This item was submitted to [Loughborough's Research Repository](#) by the author.
Items in Figshare are protected by copyright, with all rights reserved, unless otherwise indicated.

A generalized model of advertised sales

PLEASE CITE THE PUBLISHED VERSION

<https://doi.org/10.1257/mic.20170152>

PUBLISHER

American Economic Association

VERSION

VoR (Version of Record)

PUBLISHER STATEMENT

The author has the right to republish, post on servers, redistribute to lists and use any component of this work in other works. For others to do so requires prior specific permission and/or a fee. Permissions may be requested from the American Economic Association Administrative Office.

LICENCE

CC BY-NC-ND 4.0

REPOSITORY RECORD

Shelegia, Sandro, and Christopher Wilson. 2021. "A Generalized Model of Advertised Sales". Loughborough University. <https://hdl.handle.net/2134/12241229.v1>.

A Generalized Model of Advertised Sales[†]

By SANDRO SHELEGIA AND CHRIS M. WILSON*

To better understand temporary price reductions or “sales,” this paper presents a generalized “clearinghouse” framework of advertised sales and explores some applications. By viewing the firms as competing in utility and amending the conventional tiebreak rule, we allow for multiple dimensions of firm heterogeneity in complex market environments. Moreover, we (i) provide original insights into the number and types of firms that use sales, (ii) offer new results on how firm heterogeneity affects market outcomes, (iii) extend a common empirical “cleaning” procedure, and (iv) analyze a family of activities in sales markets, including persuasive advertising and obfuscation. (JEL D21, D43, L13, L25, M37)

The evidence of price dispersion within markets is overwhelming, even when products are homogeneous (as reviewed by Baye, Morgan, and Scholten 2006). Such price dispersion has a long-standing interest within the areas of industrial organization and marketing, but it is also gaining increased attention from finance, development economics, and macroeconomics.¹ Empirical findings show that much price dispersion is due to temporary price reductions or “sales” (e.g., Hosken and Reiffen 2004; Nakamura and Steinsson 2008; Kaplan and Menzio 2015; Hitsch, Hortacsu, and Lin 2019). One of the main theoretical explanations for such sales involves mixed strategies that arise from variation in consumers’ search frictions and/or the existence of moderate advertising costs.² This literature continues to offer deep insights into sales and is also receiving a renewed interest as an analytical foundation

*Shelegia: Department of Economics and Business, Universitat Pompeu Fabra, Ramon Trias Fargas 25–27, Barcelona 08005, and Barcelona GSE (email: sandro.shelegia@upf.edu); Wilson: School of Business and Economics, Loughborough University, Loughborough LE11 3TU, United Kingdom (email: c.m.wilson@lboro.ac.uk). Robert Porter was coeditor for this article. We thank the referees. We also thank Mark Armstrong, Mike Baye, Andrew Rhodes, Yossi Spiegel, Matthijs Wildenbeest, and various seminar participants. A previous version of the paper was titled, “A Generalized Model of Sales.” Shelegia acknowledges financial support from the Spanish Ministry of Economy and Competitiveness through the Severo Ochoa Programme for Centres of Excellence in R&D SEV-2015-0563, and the research grants RYC-2016-20307 and ECO2017- 89240-P.

[†]Go to <https://doi.org/10.1257/mic.20170152> to visit the article page for additional materials and author disclosure statement(s) or to comment in the online discussion forum.

¹For example, see, respectively, Woodward and Hall (2012); Allen, Clark, and Houde (2014); Jensen (2007); Nakamura et al. (2018).

²For example, see, respectively, Varian (1980), Burdett and Judd (1983), Stahl (1989), Janssen and Moraga-González (2004), Robert and Stahl (1993), Baye and Morgan (2001). See Baye, Morgan, and Scholten (2006) and Anderson and Renault (2018) for reviews.

for many broader research areas, including price comparison platforms, advertising, obfuscation, choice complexity, and several issues within macroeconomics.³

However, while the predictions of such mixed strategy sales models are frequently consistent with empirical evidence, they often struggle to fully explain the observed differences in firms' pricing and advertising behaviors.⁴ In particular, their restricted ability to allow for firm heterogeneity constrains the theoretical and empirical understanding of sales and inhibits the wider related literatures. Indeed, as Baye and Morgan (2009, 1151) state, "little is known about asymmetric models within this class. Breakthroughs on this front would not only constitute a major theoretical advance, but permit a tighter fit between the underlying theory and empirics."

