU, U.K. (e-mail: j.y.liu@lboro.ac.uk).

offspring. As Gupta and Ong [23] pointed out, accurate probability models are useful for improving the global exploration of MAs. In this sense, suitable probability models are critical to learning-based MAs, which in general can be represented in different forms, such as a probability distribution or other learning-based networks [24], depending on the problem being solved. The above works motivate us to propose an MA based on probability learning (MA/PL) to solve MKP, in which a problem-specific probability model is introduced.

There are various kinds of probability models in existing learning-based metaheuristics, such as estimation of distribution algorithm (EDA) [25]–[28], cross-entropy algorithm (CEA) [29] and ant colony optimization (ACO) [30]. In MA/PL, a simple probability model (i.e., probability distribution) is used. The probability model of MA/PL is different from EDA, CEA and ACO in that it considers the problem-specific knowledge of MKP. Based on the structures of MKP, the probability model of MA/PL can reveal the inherent relationships among items. We not only consider individual items, but also the dependency of two conjoint items. Two kinds of probability distribution are used in MA/PL: 1) the marginal probability distribution (MPD) for each item to be selected; 2) the joint probability distribution (JPD) for two conjoint items to be selected simultaneously. Whereas MPD reflects the univariate relationships of items, JPD reflects the connections of conjoint items. Complementing each other, MPD and JPD work together to improve the global exploration ability of MA/PL. Very recently, Cai *et al.* [31]–[32] proposed a novel dual population framework which used an external archive (ExA) to store elite solutions found during the search process. It can make good use of the information from ExA for generating high-quality solutions. In MA/PL, we introduce an ExA as well, which is used to build MPD and JPD.

Infeasible solutions might be generated during the search process of MA/PL. In this case, introducing suitable constraint handling techniques is crucial for the search performance [33]. In existing works, the concept of *utility* has been shown useful for designing repair operators for MKP. In [34]–[35], two novel utility-based repair operators for MKP, i.e., RO2 and RO3, were proposed. RO2 and RO3 are guided by the pseudo-utility ratio of each item that is related to the objective function and the constraint violation. It is not uncommon, however, that the values of the objective function and the constraint violation are in different magnitude. Thus, normalization is necessary. In this paper, we advance existing utility-based repair operators and embed the logarithmic utility function (LUF) [36] into the proposed repair operator. LUF can obtain a composite value by weighting the preference scales of the objective function and the constraint violation. Thereby, we propose an LUF-based repair operator (RO_LUF). RO_LUF dynamically considers the profits of items and their contributions to the current overload of each dimension, so it is different from RO2 and RO3.

Local search is another important component of MAs. Given any feasible solution of MKP, local search can be viewed as an inverse process of the repair operator and so may be guided by an LUF similar to that used in the repair operator.

We propose a simple yet effective local search based on LUF (LS_LUF), where neighboring solutions are generated according to another LUF that considers the profits of the items and spare capacity they need in each dimension. Such a problem-specific LS_LUF intends to enhance the local exploitation ability of MA/PL.

The framework of the proposed MA/PL is given in Fig. 1. As shown in Fig. 1, it starts with a randomly generated initial population. The individuals in the population are repaired and improved using RO_LUF and LS_LUF and then the best ones among them are selected to form the initial ExA. Next, the probability models MPD and JPD are built by using ExA, and used to generate a population of the next generation. The individuals in the new population are repaired and improved, the resulting feasible individuals are used to update ExA, and the iteration continues until some stopping criteria are satisfied.

Fig. 1. Sketch of the proposed MA/PL for MKP

We summarize the contributions of this paper as follows.

1) Based on features of MKP, we propose two LUFs considering the profit of each item and the overload/remaining capacity of each dimension. The LUFs are used to measure the tradeoff between the objective and the capacity constraint when performing the repair operation and the local search operation.

2) We propose the probability learning rules for MPD and JPD, which are derived from the competitive learning and the binary Markov chain. Especially JPD considers the dependency of two conjoint items in generating solutions of MKP. Integrating MPD and JPD strengthens the global search of MA/PL.

3) We introduce a random route in MA/PL's sampling operator, such that the overall dependency of items can be potentially revealed based on the dependency of two conjoint items.

The rest of the paper is organized as follows. Section II introduces related work on MA and MKP. In Section III, based on the solution representation of MKP, the proposed RO_LUF and LS_LUF are described. Section IV presents MA/PL in detail. Results of experiments on 179 benchmark instances and a real-life case study are presented in Sections V and VI. Finally, Section VII draws conclusions and proposes future work.

II. RELATED WORK

MAs have been successfully adopted to solve various optimization problems. A comprehensive review of MAs can be found in Chen *et al.* [37]. MA is naturally suitable for combi-

natorial optimization problems, due to the fact that local search can provide basic building blocks in discrete search domains. There have been only a few studies that use MA to solve MKP. Özcan and Başaran [17] proposed a genetic algorithm (GA) with hill climbing heuristic. Rezoug *et al.* [18] proposed two GA-based MAs which incorporated a stochastic local search and a simulated annealing. Sørensen and Sevaux [19] reported an MA with a population management strategy. These works show that MAs can achieve good performance for MKP and so motivate the proposed MA-based search model.

Machine learning has shown powerful ability for enhancing the search performance of MAs. Ong and Keane [21] proposed a Meta-Lamarckian learning to improve the local search of MAs. Rakshit *et al.* [22] reported an adaptive MA based on Q-Learning for multi-robot path planning. Indeed, the foremost highlight of learning-based MAs is the probability model. Different kinds of probability models have been proposed in previous learning-based metaheuristics. Liang *et al.* [25] proposed an EDA with a Gaussian probability model for the function optimization. In [26]–[27], EDAs with Gaussian probability models were used to solve multimodal problems. Shim *et al.* [28] integrated an underlying EDA into the multi-objective search model. The above works show that suitable probability models should reflect the features of problems studied. These also motivate the proposed probability models of MA/PL.

Research on MKP started in the 1950s, from seminal works of Lorie and Savage [38], Markowitz and Manne [39], Loulou and Michaelides [40], and Magazine and Oguz [41]–[42]. In [43], a comprehensive review of studies on MKP is provided. Generally, solution methods for MKP can be classified as optimization-based algorithms and metaheuristics.

There are a number of optimization-based algorithms proposed for solving MKP which apply optimization methods as part of the solution process. Hill *et al.* [44] proposed a problem reduction heuristic for MKP, where they first formulated MKP as a Lagrangian dual problem and then presented a heuristic for solving the estimated core problem. Yoon *et al.* [45] reported a Lagrangian method using the concept of Lagrangian capacity/relaxation for MKP. Rong *et al.* [46] solved a kind of multicriteria MKP with k -min objectives using hybrid algorithms. Other optimization-based methods, such as branch and bound [1], [7] and collaborative methods [8], were also proposed to solve MKP. Most of the above algorithms can only get optimal solutions for instances of small or medium sizes. Although some of them can obtain near optimal solutions for large-sized ones, such as [8], [44]–[45], the computation time may increase dramatically when the scale of instances increases.

Metaheuristics are more suitable for solving large scale MKP. Chih [47] reported a particle swarm optimization (PSO) with a self-adaptive check and a repair operator (SACRO) for MKP. Zhang *et al.* [48] solved MKP using a binary artificial algae algorithm (BAAA) derived from the artificial algae algorithm. Zouache *et al.* [49] presented a quantum-inspired firefly algorithm with PSO (QIFAPSO) for MKP. López *et al.* [50] proposed a hybrid algorithm with a binary PSO and ge-

netic operators (HPSOGO) for MKP. Bhattacharjee and Sarmah [51] proposed two metaheuristics for MKP, i.e., BCSA and BFA, based on cuckoo search and firefly algorithm in continuous domains. Other metaheuristics were also used to solve MKP, such as binary differential search (TR-BDS) [52], binary harmony search (NBHS2) [53], k -means transition ranking cuckoo search (KMTR-Cuckoo) and black hole (KMTR-BH) [54], hybrid binary differential evolution with RO3 (HBDE-RO3) [35], modified multi-verse optimization (MMVO) [60] and teaching-learning based optimization (TLBO) [61]. The above papers showed advantages of metaheuristics for MKP.

However, for MKP, studies that incorporate machine learning into metaheuristics are relatively unexplored. Wang *et al.* [34] presented a hybrid EDA (HEDA) for MKP, which adopted a probability matrix to record the distribution of elite solutions. Arin and Rabadi [55] proposed a randomized priority search model (Meta-RaPS) for MKP, whereby presented a Meta-RaPS EDA and a Meta-RaPS Q by integrating EDA and Q-learning into Meta-RaPS. Rojas-Morales *et al.* [56] presented an ant knapsack algorithm based on opposite learning for MKP. The above works show that there is still a gap between machine learning and metaheuristics for solving MKP.

III. PROPOSED PROBLEM-DEPENDENT HEURISTICS

In this section, we describe the solution representation and the problem-dependent heuristics, i.e., RO_LUF and LS_LUF that are used as repair and local search operators in MA/PL.

A. Solution Representation

In MA/PL, a string of N bits is used to represent a solution. Value 1 of a bit represents that the item is selected while value 0 means not selected. Given any solution, it may or may not be feasible. If infeasible, we can repair it by taking some items out. For feasible solutions we can try to improve it by adding more items in. In both operations, we propose to choose items based on LUFs. The idea of LUF stems from the utility concept in quality management areas [36]. We define an LUF as a metric that measures the effect of a given item on the total profit and constraints, and use LUFs to determine the priority of candidate items in RO_LUF and LS_LUF. Fig. 2 shows an infeasible solution π and candidate items in it for RO_LUF, as well as a feasible solution π' and candidate items in it for LS_LUF.

Fig. 2. Illustration of candidate items for RO_LUF and LS_LUF

B. Proposed Repair Operator Based on LUF (RO_LUF)

The repair operator for infeasible solutions can be seen as a readjustment of the load in each dimension. It is necessary to consider the overload in each dimension. Let $\mathbf{ol} = [ol_1, \dots, ol_M]$ be the overloads in the M dimensions for an infeasible solution $\boldsymbol{\pi} = [\pi_1, \dots, \pi_N]$. Each element of $\mathbf{ol}$ can be calculated as

$$ol_j = \sum_{i \in \mathcal{N}} w_{ij} \pi_i - W_j, j \in \mathcal{M}, \quad (4)$$

where the first term is the load in dimension j . Obviously, a negative or zero value of ol_j indicates that the capacity constraint in dimension j is satisfied.

In case ol_j is positive for some j , the solution is infeasible. To repair the solution, we take out items one by one until the solution becomes feasible. Denote Cr as the set of candidate items to be taken out (for example, $Cr = \{1, 2, 5, 6\}$ in Fig. 2). To determine the priority of the candidate items, we calculate an LUF value for each item $i \in Cr$ using two attributes of the item. One is its profit p_i , another reflects its impact on the capacities in all dimensions, which is defined as

$$\psi_i = \sum_{j \in \mathcal{M}} \max \{ \varepsilon, ol_j - w_{ij} \}, \forall i \in Cr, \quad (5)$$

where we set $\varepsilon = 0.001$ to ensure $\psi_i > 0$. Then, the value of LUF in RO_LUF is defined as

$$\sigma_i = \lambda_1 \cdot \log(\max_{i \in Cr} \{p_i\} / p_i) + \lambda_2 \cdot \log(\max_{i \in Cr} \{\psi_i\} / \psi_i), \quad (6)$$

where λ_1 and λ_2 are coefficients. Without loss of generality, we set $\lambda_1 = \lambda_2 = 0.5$ to represent the equal importance of the two attributes for given items. Obviously, when changing the bits of items in Cr from 1 to 0, an item that has larger LUF will result in smaller decrease of total profit while making more contribution in getting the capacities satisfied.

In RO_LUF, the item in Cr with the largest LUF is taken out, the related bit is changed from 1 to 0, and this item is taken out of Cr . If the solution is still infeasible, the values of ψ_i and σ_i are recalculated to identify the next item to be taken out. This process continues until the solution becomes feasible. The proposed RO_LUF is given in Algorithm 1.

Algorithm 1: Repair Operator Based on LUF (RO_LUF)

Input: infeasible solution $\boldsymbol{\pi}$
Output: feasible solution $\boldsymbol{\pi}'$, objective $f(\boldsymbol{\pi}')$

- 1: Initialize the set of candidate items Cr ; // as shown in Figure 3
- 2: Initialize remaining capacities of knapsacks $\mathbf{rc} = [rc_1, \dots, rc_M]$; //Equation (5)
- 3: Assign attributes ξ_i and ψ_i to each item $i \in Cr$; // Equations (6) and (7)
- 4: **Do**
- 5: Calculate the LUF σ_i for each item $i \in Cr$; //Equation (8)
- 6: Select an item s from Cr with the largest LUF; if there is a tie, select it arbitrarily;
- 7: Set $\pi_s = 0$; //change the bit of item s
- 8: Set $Cr = Cr \setminus \{s\}$; //update Cr
- 9: Set $rc_j = rc_j - w_{sj}, j \in \mathcal{M}$; //update remaining capacity of each knapsack
- 10: Assign attributes ξ_i and ψ_i to each item $i \in Cr$; // Equations (6) and (7)
- 11: **While** $(\sum_{i \in Cr} \psi_i = 0)$
- 12: Set $\boldsymbol{\pi}' = \boldsymbol{\pi}$ and calculate $f(\boldsymbol{\pi}')$;
- 13: **Return** $\boldsymbol{\pi}', f(\boldsymbol{\pi}')$

In the illustration example in Fig. 2, RO_LUF is applied on the infeasible solution $\boldsymbol{\pi}$. After one iteration taking out item 1 that has the largest LUF, a feasible solution $\boldsymbol{\pi}'$ is obtained.

C. Proposed Local Search Based on LUF (LS_LUF)

As an inverse process of the repair operator, we here consider the spare capacity in each dimension. For a feasible solution $\boldsymbol{\pi}' = [\pi'_1, \dots, \pi'_N]$, let $\mathbf{sc} = [sc_1, \dots, sc_M]$ be the spare capacities in the M dimensions. Each element can be calculated as

$$sc_j = W_j - \sum_{i \in \mathcal{N}} w_{ij} \pi'_i, \forall j \in \mathcal{M}. \quad (7)$$

Let Cl be the set of items not in the knapsack (for example, $Cl = \{1, 3, 4\}$ in Fig. 2), such that adding each of them alone does not result in capacity violation

$$Cl = \{i | \pi'_i = 0 \wedge sc_j - w_{ij} \geq 0, \forall i \in \mathcal{N} \text{ for all } j \in \mathcal{M}\}. \quad (8)$$

We calculate another LUF for item $i \in Cl$, again using two attributes, the profit p_i and a specially defined τ_i

$$\tau_i = \sum_{j \in \mathcal{M}} (sc_j - w_{ij}) + \varepsilon, i \in Cl, \quad (9)$$

where $\varepsilon = 0.001$ is used to guarantee $\tau_i > 0$. Then, this LUF for item $i \in Cl$ is calculated as

$$\delta_i = \lambda_1 \cdot \log(\max_{i \in Cl} \{p_i\} / p_i) + \lambda_2 \cdot \log(\max_{i \in Cl} \{\tau_i\} / \tau_i), \quad (10)$$

where $\lambda_1 = \lambda_2 = 0.5$ are two coefficients. In LS_LUF, we identify the item with the smallest LUF and change its bit from 0 to 1.

LS_LUF is also an iterative procedure, each time choosing one item to add in, then recalculating spare capacity of each dimension, updating Cl , and recalculating τ_i and δ_i , until Cl is empty. This procedure is given in Algorithm 2.

Algorithm 2: Local Search Based on LUF (LS_LUF)

Input: feasible solution $\boldsymbol{\pi}'$, objective $f(\boldsymbol{\pi}')$
Output: improved solution $\boldsymbol{\beta}$

- 1: Initialize spare capacities of the M dimensions $\mathbf{sc} = [sc_1, \dots, sc_M]$; //Eq. (7)
- 2: Initialize the set of candidate items Cl ; // Eq. (8)
- 3: **While** $(Cl \neq \emptyset)$ **do**
- 4: Calculate τ_i of each item $i \in Cl$; // Eq. (9)
- 5: Calculate the LUF δ_i for each item $i \in Cl$; //Eq. (10)
- 6: Select an item s from Cl with the smallest LUF; break ties arbitrarily;
- 7: Set $\pi'_s = 1$; //change the bit of item s
- 8: Set $sc_j = sc_j - w_{sj}, j \in \mathcal{M}$; //update spare capacity of each dimension
- 9: Set $f(\boldsymbol{\pi}') = f(\boldsymbol{\pi}') + p_s$; //update the objective
- 10: Update Cl by removing all such item i that $sc_j - w_{ij} < 0$ for any $j \in \mathcal{M}$;
- 11: **End do**
- 12: Set $\boldsymbol{\beta} = \boldsymbol{\pi}', f(\boldsymbol{\beta}) = f(\boldsymbol{\pi}')$;
- 13: **Return** $\boldsymbol{\beta}, f(\boldsymbol{\beta})$

Note that we use LS_LUF to improve only the feasible solutions that are generated by repairing infeasible solutions with RO_LUF, such as $\boldsymbol{\pi}'$ generated by repairing $\boldsymbol{\pi}$ in Fig. 2. This is because, in our preliminary tests, we observed that executing LS_LUF for all feasible solutions produced by MA/PL's global search is very likely to cause a premature convergence.

IV. MA/PL FOR MKP

In MP/PL, we use two learning rules: 1) the learning rule based on the competitive learning for MPD; 2) the learning rule based on the proposed binary Markov chain for JPD. Using the learning rules, the dependency of items can be captured when a new population is produced by using a random sampling route.

A. ExA and Updating Mechanism

ExA is updated at each generation of MA/PL, so that it always contains high-quality solutions. Let PS be the population size of MA/PL, AL the size of ExA, $\pi^p(gen) = \{\pi_1^p(gen), \dots, \pi_{PS}^p(gen)\}$ the population of MA/PL at generation gen , and $\pi^e(gen) = \{\pi_1^e(gen), \dots, \pi_{AL}^e(gen)\}$ the set of ExA at generation gen . ExA can be initialized and updated by Algorithm 3.

Algorithm 3: Initialization and Updating of ExA

Input: $\pi^p(gen), \pi^e(gen)$
Output: $\pi^e(gen)$

```

1: If ( $gen = 0$ ) then
2:   Find the best  $AL$  individuals from  $\pi^p(gen)$ ;
3:   Record these best  $AL$  individuals to  $\pi^e(gen)$ ; //initialize the ExA
4:   Rank individuals of  $\pi^e(gen)$  in descending order of their objective values;
5: Else
6:   Find the best individual  $\pi^{th}$  in  $\pi^p(gen)$ ;
7:   If ( $f(\pi^{th}) > f(\pi_{AL}^e(gen))$ ) then
8:     Remove  $\pi_{AL}^e(gen)$  from ExA
9:     Insert  $\pi^{th}$  into a proper position of the ranked list of individuals in ExA.
10:  End If
11: End If
12: Return  $\pi^e(gen)$ 

```

In Algorithm 3, ExA is incrementally updated by replacing the worst individual in it with the best in the current population if the latter is better. Thus, the information of ExA that reflects elite solutions can be gradually enriched. No more than one individual from the current population is added to ExA to avoid premature convergence. MPD and JPD are all based on ExA, so it can be seen as a set of training data for our learning rules.

B. Learning Rule Based on Competitive Learning for MPD

The ideas of the proposed learning rule for MPD derive from competitive learning [57]. The competitive learning can make good use of the information of high-quality samples in learning probability models. We represent MPD as the union of the univariate marginal distributions of all items. Then the existing learning rule in [34] and [57] is used. However, we learn MPD from ExA rather than from elite individuals in current population, which is different from [34]. Let $MPr(gen)$ be the union of the marginal distribution at generation gen . It is defined as

$$MPr(gen) = \begin{bmatrix} mpr_{0,1}(gen) \dots mpr_{0,i}(gen) \dots mpr_{0,N}(gen) \\ mpr_{1,1}(gen) \dots mpr_{1,i}(gen) \dots mpr_{1,N}(gen) \end{bmatrix}_{2 \times N}, \quad (11)$$

where $\sum_{k=0}^1 mpr_{k,i}(gen) = 1$ ($i \in \mathcal{N}$) and $mpr_{k,i}(gen)$ represents the probability value for the bit of item i to be k . Note that $MPr(gen)$ is initialized with $mpr_{k,i}(gen=0) = 0.5$ ($i \in \mathcal{N}$ and $k \in \{0,1\}$). Let LR be the learning rate of MA/PL. The competitive learning method for estimating MPD is given in Algorithm 4.

From Algorithm 4, it can be seen that MPDs between two consecutive generations are connected with the learning rate LR , and the probability values of items are gradually moved towards $TmpPro/AL$ obtained from elite solutions (Line 6). This is similar to the weight update rule in competitive learning. In this case, MPD is not only determined by its previous statuses, but also related to the information of the current ExA (Lines 3 to 6). Thus, the information of elite solutions can be adequately used by MA/PL, so as to predict promising search regions.