In response, this paper presents a substantially generalized clearinghouse framework of advertised sales (e.g., Baye and Morgan 2001; Baye, Morgan, and Scholten 2004a) and explores some example applications. Our main contribution is methodological: while its modeling assumptions sometimes differ to existing research, our framework can extend many of the past literature's sales predictions to more complex market settings while allowing for multiple dimensions of firm heterogeneity. Thus, we hope that our framework will help open up the future analysis of many remaining questions within the sales literature. As some examples, the paper then provides original insights into the number and type of firms that use sales, offers new predictions about the effects of firm asymmetries on market outcomes, and illustrates how the framework can be used to (i) extend a cleaning procedure that is commonly used in the empirical literature, and (ii) analyze a family of activities in sales markets, including persuasive advertising and obfuscation.

As reviewed by Baye, Morgan, and Scholten (2006), the original clearinghouse framework considers a symmetric market with a single homogeneous good. Consumers are potentially split into nonshoppers, who are only willing to buy from a designated firm, and shoppers, who can buy from any firm. Firms choose their price and whether to inform consumers of this price via some advertising channel. As consistent with observed sales behavior, the equilibrium involves each firm randomizing between selecting a high price without advertising, and advertising a lower price drawn from some support. The seminal model of sales by Varian (1980) obtains as the limiting case when advertising costs tend to zero.

We modify this clearinghouse framework in two important respects. First, we recast the firms as competing in (net) utility rather than prices. By drawing on the (symmetric, pure strategy) model of competition in the utility space by Armstrong and Vickers (2001), we let each firm i select its utility, u_i , with a per-consumer profit function, $\pi_i(u_i)$, that depends upon the firms' underlying demand, products, costs, and pricing technology. With little increase in computation, this facilitates a high

³For reviews and recent examples, see Moraga-González and Wildenbeest (2012), Armstrong (2015), Spiegler (2016), and Ronayne (2020). For macroeconomic applications to nominal rigidities, output fluctuations, and monetary policy, see Guimaraes and Sheedy (2011), Kaplan and Menzio (2016), and Burdett and Menzio (2018).

⁴For example, Lach (2002); Baye, Morgan, and Scholten (2004a, 2004b); Lewis (2008); Chandra and Tappata (2011); Wildenbeest (2011); Giulietti, Waterson, and Wildenbeest (2014); Lach and Moraga-González (2017); Zhang, Chan, and Xie (2018); and Pennerstorfer et al. (2020). Also see Potters and Suetens (2013) for experimental evidence.

level of generality across complex market settings, involving downward-sloping demand, multiple products, or two-part tariffs, where the analysis of sales would often be otherwise impenetrable.

Second, we make a subtle change to the tiebreak rule when the shoppers are indifferent over which firm to trade with. The existing literature assumes shoppers (i) trade exclusively with advertising firm(s) in any tie between advertising and nonadvertising firms, and (ii) mix between the tied firms with equal probability in any other form of tie. Although consistent with an advertising channel involving a price comparison platform where shoppers face additional visit costs to buy from nonlisted firms (Baye and Morgan 2001), it turns out that this traditional tiebreak rule impedes the analysis of sales under firm heterogeneity. In particular, it causes a substantial loss in tractability by (i) making the exact form of equilibrium become dependent upon parameter subcases, and (ii) prompting the existence of mass points in some firms' advertised price distributions.⁵

To resolve this problem, we take a different approach within a setting where shoppers receive all adverts before making their visit decisions. Here, we are free to select any tiebreak rule because shoppers should be willing to buy from any advertising or nonadvertising firm with the same expected utility. Moreover, rather than treating the tiebreak rule as a modeling assumption, we are free to specify the tiebreak rule *as part of equilibrium* in line with Simon and Zame's (1990) concept of endogenous sharing rules (which they use to offer general results about equilibrium existence in n -player games). Under our assumptions, we show that the equilibrium tiebreak rule is uniquely defined for any firm that uses sales. This rule partially offsets any firm heterogeneities and ensures that all firms that use sales have the same incentive to employ a common upper bound in advertised utilities. In symmetric settings, it coincides with the traditional tiebreak rule. However, in asymmetric settings, it offers a unique level of tractability by removing any parameter subcases and eliminating any mass points in advertised utility distributions. As such, it allows us to simultaneously permit (i) any variation in firms' shares of nonshoppers, (ii) any variation in firms' advertising costs, and (iii) considerable variation in firms' profit functions. In equilibrium, firms may vary in advertising probabilities, utility distributions, and profits depending on the level and form of heterogeneity.⁶

Sections I and II introduce the framework and equilibrium analysis. We first present the equilibrium under duopoly and show how it can generalize many predictions from the previous literature to more complex market settings with multiple forms of heterogeneity.⁷ In addition, we offer further new insights by characterizing some

⁵For instance, even in a simple unit demand duopoly where firms only differ in their shares of nonshoppers, the equilibrium in Arnold et al. (2011) has two parameter subcases and one firm uses a mass point in advertised prices. Furthermore, the equilibrium does not converge to standard sales equilibria as advertising costs tend to zero (e.g., Narasimhan 1988).

⁶The variation in profit functions is subject to a condition that is implicit within all of the existing literature: each firm would offer the same utility under monopoly, $u_i^m = u^m \geq 0 \forall i$. This does not restrict each firm's monopoly profit and is innocuous in several market settings, including unit demand.

⁷Among many others, these include symmetric models such as Varian (1980); Simester (1997); Baye, Morgan, and Scholten (2004a, 2006); and asymmetric models, such as Narasimhan (1988); Baye, Kovenock, and de Vries (1992); Kocas and Kiyak (2006); and Wildenbeest (2011).

common forms of sales that have remained unstudied within the clearinghouse literature, including cases where firms use two-part tariffs or nonprice variables such as package size (e.g., $X\%$ Free). We then present the equilibrium for $n > 2$ firms. Here, the previous literature with heterogeneous firms is particularly scant: in a simple setting of unit demand and zero advertising costs, it suggests that only two firms can ever engage in sales behavior (Baye, Kovenock, and de Vries 1992; Kocas and Kiyak 2006; and Shelegia 2012). In contrast, and in better line with typical empirical findings (e.g., Lach 2002, Lewis 2008, Chandra and Tappata 2011), our framework explains how any number of heterogeneous firms $k^* \in [2, n]$ can engage in equilibrium sales. In particular, we demonstrate how a rise in the cost of informative advertising can lead to an *increase* in the number of firms that use advertised sales. Intuitively, despite the direct cost increase, higher advertising costs can prompt more firms to use sales by softening competition for the shoppers. Thus, if the costs of informative advertising fall in the digital era, we predict that fewer firms will engage in sales behavior. Finally, we provide a broad characterization of the types of firms that are most likely to use advertised sales. *Ceteris paribus*, these are firms with relatively low shares of loyal consumers, low advertising costs, and high profitability (under reasonable assumptions on market conditions). These results offer some clear empirical predictions, but they currently remain untested within the literature.⁸

Section III explores some applications to illustrate how our framework can be used for future research. Section IIIA uses the framework to assess and extend a common procedure within the large empirical literature on sales and price dispersion. This cleaning procedure attempts to remove the effects of firm-level heterogeneities from raw price data by retrieving the residuals from a price regression involving firm-level fixed effects.⁹ Wildenbeest (2011) verifies the theoretical validity of the procedure in a setting of unit demand and zero advertising costs where the firms differ in quality and costs, but share the same value-cost margin. However, our more general framework shows how the procedure is invalid (i) for downward-sloping demand (because the relationship between firms' offered prices and utilities becomes nonlinear), and (ii) under unit demand outside Wildenbeest's condition (because the firms no longer offer the same average utility). Moreover, we then offer the basis for modified methodologies that may be applied instead.