Algorithm 4: Learning Rule based on Competitive Learning for MPD

Input: $\pi^e(gen-1), MPr(gen-1)$
Output: $MPr(gen)$

```

1: For  $i = 1$  to  $N$  do
2:   Set  $TmpPro = 0$ ;
3:   For  $k = 1$  to  $AL$  do
4:     Set  $TmpPro = TmpPro + \pi_{k,i}^e(gen-1)$ ; //calculate the total number of bit 1
5:   End For
6:   Set  $mpr_{1,i}(gen) = (1 - LR) \cdot mpr_{1,i}(gen-1) + LR \cdot TmpPro / AL$ ; //competitive learning
7:   Set  $mpr_{0,i}(gen) = 1 - mpr_{1,i}(gen)$ ; //competitive learning
8: End For
9: Return  $MPr(gen)$ 

```

C. Learning Rule Based on Binary Markov Chain for JPD

The purpose of JPD is to get the dependency of two conjoint items. Our learning rule for JPD is motivated from the binary signal source in telegraph systems [58]. The telegraph system outputs a series of pulses with high (H) and low (L) symbols (i.e., an N -bit string) in a temporal order. It can be modeled by a Markov source with given memory length. The memory length defines the dependency structure of signals. If we only consider a dependency of two conjoint symbols, the memory length is 1 (i.e., first-order Markov source). The first-order Markov source can be analyzed by a binary Markov chain. As shown in Fig. 3, the dependency of two conjoint symbols can be captured using the binary Markov chain, such as the $(i-1)$ th and the i th signals.

Fig. 3. First-order Markov source and dependency of symbols

It is interesting that the output of the first-order Markov source is equivalent to the solution representation of MKP (see Fig. 4), when regarding the “temporal order” as the “spatial order”. Thus, it is reasonable to extract the bivariate dependency of conjoint items by using a binary Markov chain.

Fig. 4. Output of Markov source and MKP's solution

In Fig. 4, the probability distributions of points 2 to N are often different, which are determined by the features of MKP. Therefore, the non-stationary binary Markov chain is used in our learning rule, as shown in Fig. 5. There are $N - 1$ pairs of conjoint items for the order $2 \rightarrow \dots \rightarrow N$. For each point of 2 to N , we first calculate the one-step transition probability matrix (OSTPM) from ExA and in turn obtain JPD based on OSTPM.

Let $pro_{i,p}$ ($i \in \{2, \dots, N\}$ and $p \in \{1, \dots, 4\}$) be the OSTPM for the transitions of states on the i th point, where $pro_{i,1} : (0 \rightarrow 0)$, $pro_{i,2} : (0 \rightarrow 1)$, $pro_{i,3} : (1 \rightarrow 1)$ and $pro_{i,4} : (1 \rightarrow 0)$. Then, OSTPM for the i th point can be calculated using Algorithm 5.

Fig. 5. Spatial order and non-stationary binary Markov chain (N items)**Algorithm 5: Calculation of One-step Transition Probability Matrix**

Input: $\pi^e(\text{gen} - 1)$, spatial point i
Output: $pro_{i,k}, k=1, \dots, 4$

```

1: Set  $pro_{i,k} = 0$  (for  $k=1, \dots, 4$ ),  $count_0 = 0$ ,  $count_1 = 0$ ; //initialization
2: For  $ii = 1$  to  $AL$  do
3:   If  $(\pi_{ii,i-1}^e(\text{gen} - 1) = 0)$  then
4:     Set  $count_0 = count_0 + 1$ ; //number of individuals for  $\pi_{ii,i-1}^e(\text{gen} - 1) = 0$ 
5:     If  $(\pi_{ii,i}^e(\text{gen} - 1) = 0)$  then
6:       Set  $pro_{i,1} = pro_{i,1} + 1$ ; Else Set  $pro_{i,2} = pro_{i,2} + 1$ ;
7:     End If
8:   Else
9:     Set  $count_1 = count_1 + 1$ ; //number of individuals for  $\pi_{ii,i-1}^e(\text{gen} - 1) = 1$ 
10:    If  $(\pi_{ii,i}^e(\text{gen} - 1) = 1)$  then
11:      Set  $pro_{i,3} = pro_{i,3} + 1$ ; Else Set  $pro_{i,4} = pro_{i,4} + 1$ ;
12:    End If
13:  End If
14: End For
15: If  $(count_0 \neq 0)$  then
16:   Set  $pro_{i,1} = pro_{i,1} / count_0$  and  $pro_{i,2} = pro_{i,2} / count_0$ ; //probability value
17: End If
18: If  $(count_1 \neq 0)$  then
19:   Set  $pro_{i,3} = pro_{i,3} / count_1$  and  $pro_{i,4} = pro_{i,4} / count_1$ ; //probability value
20: End If
21: Return  $pro_{i,k}$ , for  $k=1, \dots, 4$ 

```

Based on OSTPM, one can calculate JPD for each spatial point $i \in \{2, \dots, N\}$. Let $JPr^i(\text{gen}) = [jpr_{k,l}^i(\text{gen})]_{2 \times 2}$ be the JPD for spatial point i at generation gen . Then, it satisfies

$$\sum_{k \in \{0,1\}} \sum_{l \in \{0,1\}} jpr_{k,l}^i(\text{gen}) = 1, \quad (12)$$

where $jpr_{k,l}^i(\text{gen})$ is the probability of $\pi_{i-1}=k$ and $\pi_i=l$ at generation gen . Apparently, $JPr^i(\text{gen})$ can be seen as a discrete joint distribution of conjoint items $i-1$ and i in the order of $2 \rightarrow \dots \rightarrow N$. Then, we can calculate $jpr_{k,l}^i(\text{gen})$ as

$$jpr_{k,l}^i(\text{gen}) = \begin{cases} \left(\frac{1}{AL} \cdot \left(AL - \sum_{ii \in \{1, AL\}} \pi_{ii,i-1}^e(\text{gen}-1) \right) \right) \cdot pro_{i,\rho} & k=0 \\ \left(\frac{1}{AL} \cdot \sum_{ii \in \{1, AL\}} \pi_{ii,i-1}^e(\text{gen}-1) \right) \cdot pro_{i,\rho} & \text{otherwise} \end{cases}, \quad (13)$$

where ρ is set to 1, 2, 3 and 4 for conditions $(k=0) \wedge (l=0)$, $(k=0) \wedge (l=1)$, $(k=1) \wedge (l=1)$ and $(k=1) \wedge (l=0)$ respectively.

D. Sampling Operator Based on MPD and JPD

Accurate probability models are beneficial to improve the global exploration of learning-based MAs [23]. Indeed, there are many probability models that can capture the precise dependency of all random variables, such as Bayesian networks. However, such probability models in general take more computation time. Hence, we only consider the dependency of two conjoint items in the proposed JPD. Besides, there may be a dependency between any pair of items. In this case, many such dependencies would not be considered if one fixed sampling route is always used. Thus, we propose to take a random sampling route, so that the dependency between any items can be

potentially considered. In our sampling operator, we first generate a random permutation of integers $1, \dots, N$: $r = [r_1, \dots, r_N]$ and then use it as the sampling route, as shown in Fig. 6.

Fig. 6. Illustration of sampling route ($N = 5$)

Let $cpro(\pi_{r_i}=l | \pi_{r_{i-1}}=k), i \in \{2, \dots, N\}$ be the conditional probability of $\pi_{r_i}=l$ given $\pi_{r_{i-1}}=k$ in terms of sampling route r . We can calculate it via the conditional probability formula as

$$cpro(\pi_{r_i}=l | \pi_{r_{i-1}}=k) = jpr_{k,l}^{r_i}(\text{gen}) / \sum_{k \in \{0,1\}} jpr_{k,l}^{r_i}(\text{gen}). \quad (14)$$

Afterwards, the offspring can be produced by integrating MPD and JPD. The sampling operator is given in Algorithm 6.

Algorithm 6: Sampling Operator of MA/PL (Based on MPD and JPD)

Input: $\pi^e(\text{gen} - 1)$
Output: $\pi^e(\text{gen})$

```

1: Estimate  $MPR(\text{gen})$  using competitive learning method; //Algorithm 4
2: For  $k = 1$  to  $PS$  do
3:   Randomly generate a sampling route  $r = [r_1, \dots, r_N]$ ;
4:   Set  $\pi_{k,i}^e(\text{gen}) = 0$ , for  $i = 1, \dots, N$ ; //initialize each individual with bit 0
5:   For  $i = 1$  to  $N$  do
6:     If  $(i = 1)$  then //the first bit of sampling route  $r$ 
7:       If  $(\text{random}[0,1] \leq mpr_{1,r_1}(\text{gen}))$  then //based on the MPD
8:         Set  $\pi_{k,r_1}^e(\text{gen}) = 1$ ;
9:       End If
10:      Else //the other bits of sampling route  $r$ 
11:        Calculate one-step transition probability matrix  $pro_{r_i, \rho=1, \dots, 4}$ ; //Algorithm 5
12:        Calculate JPD  $JPr^i(\text{gen})$  based on  $pro_{r_i, \rho=1, \dots, 4}$  //Eq. (13)
13:        Obtain conditional probability  $cpro(\pi_{r_i}=l | \pi_{r_{i-1}}=\pi_{k,r_{i-1}}^e(\text{gen}))$ ; //Eq. (14)
14:        Set  $TmpPro = cpro(\pi_{r_i}=1 | \pi_{r_{i-1}}=\pi_{k,r_{i-1}}^e(\text{gen})) \cdot mpr_{1,r_i}(\text{gen})$ ;
15:        Set  $TmpPro = TmpPro / \sum_{l \in \{0,1\}} cpro(\pi_{r_i}=l | \pi_{r_{i-1}}=\pi_{k,r_{i-1}}^e(\text{gen})) \cdot mpr_{l,r_i}(\text{gen})$ ;
16:        If  $(\text{random}[0,1] \leq TmpPro)$  then //based on integration of MPD and JPD
17:          Set  $\pi_{k,r_i}^e(\text{gen}) = 1$ ;
18:        End If
19:      End If
20:    End For
21:  End for
22: Return  $\pi^e(\text{gen})$ 

```

In Algorithm 6, a sampling route is first generated (Line 3), which is helpful in capturing more potential dependency among items. Then, the first bit of the sampling route is generated from MPD (Lines 6 to 11), while other bits are generated by integrating JPD and MPD (Lines 11 to 20). Thus, MPD and JPD can complement each other in the global search of MA/PL.

E. Overall Procedure of Proposed MA/PL

Let $genMax$ be the maximum generations and $gbest(\text{gen})$ the best feasible individual found so far. The overall procedure of MA/PL is given in Algorithm 7. It shows that MPD and JPD are all estimated from ExA (Lines 5 and 10), which is useful to get more information of promising search regions. Moreover, we use RO_LUF and LS_LUF to repair and improve new solutions (Lines 4 and 9). With this precise model, MA/PL can hopefully achieve a good balance between exploration and exploitation.