Section IIIB uses the framework to study a family of games where each firm's share of nonshoppers is determined endogenously as a function of the firms' actions prior to sales competition. Among other examples, such actions are consistent with forms of persuasive advertising, sales force methods, and obfuscation. Starting with Chioveanu (2008), Carlin (2009), and Wilson (2010), some related streams of

⁸Existing empirical studies often focus on different factors affecting firms' use of sales, such as market information, competition, or rivals' behavior (e.g., Shankar and Bolton 2004, Ellickson and Misra 2008, Lewis 2008, and Chandra and Tappata 2011).

⁹The residuals are then used in (i) reduced-form studies of price dispersion (e.g., Sorensen 2000; Brown and Goolsbee 2002; Lach 2002; Barron, Taylor, and Umbeck 2004; Lewis 2008; Chandra and Tappata 2011; Sherman and Weiss 2017; and Pennerstorfer et al. 2020), or (ii) structural estimations (e.g., Wildenbeest 2011; Moraga-González, Sándor, and Wildenbeest 2013; Allen, Clark, and Houde 2014; Giulietti, Waterson, and Wildenbeest 2014; and An et al. 2017).

literature have become popular in recent years.¹⁰ However, almost all such models are based upon simple market settings with zero costs of informative advertising. An exception is Baye and Morgan (2009), but due to the consequent difficulties of analyzing asymmetries, they are unable to consider all possible subgames. Instead, they show the existence of a continuum of symmetric Nash equilibria and a unique symmetric equilibrium in *secure strategies*. In contrast, our framework can characterize the unique symmetric *subgame perfect* Nash equilibrium across a general market setting. Moreover, while their equilibria imply that an increase in the cost of informative advertising can have a positive, negative, or zero effect on the prior actions, our framework offers a unique empirically testable prediction. As the costs of informative advertising decrease, sales markets should experience (i) a reduction in loyalty-enhancing actions, such as persuasive advertising, and (ii) an increase in loyalty-reducing actions, such as some forms of obfuscation. This offers a first theoretical connection between the costs of informative advertising and equilibrium levels of persuasive advertising, and gives a new advertising-costs-based explanation for why firms increase their obfuscation tactics in response to advances in digital technology (e.g., Ellison and Ellison 2009).

Finally, Section IIIC provides some new comparative static results that could be utilized for future research regarding the effects of *firm-level* characteristics on sales and market performance. For instance, standard results show that an industry-wide increase in advertising costs deters the use of sales and raises firms' profits. However, we can isolate the effects of an increase in a single firm's advertising costs—we show that firms still reduce their use of sales, but that it is rival rather than own advertising costs that matter in determining profits. Similarly, we isolate the effects of an increase in an individual firm's share of nonshoppers. In contrast to results under the traditional tiebreak rule (e.g., Arnold et al. 2011), this induces the firm to set lower average utility offers as more consistent with standard results under zero advertising costs (e.g., Narasimhan 1988). Lastly, we study changes in firms' profit functions or profitability. These results are new even in a symmetric industry-wide setting—an industry-wide increase in profitability, such as a reduction in costs or an increase in per-consumer demand, will always increase firms' use of sales. Further, an increase in a single firm's profitability will increase its sale probability and prompt it to use higher average offers in most common market settings.

Related Literature.—Armstrong and Vickers (2001) introduced competition in utility to study price discrimination in a symmetric, pure strategy setting. In contrast, we transfer their utility approach into an asymmetric (clearinghouse) model to study mixed strategy sales. Some past sales papers have referred to competition in utility (Simester 1997; Hosken and Reiffen 2007; Wildenbeest 2011; Dubovik and Janssen 2012; Anderson, Baik, and Larson 2015). However, they only use it to compute equilibria in specific settings, and do not use the associated profit function, $\pi(u)$, to explore any general results or implications. In a recent paper, Armstrong

¹⁰For a review, and for some wider related models, see Grubb (2015) and Spiegler (2016).

and Vickers (2019) take a different line. They use a dual approach by writing an individual consumer's surplus as a function of the associated per-consumer profit to analyze the effects of price discrimination within a duopoly sales model with asymmetric shares of nonshoppers. Some other work also exists in nonclearinghouse settings. First, Anderson, Baik, and Larson (2015) allow for firm heterogeneity in a model where firms must advertise to earn positive profits, and where all consumers are shoppers. Contrary to us, they find that only two firms can ever use advertised sales when firms are heterogeneous. As such, they cannot analyze how market factors affect the number and type of firms that use sales, or connect to the larger theoretical or empirical clearinghouse literature. Instead, they focus on some interesting results regarding equilibrium selection and welfare. Second, some papers consider clearinghouse-style frameworks but under an assumption of horizontally differentiated products (e.g., Galeotti and Moraga-González 2009, Moraga-González and Wildenbeest 2012). These papers exhibit pure strategy price equilibria without price dispersion, and therefore do not share our focus on sales.

I. Model

Let there be $n \geq 2$ firms, $i \in \{1, \dots, n\}$. Also suppose there is a unit mass of risk-neutral consumers that have a zero outside option. Each firm i competes by choosing a utility offer (net of any associated payments), $u_i \in \mathbb{R}_{\geq 0}$. All consumers have identical preferences and so each consumer values firm i 's offer at precisely u_i .¹¹

The maximum possible profit that firm i can extract per consumer when providing an offer, u_i , is defined as $\pi_i(u_i)$. Following Armstrong and Vickers (2001), the exact source of utility and form of profit function can depend upon a rich set of demand, product, and cost conditions.¹² We assume that $\pi_i(u_i)$ is independent of the number of consumers served.¹³ Further, to ensure that any sales equilibrium is well-behaved, we make some mild technical assumptions: (i) $\pi_i(u_i)$ is strictly quasi-concave in u_i with a unique maximizer at firm i 's monopoly utility level, $u_i^m \in [0, \infty)$, (ii) $\pi_i(u_i^m) \equiv \pi_i^m > 0$, (iii) $\pi_i(u_i)$ is twice continuously differentiable for all $u_i > u_i^m$, and (iv) there exists a finite break-even utility $u_i > u_i^m$, where $\pi_i(u_i) = 0$.

Consumers are initially uninformed about firms' utility offers. However, each firm can choose whether or not to advertise in order to inform consumers of its offer, $\eta_i \in \{0, 1\}$. In line with some previous versions of the clearinghouse model

¹¹ This assumption of identical preferences is standard within the mixed strategy sales literature. If, in contrast, consumers' preferences were sufficiently heterogeneous, then any mixed strategy sales equilibrium would be replaced by a pure strategy nonsales equilibrium.

¹² As two simple examples, consider the following, where firm i sells a single good at price p_i with marginal cost c_i . First, suppose each consumer has a unit demand and values firm i 's good at V_i . Firm i 's utility offer is the associated consumer surplus, $u_i = V_i - p_i$, while its profits per consumer equal $\pi_i(u_i) = V_i - c_i - u_i$. Second, let each consumer have a linear demand for firm i 's good, $q_i(p_i) = a_i - b_i p_i$. Firm i 's utility offer equals $u_i = (a_i - b_i p_i)^2 / 2b_i$, and by using $p_i = (a_i - \sqrt{2b_i u_i}) / b_i$, one can write $\pi_i(u_i) = (1/b_i)[a_i - b_i c_i - \sqrt{2b_i u_i}][\sqrt{2b_i u_i}]$. Further examples including more general downward-sloping demand, multiproduct firms, two-part tariffs, and nonprice sales are detailed in Appendix A.

¹³ Similar to Armstrong and Vickers (2001), this is needed for the profit function to remain well-defined. However, it rules out some empirically relevant features such as scale economies or capacity constraints.

(e.g., Baye, Morgan, and Scholten 2004a), we assume (i) all advertising must be truthful, (ii) any advert is observed by all relevant consumers, and (iii) rms advertising costs are exogenous. However, in contrast to the previous literature, we also assume that (iv) advertising costs can differ across rms, as consistent with different advertising capabilities or channels, and (v) each rm's advertising cost is strictly positive, $A_i > 0 \forall i$.¹⁴