F. Time Complexity of Proposed MA/PL

The time complexities of calculating the objective function, initializing population and sampling new individuals are all

$O(PS \cdot N)$. In the worst case, the time complexities of RO_LUF and LS_LUF are all $O(PS \cdot N \cdot (3 \cdot N + M))$. The time complexities of initializing and updating ExA are all $O(AL)$. The time complexity of learning and initializing MPD are all $O(N)$. The time complexity of learning JPD is $O((N-1) \cdot AL + N)$. Thus, the total time complexity is $genMax \cdot \{O(PS \cdot N^2 + PS \cdot N \cdot M) + O(N \cdot AL)\}$.

Algorithm 7: Overall Procedure of Proposed MA/PL

Input: PS, AL, LR
Output: $gbest(genMax)$

```

1: For  $gen = 0$  to  $genMax$  do
2:   If ( $gen = 0$ ) then
3:     Initialize the population  $\pi^p(gen)$  and  $\mathbf{MP}^r(gen)$ ;
4:     Perform RO_LUF and LS_LUF; //perform the repair and local search operators
5:     Initialize ExA  $\pi^e(gen)$  from  $\pi^p(gen)$ ; //Algorithm 3
6:     Initialize global best feasible individual  $gbest(gen)$ ;
7:   Else
8:     Perform the sampling operator and generate  $\pi^s(gen)$ ; //Algorithms 4, 5 and 6
9:     Perform RO_LUF and LS_LUF; //perform the repair and local search operators
10:    Update ExA  $\pi^e(gen)$ ; //Algorithm 3
11:    Update global best feasible individual  $gbest(gen)$ ;
12:   End If
13: End For
14: Return  $gbest(genMax)$ 

```

V. EXPERIMENTAL RESULTS AND COMPARISONS

A. Experimental Setup and Parameter Settings

To verify the performance of the proposed MA/PL, we test it on four sets of well-known benchmark data for MKP. The sets are related to different sizes. Test set I contains 11 large scale instances with $M=15$ to 100 and $N=100$ to 2500 (available from <http://hces.bus.olemiss.edu/tools.html>). Test set II contains 20 medium scale instances with $M=5$ and 10 and $N=100$ (available from ORLIB [59]). Test set III contains 58 small scale instances from $M=2$ to 30 and $N=10$ to 105 (available from ORLIB). Test set IV consists of 90 large scale instances with $M=5$ to 30 and $N=500$ (available from ORLIB). Therefore, there are a total of 179 instances. MA/PL is coded in C++ and run on a PC with Intel Core-i7 3.4 GHz processor and 16 GB memory.

MA/PL is independently run 30 times for each instance. Results are reported with 8 metrics, i.e., *BEST*, *AVG*, *SD*, *SR*, *Min.Dev*, *Ave.Dev*, *Max.Dev* and *Var.Dev*. *BEST*, *AVG*, *SD* and *SR* are the best value, mean value, standard deviation and success rate. *Min.Dev*, *Ave.Dev* and *Max.Dev* are the minimum, average and maximum percentage deviations from best-known solutions (BKS). *Var.Dev* is the variance of percentage deviations. We note that the values of BKS for all the instances are collected from the most currently published works for MKP (available at <http://dx.doi.org/10.13140/RG.2.2.14761.95841>). In Tables I to XIV, each number in the row #Best is the total number of instances for which the relevant algorithm obtained the best result in terms of different quality metrics. Besides, we highlight the best quality metric of each instance found by relevant algorithms in bold, and mark the column of *BEST* of certain instances with symbol “*” if BKS is attained. To check the significance of the differences between MA/PL and other algorithms, the p -values based on the non-parametric Friedman test at the 95% confidence level (CI) are reported as well.

In subsections V-C and V-D we adopt the results of existing algorithms from relevant references. The average CPU times (s) (i.e., CPU (s)) of MA/PL are listed for information. Due to the differences in programming environments and computer con-

figurations, it is hard to accurately compare CPU times of other algorithms. However, we find that CPU times of MA/PL are all acceptable and thus running times will not be further discussed.

The parameters of MA/PL include the population size PS , the size of ExA AL and the learning rate LR . To obtain the appropriate parameters, relevant experiments based on the design-of-experiment (DOE) are carried out. We set the parameters of MA/PL as follows: $PS=150$, $AL=20$ and $LR=0.01$. More details about the parameter settings of MA/PL can be found at <http://dx.doi.org/10.13140/RG.2.2.23150.56644>.

B. Effectiveness of MA/PL's Components

1) Effectiveness of proposed learning rules and local search

To study the impact of the proposed learning rules, random sampling route and local search, we introduce the following variants. 1) MA/PL_V1: it is different from MA/PL in that the learning method for JPD, the random sampling route and LS_LUF are removed. That is, MA/PL_V1's search behavior is only guided by the learning and sampling operators of MPD. 2) MA/PL_V2: it is the same as MA/PL_V1 except that the learning method for JPD is added. 3) MA/PL_V3: it is obtained by replacing the fixed sampling route in MA/PL_V2 by random sampling route. Obviously, the comparisons of these variants and MA/PL can reveal the effectiveness of the learning rule for JPD, the random sampling route and LS_LUF. For fair comparison, we first run MA/PL with $genMax = 5000$ and record the CPU time. Then, the other variants are run for the same CPU time. Note that parameters in the variants are the same as in MA/PL. Comparison results are listed in Table I. In terms of the percentage deviations from BKS, i.e., *Dev* (%), the related violin plots are given in Fig. 7.

TABLE I
COMPARISONS OF MA/PL_V1, MA/PL_V2, MA/PL_V3 AND MA/PL

Instance	MA/PL_V1		MA/PL_V2		MA/PL_V3		MA/PL	
	<i>AVG</i>	<i>SD</i>	<i>AVG</i>	<i>SD</i>	<i>AVG</i>	<i>SD</i>	<i>AVG</i>	<i>SD</i>
Hp1	3417.1	3.49	3417.5	2.51	3418.0	0.00	3418.0	0.00
Hp2	3181.4	7.62	3184.8	4.49	3184.9	4.07	3186.0	0.00
Pb1	3088.1	4.76	3088.9	5.92	3090.0	0.00	3090.0	0.00
Pb2	3183.2	6.39	3183.2	6.39	3184.0	5.16	3186.0	0.00
Pb4	94799.0	6.00	95168.0	0.00	95168.0	0.00	95168.0	0.00
Pb5	2137.8	6.64	2139.0	0.00	2139.0	0.00	2139.0	0.00
Pb6	769.9	6.62	774.6	4.20	775.5	2.51	776.0	0.00
Pb7	1034.2	0.37	1033.2	2.94	1034.1	0.50	1035.0	0.00
Pet2	87061.0	0.00	87061.0	0.00	87061.0	0.00	87061.0	0.00
Pet3	4015.0	0.00	4015.0	0.00	4015.0	0.00	4015.0	0.00
Pet4	6120.0	0.00	6120.0	0.00	6120.0	0.00	6120.0	0.00
Pet5	12399.0	3.00	12399.3	2.49	12400.0	0.00	12400.0	0.00
Pet6	10615.7	5.15	10617.5	2.51	10618.0	0.00	10618.0	0.00
Pet7	16527.0	14.59	16526.8	18.40	16531.8	8.04	16537.0	0.00
10.100.00	23003.0	39.00	23014.8	30.23	23023.8	30.14	23046.8	15.82
10.100.01	22703.9	37.37	22710.6	51.77	22729.7	27.47	22750.7	28.17
10.100.02	22027.1	66.25	22027.5	46.92	22045.8	47.54	22105.1	28.10
10.100.03	22602.1	49.81	22596.1	39.46	22603.0	50.98	22714.0	12.44
10.100.04	22569.4	103.43	22566.1	96.72	22589.7	124.26	22726.7	39.59
10.100.05	22555.7	29.73	22573.3	54.69	22577.9	47.48	22653.9	43.92
10.100.06	21700.3	20.61	21703.0	5.21	21705.5	8.08	21804.0	54.43
10.100.07	22425.6	83.92	22429.8	49.23	22450.6	60.88	22536.7	39.08
10.100.08	22277.5	27.52	22281.9	44.92	22289.9	38.20	22508.1	15.80
10.100.09	22547.2	30.93	22547.2	30.93	22555.3	26.85	22558.2	14.90
#Best	3	4	5	6	9	10	24	21
p -value	0.000		0.000		0.000			

Note: The p -values of “MA/PL_V2 vs. MA/PL_V1” and “MA/PL_V3 vs. MA/PL_V2” are 0.012 and 0.000, respectively.

From Table I, we can see that MA/PL_V2 outperforms MA/PL_V1, clearly showing that the integration of MPD and JPD enhances the search ability. That is, considering the dependency of conjoint items leads to better the global exploration. Meanwhile, MA/PL_V3 outperforms MA/PL_V2 for almost all the instances in terms of *AVG*. Especially, there are a total of 13 instances for which the *SD* values of MA/PL_V3 are smaller than those of MA/PL_V2. Thus, by capturing the potential dependencies among all items, the random sampling route improves the effectiveness and the robustness of MA/PL. Moreover, MA/PL outperforms MA/PL_V3 which demonstrates the effectiveness of the proposed LS_LUF. From Fig. 7, it is clear that the distributions of MA/PL_V3 are better than MA/PL_V1 and MA/PL are better than MA/PL_V3, in terms of both the dispersions and the interquartile ranges.

Fig. 7. Violin plots for considered comparisons

In addition, we also observe the 95% CI for the solutions of MA/PL and the three variants for the other instances, i.e., Hp2, Knap 28, MK_gk04, Pb6, Pet7 and Sento2. The related CI's are shown in Fig. 8. We can see that MA/PL can obtain relatively more consistent solutions than the other three variants. These CI's also reveal the performance of MA/PL's components.

Fig. 8. Error bars (95% CI) for MA/PL (V1~V3) and MA/PL

To further reveal the effectiveness of the proposed LS_LUF, two variants of MA/PL that use advanced local search methods in existing works for MKP are introduced. The two variants are MA/PL_NLS and MA/PL_ILS, which are the same as MA/PL

except that LS_LUF is replaced by the neighborhood local search [62] and the intensity-based local search [34]. The same CPU times used in Table I are adopted for each algorithm. Results are given in Table II. In addition, in Fig. 9, we give the 95% CI for the solutions of MA/PL and the two variants for the same instances that are shown in Fig. 8. The results of Table II and Fig. 9 reveal the competitiveness of the proposed LS_LUF.

2) Effectiveness of proposed repair operator

To verify the effectiveness and efficiency of the proposed RO_LUF, we compare MA/PL with two variants, namely, MA/PL_RO2 and MA/PL_RO3. They are the same as MA/PL except that RO_LUF is replaced by RO2 [34] and RO3 [35]. RO2 and RO3 are all the most effective repair heuristics for MKP in existing works. Note that the same stopping criterion (*genMax*=5000) is used for two variants and MA/PL. Relevant results on Test set II are reported in Table III. Meanwhile, the average CPU times for three algorithms are given in Fig. 10.