There are two types of consumers, nonshoppers and shoppers, in respective proportions, $\theta \in (0, 1)$ and $(1 - \theta)$. Nonshoppers ignore all adverts. They simply visit their designated local rm and buy according to their underlying demand function, or exit. Our framework allows the rms to have asymmetric shares of nonshoppers, $\theta_i > 0$, with $\sum_{i=1}^n \theta_i = \theta$. In contrast, the remaining shopper consumers pay attention to adverts and can buy from any rm. However, to simplify exposition, we assume that shoppers can only visit one rm. Hence, shoppers choose between (i) visiting an advertising rm to buy from its known utility offer, (ii) visiting a nonadvertising rm to discover its utility offer and potentially buy, or (iii) exiting the market immediately.¹⁵

We analyze the following game. In stage 1, each rm chooses its utility offer, $u_i \in \mathbb{R}_{\geq 0}$, and its advertising decision, $\eta_i \in \{0, 1\}$. To allow for mixed strategies, define (i) $\alpha_i \in [0, 1]$ as rm i 's advertising probability, (ii) $F_i^A(u)$ as rm i 's utility distribution when advertising, and (iii) $F_i^N(u)$ as rm i 's utility distribution when not advertising, both on support $\mathbb{R}_{\geq 0}$. In stage 2, consumers observe any adverts, form beliefs about the (expected) utility provided by any nonadvertising rm, u_i^e , and then make their visit and purchase decisions in accordance with the strategies outlined above.

We define a tie as any situation where the shoppers are indifferent over visiting a set of rms, T , where $|T| \in [2, n]$. Any rm within the tied set, $i \in T$, must have advertised (or be expected to offer) a common level of utility, u , while any rm outside the tie, $j \notin T$, must have advertised (or be expected to offer) a utility strictly lower than u . A tiebreak rule then assigns the probability (or proportion) with which the shoppers visit each tied rm. In particular, for any $i \in T$, let rm i 's tiebreak probability, $x_i(\eta, u, T) \in [0, 1]$, depend upon the tied rms advertising decisions, $\eta = \{\eta_i\}_{i \in T}$, the tied utility level, u , and the set of tied rms, T , where $\sum_{i \in T} x_i(\eta, u, T) = 1$.

The existing literature assumes that shoppers (i) trade exclusively and symmetrically with advertised rm(s) in any tie between advertised and nonadvertised rms, and (ii) trade symmetrically with all tied rms in any other form of tie. This can be summarized as follows for $n = 2$: $x_1((1, 0), u, \{1, 2\}) = 1$, $x_1((0, 1), u, \{1, 2\}) = 0$, and $x_1((1, 1), u, \{1, 2\}) = x_1((0, 0), u, \{1, 2\}) = 0.5$ for all u . In contrast, while we also assume that $x_i(\cdot)$ is independent of u , we depart from the literature's approach in two ways.

¹⁴ Strictly positive advertising costs help ensure that each rm refrains from advertising with positive probability in our later equilibrium. This is needed for our tiebreak rule to be effective in providing tractability. See footnote 22 for more.

¹⁵ These assumptions can be substantially generalized by allowing shoppers to visit rms sequentially provided that (i) the cost of any first visit is not too large, and (ii) each shopper may only purchase from a single rm (one-stop shopping). For technical details see online Appendix C2.

First, we assume that the tiebreak probabilities are independent of firms' advertising decisions. In particular, Assumption X lets the tiebreak probabilities depend only on the set of tied firms, T . While this assumption may be restrictive in some situations, it permits sufficient flexibility for us to manipulate the tiebreak rule. Furthermore, Assumption X remains consistent with our context where shoppers receive all adverts before making their visit decisions and so have no reason to favor advertising firms:¹⁶

$$(\text{Assumption X}) \quad x_i(\eta, u, T) = x_i(T) \quad \forall \eta, u.$$

Second, instead of assuming that the tiebreak probabilities are otherwise symmetric, $x_i(T) = |T|^{-1}$, we build on the concept of endogenous sharing rules (Simon and Zame 1990) to specify them endogenously as part of the game equilibrium. Hence, we focus on perfect Bayesian equilibria (PBE), where in addition to specifying the players' equilibrium strategies and beliefs, we also specify the profile of equilibrium tiebreak probabilities, $x^*(T) = \{x_1^*(T), \dots, x_n^*(T)\}$, for all possible T . As detailed below, this approach, together with Assumption X, will allow us to manipulate the tiebreak probabilities to help improve equilibrium tractability.