TABLE II
COMPARISONS OF MA/PL NLS, MA/PL_ILS AND MA/PL

Instance	MA/PL NLS		MA/PL_ILS		MA/PL	
	<i>AVG</i>	<i>SD</i>	<i>AVG</i>	<i>SD</i>	<i>AVG</i>	<i>SD</i>
Hp1	3416.7	3.83	3416.9	3.22	3418.0	0.00
Hp2	3180.5	9.88	3183.5	5.91	3186.0	0.00
Pb1	3090.0	0.00	3089.9	0.97	3090.0	0.00
Pb2	3182.6	6.56	3183.2	6.06	3186.0	0.00
Pb4	95167.9	0.51	94797.5	9.62	95168.0	0.00
Pb5	2138.9	0.37	2138.8	0.91	2139.0	0.00
Pb6	772.7	6.02	772.8	5.86	776.0	0.00
Pb7	1034.6	0.50	1033.9	2.22	1035.0	0.00
Pet2	87061.0	0.00	87061.0	0.00	87061.0	0.00
Pet3	4015.0	0.00	4015.0	0.00	4015.0	0.00
Pet4	6120.0	0.00	6120.0	0.00	6120.0	0.00
Pet5	12398.9	3.09	12399.3	1.86	12400.0	0.00
Pet6	10616.5	4.17	10616.6	3.97	10618.0	0.00
Pet7	16533.7	6.89	16532.8	7.75	16537.0	0.00
10.100.00	23028.9	30.47	23025.9	32.62	23046.8	15.82
10.100.01	22724.8	34.73	22722.9	51.61	22750.7	28.17
10.100.02	22049.8	77.84	22061.2	62.58	22105.1	28.10
10.100.03	22691.1	53.64	22700.1	45.02	22714.0	12.44
10.100.04	22708.7	82.10	22701.2	67.14	22726.7	39.59
10.100.05	22611.9	77.62	22606.2	55.79	22653.9	43.92
10.100.06	21747.3	75.01	21753.7	73.76	21804.0	54.43
10.100.07	22483.8	100.09	22502.2	70.28	22536.7	39.08
10.100.08	22422.0	112.19	22425.3	117.37	22508.1	15.80
10.100.09	22530.6	40.68	22533.3	39.80	22558.2	14.90
#Best	4		3		24	24
<i>p</i> -value	0.000		0.000

Fig. 9. Error bars (95% CI) for MA/PL_NLS (ILS) and MA/PL

TABLE III
COMPARISONS OF MA/PL RO2/3 AND MA/PL

Instance	BKS	MA/PL RO2		MA/PL RO3		MA/PL	
		BEST	AVG	BEST	AVG	BEST	AVG
5.100.00	24381	24381	24336.97	24381	24380.20	24381*	24380.67
5.100.01	24274	24274	24250.50	24274	24269.10	24274*	24274.00
5.100.02	23551	23538	23521.23	23538	23528.63	23538	23519.97
5.100.03	23534	23486	23477.37	23501	23422.93	23534*	23486.00
5.100.04	23991	23966	23945.13	23966	23639.23	23991*	23962.43
5.100.05	24613	24601	24574.53	24598	24546.10	24613*	24588.90
5.100.06	25591	25591	25558.23	25591	25558.27	25591*	25558.33
5.100.07	23410	23410	23410.00	23410	23375.83	23410*	23376.17
5.100.08	24216	24216	24203.60	24216	24207.27	24216*	24208.40
5.100.09	24411	24357	24314.47	24390	24346.70	24411*	24371.37
10.100.00	23064	23055	23055.00	23055	23045.43	23057	23046.83
10.100.01	22801	22753	22676.8	22796	22635.70	22801*	22750.73
10.100.02	22131	22131	22106.30	22131	22109.93	22131*	22105.13
10.100.03	22772	22718	22660.17	22763	22683.53	22763	22714.03
10.100.04	22751	22654	22632.30	22751	22636.37	22751*	22726.73
10.100.05	22777	22702	22626.60	22777	22637.30	22777*	22653.90
10.100.06	21875	21786	21712.47	21791	21771.97	21856	21803.97
10.100.07	22635	22551	22508.17	22564	22527.27	22635*	22536.67
10.100.08	22511	22423	22365.07	22489	22484.43	22511*	22508.07
10.100.09	22702	22702	22520.37	22559	22542.40	22561	22558.23
#Best	8	2		10	2	19	16
p-value		0.002	0.007	0.002	0.000		

Table III shows that MA/PL is better than MA/PL RO2/3 for almost all the instances, regarding *BEST* and *AVG*. Hence, RO_LUF is beneficial to enhance the overall solution quality of MA/PL. Besides, as shown in Fig. 10, MA/PL takes less CPU time than MA/PL RO2/3 although the same stopping criterion is adopted. These results prove the effectiveness and efficiency of the proposed RO_LUF. In terms of *Dev* (%), the violin plots of MA/PL and MA/PL RO2/3 are given in Fig. 11, showing that MA/PL can obtain more robust solutions than the variants.

Fig. 10. Average CPU times for MA/PL RO2/3 and MA/PL

Fig. 11. Violin plots for solutions of MA/PL RO2/3 and MA/PL

3) Discussions on coefficients for RO_LUF and LS_LUF

The coefficients λ_1 and λ_2 applied for calculating LUFs play

an important role on the performance of RO_LUF and LS_LUF. To study the impacts of λ_1 and λ_2 , we introduce four variants of MA/PL, i.e., MA/PL_A, MA/PL_B, MA/PL_C and MA/PL_D, with respect to different combinations of (λ_1, λ_2) . The values of (λ_1, λ_2) for these four variants are (0.1, 0.9), (0.3, 0.7), (0.7, 0.3) and (0.9, 0.1). We then assess the effectiveness and the efficiency of MA/PL and the four variants. The comparative results in terms of *AVG* are reported in Table IV. In addition, we give trends of *SD* in Fig. 12. Note that the four variants are executed with *genMax*=5000 which is the same as MA/PL.

TABLE IV
COMPARISONS OF FOUR MA/PL (A~D) AND MA/PL

Instance	MA/PL A	MA/PL B	MA/PL C	MA/PL D	MA/PL
	AVG	AVG	AVG	AVG	AVG
5.100.00	24338.8	24341.8	24348.8	24341.6	24380.67
5.100.01	24236.0	24233.9	24243.2	24238.8	24274.00
5.100.02	23508.2	23509.1	23508.2	23509.1	23519.97
5.100.03	23440.3	23439.7	23460.5	23454.7	23486.00
5.100.04	23915.2	23924.5	23941.3	23937.2	23962.43
5.100.05	24574.9	24573.5	24574.5	24572.6	24588.90
5.100.06	25503.1	25475.4	25517.3	25505.3	25558.33
5.100.07	23353.6	23349.4	23350.4	23348.7	23376.17
5.100.08	24165.1	24196.3	24206.0	24206.4	24208.40
5.100.09	24325.4	24331.0	24333.3	24335.5	24371.37
10.100.00	23026.8	23029.0	23036.9	23036.9	23046.83
10.100.01	22723.4	22714.6	22735.9	22736.2	22750.73
10.100.02	22075.0	22067.9	22085.3	22080.8	22105.13
10.100.03	22664.2	22660.3	22662.7	22666.2	22714.03
10.100.04	22659.4	22670.9	22652.3	22669.7	22726.73
10.100.05	22620.8	22630.3	22630.3	22627.1	22653.90
10.100.06	21767.8	21766.1	21768.1	21768.1	21803.97
10.100.07	22501.8	22502.8	22515.4	22506.9	22536.67
10.100.08	22450.0	22453.5	22446.9	22451.1	22508.07
10.100.09	22558.2	22552.7	22556.8	22560.5	22558.23
#Best	0	0	0	1	19
p-value	0.000	0.000	0.000	0.000

Fig. 12. Trends of SD for MA/PL (A~D) and MA/PL

Table IV shows that the values of *AVG* of MA/PL are much better than those of the four variants for almost all the used instances. The results verify that in RO_LUF and LS_LUF the equal weight of the two attributes for candidate items is helpful in enhancing the effectiveness of MA/PL. Meanwhile, in Fig. 12, MA/PL has relatively smaller *SD* for each instance, so setting the equal coefficient to the two attributes is beneficial to enhance the robustness of MA/PL as well. This can be due to the fact that for the proposed LUFs the different magnitude of the two attributes has already been normalized before using them. Thereby, the equal coefficient of them is more reliable to reflect the problem-specific relationships of the two attributes.

Moreover, we draw the trend of CPU (s) for each algorithm

in Fig. 13. It is clear that algorithms with larger λ_2 tend to take less CPU time. This is because, during the search process of MA/PL, a larger λ_2 that corresponds to a smaller λ_1 will emphasize the role of the constraint violation but weaken the impact of the objective. Thereby, for MA/PL with a larger λ_2 , the time complexities of RO_LUF and LS_LUF can be much lower than the worst case $O(PS \cdot N \cdot (3 \cdot N + M))$, compared with ones using smaller λ_2 . Especially, the trend line of MA/PL in Fig. 13 is located in the relatively middle area among all the lines. However, the CPU time for MA/PL is quite acceptable as we can see in this section. Therefore, we set $(\lambda_1, \lambda_2) = (0.5, 0.5)$ considering both the effectiveness and efficiency of MA/PL.

Fig. 13. Trends of CPU time for MA/PL (A~D) and MA/PL

C. Comparisons with Learning-based Metaheuristics

We compare MA/PL with four learning-based metaheuristics for MKP, i.e., HEDA1 [34], HEDA2 [34], Meta-RaPS EDA [55] and Meta-RaPS Q [55]. HEDA1 and HEDA2 use the EDA-based learning scheme. Whereas HEDA2 uses RO2 as the repair operator, HEDA1 uses a repair operator based on LP. Meta-RaPS EDA and Meta-RaPS Q adopt a model of memoryless metaheuristic (i.e., Meta-RaPS). Meta-RaPS EDA and Meta-RaPS Q employ EDA and Q-learning as the underlying algorithms. These algorithms are the most currently reported learning-based ones for MKP. The stopping criterion of MAPL is $genMax=5000$. Relevant results are listed in Tables V to VI.

TABLE V
COMPARISONS OF HEDA1, HEDA2 AND MA/PL

Instance	HEDA1		HEDA2		MA/PL		CPU (s)
	Ave. Dev	Var. Dev	Ave. Dev	Var. Dev	Ave. Dev	Var. Dev	
Mk_gk01	0.32	0.08	0.94	0.66	0.23	0.00	20.07
Mk_gk02	0.20	0.12	0.98	0.40	0.25	0.00	22.77
Mk_gk03	0.20	0.10	1.15	0.78	0.40	0.00	40.83
Mk_gk04	0.34	0.29	1.27	0.71	0.31	0.00	52.73
Mk_gk05	0.28	0.22	1.50	0.75	0.23	0.00	57.19
Mk_gk06	0.44	0.23	1.44	1.16	0.35	0.00	78.95
Mk_gk07	0.72	0.82	1.85	0.99	0.28	0.00	211.11
Mk_gk08	0.77	1.10	1.73	0.29	0.40	0.00	348.42
Mk_gk09	—	—	2.55	0.68	0.24	0.00	1204.68
Mk_gk10	—	—	2.12	0.78	0.81	0.00	1779.39
Mk_gk11	—	—	1.74	0.22	0.01	0.00	2411.12
Knap15	—	—	0.00	0.00	0.00	0.00	2.02
Knap20	—	—	0.00	0.00	0.00	0.00	2.72
Knap39	—	—	0.00	0.00	0.03	0.00	5.42
Knap28	—	—	0.00	0.00	0.00	0.00	3.76
#Best	2	0	4	4	12	15	
p-value	0.157		0.004				

TABLE VI
COMPARISONS OF META-RAPS EDA, META-RAPS Q AND MA/PL

Instance	RaPS-EDA	RaPS-Q	MA/PL	CPU (s)
	Ave. Dev	Ave. Dev	Ave. Dev	
Hp1	0.00	0.09	0.00	3.73
Hp2	0.00	0.19	0.00	4.68
Pb1	0.00	0.09	0.00	3.68
Pb2	0.00	0.52	0.00	4.47
Pet6	0.04	0.20	0.02	6.04
Pet7	0.00	0.19	0.00	7.98
Sento2	0.00	0.02	0.00	10.64
Weing6	0.00	0.06	0.00	4.13
Weing7	0.00	0.05	0.00	16.30
Weing8	0.05	0.31	0.00	18.12
Weish18	0.00	0.13	0.00	10.86
Weish22	0.00	0.07	0.00	12.72
Weish23	0.00	0.02	0.00	12.83
Weish24	0.00	0.20	0.00	12.61
Weish25	0.00	0.09	0.00	12.67
Weish26	0.00	0.09	0.00	14.39
#Best	14	0	16	
p-value	0.157		0.000	

From Table V, we can see that MA/PL outperforms HEDA1 and HEDA2. Although MA/PL's learning mechanism is similar to HEDA1 and HEDA2, its probability distribution is estimated based on ExA, rather than the elite individuals in the population. Such a learning method of MA/PL can make good use of the information of elite individuals and so speeds up the convergence. Besides, the integration of MPD and JPD can capture valuable probability information to guide the global exploration of MA/PL. We can see from Table VI that MA/PL outperforms Meta-RaPS EDA and Meta-RaPS Q. For all the instances where any of other methods found BKS, MA/PL also found BKS, while for all other instances MA/PL found better solutions. Thus, our learning method is effective for MKP. It may be worthwhile to consider embedding our learning method into other metaheuristics for MKP in future research.

D. Comparisons with Other Existing Algorithms

In this subsection, we compare MA/PL with 9 existing algorithms for MKP with respect to Test sets I to IV. These existing algorithms are listed in Table VII, which are all the most currently effective solution methods for MKP. The stopping criterion of MA/PL is $genMax = 5000$.

TABLE VII
EXISTING ALGORITHMS CONSIDERED IN THE COMPARISONS

Algorithm	Published year	Test set			
		I	II	III	IV
NBHS2 [53]	2015	√	×	×	×
TR-BDS [52]	2016	√	×	√	×
QIFAPSO [49]	2016	×	√	×	×
BCSA [51]	2017	√	×	√	×
HPSOGO [50]	2018	×	×	√	×
KMTR-Cuckoo [54]	2018	×	×	×	√
KMTR-BH [54]	2018	×	×	×	√
HBDE-RO3 [35]	2019	×	√	√	×
MMVO [60]	2019	×	√	×	×

Note: “√” (“×”) means the test set is relevant (irrelevant) to the algorithm.

1) Computational results for Test set I

The comparative results on Test set I are reported in Table VIII. We can see from Table VIII that MA/PL outperforms TR-BDS, regarding AVG and SD , for almost all the instances of

Test set I. Although the *Min. Dev* and *Ave. Dev* values of MA/PL are similar to those of NBHS2, MA/PL's *Var. Dev* values are all less than 0.01 and are all smaller than those of NBHS2. Thus, MA/PL can maintain better robustness than NBHS2. There are three instances, i.e., MK_gk03, MK_gk04 and MK_gk05 for which BCSA can provide smaller *Ave. Dev*. However, BCSA is somewhat weak in other instances and may be unavailable for large scale instances MK_gk10 and MK_gk11. Moreover, the CPU times for MA/PL are all less than 1205s from MK_gk01 to MK_gk09 which are quite acceptable. Furthermore, for large-sized instances MK_gk10 and MK_gk11, MA/PL is able to obtain high-quality solutions with smaller *AVG*, *SD* and *Var. Dev*. In view of the above analysis, the proposed MA/PL can be potentially used for

large-sized instances of MKP.

2) Computational results for Test set II

In terms of Test set II, comparative results are listed in Table IX. It is clear that MA/PL achieves a good performance for almost all the considered instances in terms of *BEST*, *AVG* and *SD*. For the significance, the *p*-values for three comparative algorithms are all smaller than 0.05 concerned with *BEST* and *AVG*, except for the *AVG* values of QIFAPSO. However, MA/PL can attain the BKS values for 15 of 20 instances which is much better than QIFAPSO. Moreover, the CPU (s) values listed in Tables IX range from 15s to 19s, which verify the efficiency of the proposed MA/PL. Therefore, the proposed MA/PL is effective for medium scale instances of MKP.

TABLE VIII
COMPARISONS OF BHS2, BCSA, TR-BDS AND MA/PL (VS. TEST SET I)

Instance	BKS	NBHS2			BCSA		TR-BDS		MA/PL					CPU (s)
		<i>Min. Dev</i>	<i>Ave. Dev</i>	<i>Var. Dev</i>	<i>Ave. Dev</i>	<i>Var. Dev</i>	<i>AVG</i>	<i>SD</i>	<i>AVG</i>	<i>SD</i>	<i>Min. Dev</i>	<i>Ave. Dev</i>	<i>Var. Dev</i>	
Mk_gk01	3766	0.19	0.29	0.09	0.23	0.09	3688.26	6.02	3757.03	1.14	0.19	0.23	<0.01	20.07
Mk_gk02	3958	0.18	0.30	0.08	0.27	0.08	3878.88	8.09	3948.00	2.75	0.08	0.25	<0.01	22.77
Mk_gk03	5650	0.16	0.44	0.12	0.17	0.07	5512.06	7.55	5633.37	4.09	0.28	0.40	<0.01	40.83
Mk_gk04	5764	0.12	0.41	0.21	0.15	0.05	5623.70	9.16	5746.20	3.64	0.16	0.31	<0.01	52.73
Mk_gk05	7557	0.16	0.39	0.11	0.17	0.08	7349.14	11.41	7539.70	4.69	0.11	0.23	<0.01	57.19
Mk_gk06	7672	0.20	0.42	0.17	0.35	0.12	7488.88	9.95	7645.10	4.23	0.25	0.35	<0.01	78.95
Mk_gk07	19215	0.15	0.23	0.06	0.70	0.12	18588.20	22.74	19161.27	6.02	0.23	0.28	<0.01	211.11
Mk_gk08	18801	0.36	0.53	0.09	0.77	0.11	18299.68	14.88	18725.33	9.40	0.27	0.40	<0.01	348.42
Mk_gk09	58085	0.18	0.26	0.06	0.98	0.08	56035.20	38.46	57598.70	25.95	0.72	0.24	<0.01	1204.68
Mk_gk10	57292	0.38	0.53	0.06	—	—	55719.48	25.50	56825.43	23.07	0.33	0.81	<0.01	1779.39
Mk_gk11	95231	0.46	0.64	0.09	—	—	93132.92	35.92	94124.03	0.94	0.01	0.01	<0.01	2411.12
#Best		6	2	0	5	0	1	0	10	11	6	6	11	
<i>p</i> -value		1.000	0.035		0.705		0.001							

TABLE IX
COMPARISONS OF HBDE-RO3, MMVO, QIFAPSO AND MA/PL (VS. TEST SET II)

Instance	BKS	HBDE-RO3			MMVO		QIFAPSO		MA/PL		SD	CPU (s)
		<i>BEST</i>	<i>AVG</i>	<i>SD</i>	<i>BEST</i>	<i>AVG</i>	<i>BEST</i>	<i>AVG</i>	<i>BEST</i>	<i>AVG</i>		
5.100.00	24381	24381	24378.1	7.66	24192	24050	24381	24312.0	24381*	24380.67	1.45	16.95
5.100.01	24274	24274	24250.4	61.01	24274	24274	24274	24206.0	24274*	24274.00	0.00	16.66
5.100.02	23551	23523	23490.7	33.44	23538	23520	23538	23513.0	23538	23519.97	14.50	16.78
5.100.03	23534	23486	23465.0	30.54	23288	23120	23484	23451.0	23534*	23486.00	16.32	17.44
5.100.04	23991	23966	23953.4	17.50	23947	23900	23991	23924.0	23991*	23962.43	7.54	16.78
5.100.05	24613	24613	24597.9	8.97	—	—	—	—	24613*	24588.90	31.36	16.85
5.100.06	25591	25591	25522.4	9.80	—	—	—	—	25591*	25558.33	34.92	16.76
5.100.07	23410	23410	23410.0	0.00	—	—	—	—	23410*	23376.17	35.55	17.04
5.100.08	24216	24216	24187.2	16.63	—	—	—	—	24216*	24208.40	5.78	16.36
5.100.09	24411	24411	24297.0	26.88	—	—	—	—	24411*	24371.37	45.19	17.18
10.100.00	23064	23055	23053.9	7.56	22805	22700	23057	22954.0	23057	23046.83	15.82	18.04
10.100.01	22801	22801	22735.4	51.55	22630	22480	22750	22649.0	22801*	22750.73	28.17	18.63
10.100.02	22131	22131	22098.2	32.76	22131	21720	22131	21977.0	22131*	22105.13	28.10	18.97
10.100.03	22772	22749	22693.2	43.87	22347	22160	22772	22566.0	22763	22714.03	12.44	17.01
10.100.04	22751	22751	22748.4	17.92	22417	22290	22654	22555.0	22751*	22726.73	39.59	15.86
10.100.05	22777	22698	22593.6	45.86	—	—	—	—	22777*	22653.90	43.92	17.29
10.100.06	21875	21856	21734.5	63.11	—	—	—	—	21856	21803.97	54.43	16.00
10.100.07	22635	22635	22550.1	20.47	—	—	—	—	22635*	22536.67	39.08	16.28
10.100.08	22511	22511	22436.9	46.40	—	—	—	—	22511*	22508.07	15.80	15.43
10.100.09	22702	22561	22471.4	37.54	—	—	—	—	22561	22558.23	14.90	15.84
#Best		4	2	7	3	2	7	0	17	17	13	
<i>p</i> -value		0.014	0.025		0.008	0.020	0.317	0.002				

3) Computational results for Test set III

We report the results regarding Test set III in Table X, which only contains the most currently proposed algorithms, i.e., HBDE-RO3 and HPSOGO. Meanwhile, the #Best of *BEST*, *AVG* and *SD* of TR-BDS, BCSA and MA/PL are given in Table XI. It is worth noting that Table X MA/PL can get the BKS values for all instances of Test set III. In addition, the *SD* values

of MA/PL are all obviously smaller than those of HBDE-RO3 and HPSOGO. Table XI shows that MA/PL can obtain more competitive performance than TR-BDS and BCSA. The above facts prove the effectiveness and robustness of our MA/PL for solving the small scale instances of MKP. More details about the comparative results associated with Test set III are available at <http://dx.doi.org/10.13140/RG.2.2.23779.71208>.