Finally, we discuss two remaining assumptions. First, while our framework offers a significant increase in generality, it cannot avoid an assumption that is implicit across the *entire* previous literature. We are the first to state it:

$$(\text{Assumption U}) \quad u_i^m = u^m \quad \forall i.$$

This does not require firms to have the same monopoly price or the same monopoly profits, only the same level of monopoly utility. However, Assumption U is not innocuous. Although it is trivially satisfied under unit demand or two-part tariffs because u_i^m is then always zero (given our assumption that all consumers have identical preferences), it is restrictive under downward-sloping demand and linear prices. Here, one must either (i) restrict attention to symmetric profit functions $\pi_i(u) = \pi(u)$, or (ii) introduce some binding lower bound on firms' utility offers, $u^{\min} \geq \min\{u_i^m, \dots, u_n^m\}$, as consistent with an unmodeled competitive fringe, or a price cap policy when firms' profit functions differ only in costs. Outside Assumption U, the power of our tiebreak approach is lost.¹⁷

Second, we let all firms have some basic potential to use advertised sales. Specifically, we let each firm i 's profits from not advertising with $u_i = u^m$ and selling only to its nonshoppers, $\theta_i \pi_i^m$, be less than its profits from advertising an offer just above u^m and gaining the shoppers, $[\theta_i + (1 - \theta)]\pi_i^m - A_i$. The resulting Assumption A is relatively mild and just ensures that each firm's advertising cost is not prohibitively large:

$$(\text{Assumption A}) \quad A_i \leq (1 - \theta)\pi_i^m \quad \forall i.$$

¹⁶Footnote 19 also later explains how Assumption X can be partially relaxed.

¹⁷Suppose $u_i^m > u^m$ for some i . In the event where all firms set their monopoly utility, the shoppers will then strictly prefer to visit firm i . As later explained, this implies there will be no ties in equilibrium. Thus, the tiebreak probabilities are redundant and cannot be used to ensure a tractable equilibrium.

II. Equilibrium Analysis

Section IIA considers some preliminary findings before Section IIB provides some results on the equilibrium tiebreak probabilities. Section IIC then completes the equilibrium analysis for duopoly ($n = 2$) before Section IID tackles the more complex case of a broader oligopoly ($n > 2$). Any formal proofs are listed in online Appendix B.

A. Preliminary Results

LEMMA 1: *In any equilibrium, each firm i must set $u_i = u^m$ if it does not advertise, $\eta_i = 0$, and set $u_i > u^m$ if it advertises, $\eta_i = 1$.*

Any firm that does not advertise cannot use its unobserved utility offer to attract more consumers. Instead, it will find it optimal to set the monopoly utility level, u^m , because its nonshoppers and any visiting shoppers cannot visit elsewhere. In addition, no firm will ever wish to advertise an offer of u^m . Specifically, any firm i that advertises u^m (with positive probability) could profitably deviate by not advertising. By doing so, it would reduce its advertising costs, $A_i > 0$, while having no impact on its units sold since $x_i(T)$ is independent of advertising decisions via Assumption X.

Lemma 1 offers several implications. First, in equilibrium, if no advert is observed from firm i , then shoppers must correctly believe $u_i^e = u^m$. Moreover, if no adverts are observed from any firm, then shoppers must believe that all the firms are tied with $u_i^e = u^m$ for all i via Assumption U. Second, if firm i advertises, firm i 's lowest advertised utility will always be strictly larger than its nonadvertised utility, u^m . Therefore, from this point forward, we will simply refer to firm i 's utility distribution *unconditional on advertising*, $F_i(u)$, where firm i sets u^m without advertising with probability $1 - \alpha_i = F_i(u^m) \in [0, 1]$, and uses advertised sales on $u > u^m$ with total probability α_i .

Firm i will then be said to use sales if it advertises an offer above u^m with positive probability, $\alpha_i > 0$. In any given equilibrium, we will refer to k^* as the number of firms that use sales and K^* as the set of firms that use sales. A sales equilibrium will be defined as any equilibrium where $k^* \geq 1$. In any given sales equilibrium, we will denote $u > u^m$ as the minimum level of u for which $F_i(u) = 1$ for all i . By adapting standard arguments, we can state the following result.