TABLE X
COMPARISONS OF HBDE-RO3, HPSOGO AND MA/PL (VS. TEST SET III)

Instance	BKS	HBDE-RO3			HPSOGO			MA/PL			CPU (s)
		SR	SD	SR	Ave.	Dev	SD	Ave.	Dev	SD	
Sento1	7772	1.00	0.00	0.25	0.00	1.00	0.00	1.00	0.00	0.00	10.34
Sento2	8722	0.91	0.29	0.10	0.00	1.00	0.00	1.00	0.00	0.00	10.64
Hp1	3418	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	3.73
Hp2	3186	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	4.68
Pb1	3090	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	3.68
Pb2	3186	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	4.47
Pb4	95168	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	3.74
Pb5	2139	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	2.71
Pb6	776	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	6.42
Pb7	1035	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	6.39
Pet2	87061	—	—	—	—	1.00	0.00	1.00	0.00	0.00	1.37
Pet3	4015	—	—	—	—	1.00	0.00	1.00	0.00	0.00	2.09
Pet4	6120	—	—	—	—	1.00	0.00	1.00	0.00	0.00	2.77
Pet5	12400	—	—	—	—	1.00	0.00	1.00	0.00	0.00	3.96
Pet6	10618	—	—	—	—	0.83	0.02	5.15	6.04	0.00	6.04
Pet7	16537	—	—	—	—	1.00	0.00	1.00	0.00	0.00	7.98
Weing1	141278	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.15
Weing2	130883	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.27
Weing3	95677	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.27
Weing4	119337	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.33
Weing5	98796	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.20
Weing6	130623	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.13
Weing7	1095445	1.00	0.00	1.00	0.11	0.23	0.00	0.00	0.00	0.00	16.30
Weing8	624319	1.00	0.00	1.00	1.34	1.00	0.00	1.00	0.00	0.00	18.12
Weish1	4554	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.67
Weish2	4536	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.65
Weish3	4115	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.70
Weish4	4561	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.62
Weish5	4514	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	4.76
Weish6	5557	0.91	3.72	1.00	0.00	1.00	0.00	1.00	0.00	0.00	6.45
Weish7	5567	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	6.41
Weish8	5605	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	6.46
Weish9	5246	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	6.43
Weish10	6339	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	8.26
Weish11	5643	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	7.82
Weish12	6339	1.00	0.00	0.35	0.00	1.00	0.00	1.00	0.00	0.00	7.73
Weish13	6159	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	7.80
Weish14	6954	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	9.38
Weish15	7486	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	9.41
Weish16	7289	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	9.29
Weish17	8633	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	9.17
Weish18	9580	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	10.86
Weish19	7698	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	13.56
Weish20	9450	1.00	0.00	0.10	0.00	1.00	0.00	1.00	0.00	0.00	10.96
Weish21	9074	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	12.08
Weish22	8947	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	12.72
Weish23	8344	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	12.83
Weish24	10220	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	12.61
Weish25	9939	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	12.67
Weish26	9584	1.00	0.00	1.00	0.21	1.00	0.00	1.00	0.00	0.00	14.39
Weish27	9819	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	14.40
Weish28	9492	1.00	0.00	1.00	0.99	1.00	0.00	1.00	0.00	0.00	14.59
Weish29	9410	1.00	0.00	—	—	1.00	0.00	1.00	0.00	0.00	14.47
Weish30	11191	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.00	14.31
#Best		46	46	35	35	53	54	54			
p-value		0.564		0.180	0.046						

TABLE XI
COMPARISONS OF TR-BDS, BCSA AND MA/PL (VS. TEST SET III)

	TR-BDS		BCSA		MA/PL		
	AVG	SD	BEST	AVG	BEST	AVG	SD
#Best	16	15	48	34	54	53	53
p-value	0.000		1.000	0.000			

4) Computational results for Test set IV

Results concerned with Test set IV are given in Tables XII to XIV. In Table XII, the *AVG* values of MA/PL are better than those of other algorithms. Meanwhile, the *SD* values of MA/PL are no worse than existing algorithms and are quite acceptable. In Tables XIII to XIV, the *BEST* and *AVG* values of MA/PL are better than those of other algorithms for almost all the instances.

Moreover, the CPU (s) values of MA/PL are less than 220s, indicating the efficiency of MA/PL for large-sized instances.

TABLE XII
COMPARISONS OF KMTR-CUCKOO, KMTR-BH AND MA/PL (VS. TEST SET IV)

Instance	KMTR-Cuckoo		KMTR-BH		MA/PL		CPU (s)
	AVG	SD	AVG	SD	AVG	SD	
5.500.00	120036.8	25.5	120029.9	25.3	120009.7	41.35	152.05
5.500.01	117570.6	57.3	117617.5	55.8	117744.3	43.51	138.48
5.500.02	120855.1	39.8	120937.9	50.3	121004.3	31.09	131.03
5.500.03	120455.7	32.1	120522.8	71.6	120688.9	38.05	156.01
5.500.04	122136.4	39.4	122165.2	50.2	122194.5	37.21	142.95
5.500.05	121824.6	35.4	121868.7	52.2	121903.0	46.72	145.91
5.500.06	118895.5	40.1	118950.0	52.9	119007.8	34.11	139.73
5.500.07	120320.4	43.0	120336.6	45.7	120467.5	32.07	153.41
5.500.08	121126.3	54.5	121161.9	91.3	121464.1	28.55	165.24
5.500.09	120335.5	62.2	120362.9	89.0	120584.6	42.81	161.92
5.500.10	218208.9	47.8	218163.7	50.7	218273.8	40.72	137.09
5.500.11	220862.3	63.9	220813.9	65.3	221078.2	34.69	129.78
5.500.12	217293.0	53.3	217254.3	51.4	217424.7	35.30	122.32
5.500.13	223455.6	45.4	223455.2	32.8	223410.9	45.96	116.56
5.500.14	218794.0	49.0	218771.5	44.0	218860.8	33.89	113.53
5.500.15	220352.7	40.8	220342.2	56.6	220377.5	46.77	127.16
5.500.16	219732.8	47.2	219717.9	60.1	219869.9	40.91	131.51
5.500.17	217928.7	55.4	217890.1	57.3	218059.6	50.41	120.25
5.500.18	216829.8	39.5	216798.8	41.2	216850.0	47.45	130.49
5.500.19	219558.9	54.9	219520.0	52.4	219607.1	39.82	134.00
5.500.20	295608.8	33.1	295628.4	48.9	295680.7	49.98	120.59
5.500.21	307914.8	59.1	307860.6	55.7	307942.4	45.96	113.77
5.500.22	299660.9	12.1	299717.8	89.9	299675.2	45.87	106.66
5.500.23	306397.3	17.1	306445.2	96.1	306340.6	46.34	118.85
5.500.24	300184.4	16.9	300202.5	26.4	300197.6	48.27	125.03
5.500.25	302435.6	24.0	302442.3	24.1	302376.3	66.82	121.49
5.500.26	301239.7	24.1	301238.3	37.9	301194.4	45.59	122.72
5.500.27	306276.4	23.8	306264.2	45.4	306281.6	68.53	116.30
5.500.28	302716.9	32.0	302721.4	22.4	302712.4	28.78	107.47
5.500.29	299766.0	45.5	299722.7	34.1	299786.4	35.72	111.23
#Best	3	12	5	1	22	17	
p-value	0.001		0.011				

TABLE XIII
COMPARISONS OF KMTR-CUCKOO, KMTR-BH AND MA/PL (VS. TEST SET IV)

Instance	KMTR-Cuckoo		KMTR-BH		MA/PL		CPU
	BEST	AVG	BEST	AVG	BEST	AVG	
10.500.00	117509	117302.8	117558	117293.0	117628	117571.2	159.11
10.500.01	119072	118936.2	119232	118980.8	118974	118928.9	153.88
10.500.02	119039	118723.0	118940	118840.5	119055	118950.4	166.51
10.500.03	118586	118433.9	118598	118516.2	118680	118614.6	154.94
10.500.04	116312	116013.8	116186	116095.5	116289	116229.6	158.44
10.500.05	119257	119065.7	119257	119113.2	119318	119248.2	148.38
10.500.06	119663	119506.6	119691	119556.7	119658	119566.1	163.87
10.500.07	118058	117760.3	118016	117907.3	118121	118036.5	157.29
10.500.08	117550	117235.0	117550	117363.0	117634	117546.6	162.53
10.500.09	118962	118514.2	118896	118739.0	119086	118981.1	150.97
10.500.10	217126	216892.0	216990	216892.0	217192	217125.7	135.13
10.500.11	218872	218592.4	218672	218592.0	218922	218861.7	137.63
10.500.12	217573	217542.3	217447	217358.8	217729	217621.9	115.63
10.500.13	216570	216469.9	216570	216484.5	216711	216654.5	108.38
10.500.14	213474	213363.5	213474	213374.0	213712	213619.1	144.96
10.500.15	214829	214702.6	214761	214638.7	214887	214721.2	143.06
10.500.16	217629	217567.1	217583	217484.2	217812	217724.1	146.39
10.500.17	219675	219554.4	219589	219496.8	219814	219740.6	156.26
10.500.18	214045	213939.4	214015	213862.7	214195	214097.0	151.19
10.500.19	220582	220515.1	220488	220391.3	220752	220688.8	144.86
10.500.20	304116	304019.1	304102	303991.6	304234	304132.7	114.55
10.500.21	302263	302156.3	302138	302078.2	302265	302167.0	122.66
10.500.22	302118	302061.6	302103	301968.2	302301	302195.8	134.62
10.500.23	300566	300498.3	300542	300480.4	300665	300606.4	122.49
10.500.24	304229	304187.7	304267	304168.7	304239	304147.7	115.83
10.500.25	301445	301332.1	301730	301461.8	301621	301559.3	131.48
10.500.26	304905	304814.6	304833	304778.1	304785	304689.3	107.56
10.500.27	296361	296288.9	296263	296194.0	296337	296227.1	104.04
10.500.28	301085	301031.2	301085	301026.0	301226	301149.1	116.26
10.500.29	306881	306786.6	306881	306786.6	306973	306834.6	106.26
#Best	3	3	4	1	23	26	
p-value	0.000	0.000	0.000	0.000			

TABLE XIV

COMPARISONS OF KMTR-CUCKOO, KMTR-BH AND MA/PL (VS. TEST SET IV)

Instance	KMTR-Cuckoo		KMTR-BH		MA/PL		CPU
	BEST	AVG	BEST	AVG	BEST	AVG	
30.500.00	115526	115341.7	115449	115236.6	115609	115480.7	214.65
30.500.01	114405	114291.5	114352	114172.2	114426	114299.9	214.96
30.500.02	116256	116093.8	116158	116065.5	116357	116231.3	215.11
30.500.03	114782	114673.9	114739	114574.4	114933	114815.9	199.56
30.500.04	115995	115872.7	115994	115881.2	116146	116027.4	215.82
30.500.05	115342	115143.0	115244	115149.5	115331	115150.6	216.66
30.500.06	113712	113527.5	113593	113433.7	113621	113524.5	209.25
30.500.07	113626	113516.5	113610	113522.0	113850	113683.7	195.16
30.500.08	114822	114633.4	114705	114684.2	115049	114889.3	199.12
30.500.09	116467	116338.6	116382	116374.5	116724	116595.7	160.12
30.500.10	217776	217619.2	217607	217574.8	217858	217792.2	167.67
30.500.11	214110	214002.7	214110	214089.5	214323	214209.7	179.88
30.500.12	215638	215494.8	215580	215506.5	215668	215539.3	184.60
30.500.13	217301	217215.8	217201	217136.0	217582	217481.4	175.30
30.500.14	215116	214992.6	215036	214974.7	215304	215206.6	160.30
30.500.15	215408	215219.7	215326	215223.6	215608	215467.7	189.97
30.500.16	215576	215486.9	215516	215449.1	215764	215626.3	168.66
30.500.17	216057	216002.6	215999	215981.6	216157	216000.9	170.98
30.500.18	217013	216886.2	216882	216867.8	217103	216987.2	164.31
30.500.19	214332	214127.3	214194	214127.3	214451	214347.6	160.09
30.500.20	301343	301241.0	301343	301200.8	301482	301350.6	156.89
30.500.21	299720	299579.9	299636	299556.8	299908	299748.6	155.93
30.500.22	304852	304748.2	304850	304774.5	304912	304854.4	141.40
30.500.23	301645	301583.2	301658	301536.7	301742	301629.4	152.39
30.500.24	304186	304106.1	304186	304082.4	304271	304185.4	155.78
30.500.25	296521	296420.5	296450	296413.0	296744	296658.8	132.81
30.500.26	302941	302843.8	302917	302841.6	303141	303002.7	153.15
30.500.27	306616	306451.3	306616	306450.7	306774	306674.1	146.96
30.500.28	302791	302565.8	302636	302550.3	302967	302881.9	153.41
30.500.29	300170	300063.8	300170	300061.5	300269	300205.0	149.02
#Best	2	2	0	0	28	28	
p-value	0.000	0.000	0.000	0.000			

Based on the results in Table XII, we draw the violin plots of KMTR-Cuckoo, KMTR-BH and MA/PL in Fig. 14, in terms of *Ave. Dev* and *SD*, showing that MA/PL is better than the other two algorithms. Therefore, there would be a large potential to use MA/PL for real-life problems that can be modeled as MKP.

Fig. 14. Violin plots for KMTR-Cuckoo, KMTR-BH and MA/PL

VI. CASE STUDY

Recently, the authors conducted a real-life project for an iron and steel company in Qian'an, Hebei, China. In this project, we focus on the production planning for the cold rolling of the iron and steel production process. Cold rolling is the most complex process in the iron and steel industry. The primary procedure of the cold rolling production planning is the order selection (OS), which involves selecting suitable orders (items) from a given order pool, on the basis of the capability of processing units (dimensions) and the due dates of orders. The sketch of OS is given in Fig. 15. We consider 5 bottleneck units in OS, i.e., acid pickling line (APL), pickling line tandem (PLT), continuous annealing 1 and 2 (CA1 and CA2) and twenty-high cold rolling mill (RCM). Note that in a given planning interval, each unit

has a limited capacity (constraint) that is determined by the maintenance activity and the already assigned load.

To improve customer satisfaction, orders that are closer to their due dates should be given higher priority to be selected. We thus model OS as an MKP where the objective to be maximized is the total inverse of the time difference between the current time and the due date of each order. Besides, constraints considered here are the limited capacity of each unit in the current planning interval. In this section, OS is used as a case study and the related factory data are collected (available from <http://dx.doi.org/10.13140/RG.2.2.21263.12964>).

Fig. 15. Sketch of order selection in cold rolling process

In this case study, MA/PL is independently run 30 times with the stopping condition $genMax = 5000$. Besides, we specially invite two experienced schedulers (i.e., Schedulers A and B) from the company to manually make the decision of OS by using the same factory data as MA/PL. The results of MA/PL and the manual method are given in Table XV (details can be found at <http://dx.doi.org/10.13140/RG.2.2.10777.36960>).

TABLE XV
RESULTS OF MA/PL AND MANUAL METHOD (VS. CASE STUDY)

Decision method	BEST	AVG	SD	#Selected order	CPU (s)
MA/PL	0.546	0.546	0.00	306	77.14
Scheduler A	0.520	0.520	—	270	—
Scheduler B	0.513	0.513	—	262	—

From Table XV, it is clear that MA/PL outperforms, in terms of *BEST* and *AVG*, the manual method. Especially, MA/PL can select 306 orders with $SD = 0.00$, which is much better than the two schedulers. Moreover, the value of CPU (s) of MA/PL is 77.14s, which is acceptable for the efficiency requirements of the company. Furthermore, we give the trend of objective found so far at each generation of MA/PL in Fig. 16, showing that MA/PL has a good convergence feature to solve OS. Toward that end, MA/PL can be seen as an effective and efficient solution method for practical applications.

Fig. 16. Trend of objective at each generation of MA/PL (case study)

VII. CONCLUSIONS AND FUTURE WORK

This paper proposed a memetic algorithm based on probability learning (i.e., MA/PL) to solve MKP. The primary high-

lights of this work lie in two aspects, i.e., the problem-dependent heuristics and the learning-based search framework of MA/PL.

Based on the features of MKP, we proposed two problem-dependent heuristics for the repair operation and local search of MA/PL, i.e., RO_LUF and LS_LUF. These heuristics fully considered the tradeoff between the objective and the violation of constraints according to the presented LUFs. The framework of MA/PL adopted two kinds of probability distribution, i.e., MPD and JPD. MPD was concerned with the univariate probability distributions of items, whereas JPD focused on the bivariate dependency of conjoint items in a given spatial order. The learning scheme of MPD and JPD were all based on elite solutions recorded in ExA. To be specific, MPD was first estimated by the canonical competitive learning. And then JPD was constructed using the proposed learning rule based on a special binary Markov chain. By integrating and complementing MPD and JPD, more accurate probability models can be constructed accordingly. Moreover, we introduced a random sampling route based on the solution structures of MKP to reveal the overall dependency among all items by means of the bivariate dependency of two conjoint items. Thereby, promising offspring of MA/PL can be produced by the integration of MPD and JPD. Results of experiments and comparisons on 179 well-known benchmark instances and a real-life case study verified the effectiveness and practical values of the proposed MA/PL.

For further research, effective metaheuristics may be designed based on machine learning and mathematical programming to address other types of knapsack problems, such as the multi-objective multidimensional knapsack problem (MOMKP), the quadratic knapsack problem (QKP) and the multi-choice knapsack problem (MCKP).

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Zuocheng Li is currently working toward the Ph.D. degree in Key Laboratory of Data Analytics and Optimization for Smart Industry (Northeastern University), Ministry of China, Shenyang, China. His research interests include evolutionary algorithms, combinatorial optimization, machine learning and their applications.

Lixin Tang (M'09-SM'14) received the B.Eng. degree in industrial automation, the M.Eng. degree in systems engineering and the Ph.D. degree in control theory and application from Northeastern University, Shenyang, China, in 1988, 1991, 1996, respectively.

He is a Fellow of Chinese Academy of Engineering, a Chair Professor of the Institute of Industrial & Systems Engineering, Northeastern University; the Director of Key Laboratory of Data Analytics and Optimization for Smart Industry (Northeastern University), Ministry of Education, China; and the Head of Centre for Artificial Intelligence and Data Science.

His research interests cover industrial big data science, data analytics and machine learning, computational intelligent optimization, production and logistics batching and scheduling and engineering applications. His research papers have appeared in academic journals such as *Operations Research*, *IIE Transactions*, *Naval Research Logistics*, *IEEE Transactions on Evolutionary Computation*, *IEEE Transactions on Power Systems*, *IEEE Transactions on Control Systems Technology*, and *European Journal of Operational Research*.

He serves as an Associate Editor of *IIE Transactions*, *IEEE Transactions on Evolutionary Computation*, *IEEE Transactions on Cybernetics*, *International Journal of Production Research*, *Journal of the Operational Research Society*, in Editorial Board of *Annals of Operations Research*, and an Area Editor of *Asia-Pacific Journal of Operational Research*.

Jiying Liu received the B.Eng. degree in industrial automation and the M. Eng. degree in systems engineering from Northeastern University, Shenyang, China, in 1982 and 1985, respectively, and the Ph.D. degree in manufacturing engineering and operations management from University of Nottingham, Nottingham, U.K., in 1993. He is a Professor of Operations

Management with the School of Business and Economics, Loughborough University, Leicestershire, U.K. He is also a Cheung Kong Scholars Visiting Chair Professor with Institute of Industrial & Systems Engineering, Northeastern University. His research interests include operations planning and scheduling problems in production; logistics and supply chains; and mathematical modeling, optimization, and heuristic methods. He has published papers in journals such as *European Journal of Operational Research*, *IIE Transactions*, *International Journal of Production Research*, *IEEE Transactions on Evolutionary Computation*, *Naval Research Logistics*, *Operations Research*, and *Transportation Research*.