LEMMA 2: *In any sales equilibrium, at least two firms use sales, $k^* \geq 2$, and for at least two firms i and j , u is a point of increase of $F_i(u)$ and $F_j(u)$ at any $u \in (u^m, \bar{u}]$. Any firm which uses sales, $i \in K^*$, has no point masses in $F_i(u)$ for $u > u^m$ and advertises with an interior probability, $\alpha_i = 1 - F_i(u^m) \in (0, 1)$. When $n = 2$, any sales equilibrium has both firms advertising on $(u^m, \bar{u}]$ without gaps.*

Thus, any firm that uses sales will set an unadvertised regular offer of u^m with probability $(1 - \alpha_i) \in (0, 1)$, together with randomized discounted offers

$u \in (u^m, u]$ with probability α_i .¹⁸ When $n = 2$, Lemma 2 demonstrates that any sales equilibrium will involve both firms using sales on the same full support $(u^m, u]$. However, when $n > 2$, similar to the insights of Baye, Kovenock, and de Vries (1992) for zero advertising costs, it implies that there may be multiple forms of sales equilibria with firms using different supports. Indeed, provided at least two firms mix on any given interval within $(u^m, u]$, other advertising firms need not be active on the same interval. Hence, to avoid these significant complications and potential multiplicities when $n > 2$, we follow Chioveanu (2008) by focusing only on sales equilibria where all advertising firms use the full convex support, $(u^m, u]$. Hence, for all i with $\alpha_i > 0$, we assume u is a point of increase of $F_i(u)$ for all $u \in (u^m, u]$.

B. Equilibrium Tiebreak Probabilities

LEMMA 3: *Ties can only occur with positive probability in equilibrium when all firms refrain from advertising, $\eta_i = 0 \forall i$.*

In contrast to the existing literature where ties are possible between nonadvertising and advertising firms, Lemma 3 shows that ties are only possible in our model when *all* firms choose not to advertise. Hence, the only tiebreak probability that can be relevant in equilibrium is $x_i(N)$ where N denotes the set of all firms, and so from this point forward, we simply denote $x_i(N) \equiv x_i$, and $x^* = \{x_1^*, \dots, x_n^*\}$ as a set of equilibrium tiebreak probabilities. This difference to the literature arises from our Assumption X, which ensures that the tiebreak rule is independent of firms' advertising decisions.¹⁹

For a given x_i^* , firm i 's expected profits from not advertising with $u_i = u^m$ equal

$$(1) \quad \pi_i^m \left[\theta_i + (1 - \theta) x_i^* \prod_{j \neq i} (1 - \alpha_j) \right].$$

Firm i will always trade with its θ_i nonshoppers, but it will also trade with the $(1 - \theta)$ shoppers if (i) all other firms also choose not to advertise, which occurs with probability $\prod_{j \neq i} (1 - \alpha_j)$, and (ii) the shoppers visit i in the subsequent tie, which occurs with tiebreak probability x_i^* .

If firm i uses sales in equilibrium, then we know from Lemma 2 that it must use an interior probability, $\alpha_i \in (0, 1)$. Hence, under the requirements of a mixed strategy equilibrium and our assumption that all advertising firms use the full support $(u^m, u]$, firm i must expect to earn the same level of equilibrium profits, Π_i , from

¹⁸This sales behavior is consistent with the empirical evidence cited in the introduction. While Hitsch, Hortacsu, and Lin (2019) find that firms use different regular prices, this need not be inconsistent with all firms using regular offers of u^m provided the firms differ in (perceived) quality or service levels.

¹⁹Assumption X can be partially relaxed. If, instead, advertising firms were assigned (slightly) lower tiebreak probabilities than nonadvertising firms, then our results would not change: advertising u^m would still be dominated and ties could still only occur when all firms refrain from advertising. However, if advertising firms were assigned significantly higher tiebreak probabilities than nonadvertising firms, we would move closer to the existing literature. Specifically, advertising u^m would not be dominated and so ties could also exist between advertising and nonadvertising firms at u^m in ways that would generate the literature's associated loss in equilibrium tractability.

(i) setting $u_i = u^m$ and not advertising, and (ii) advertising any $u_i \in (u^m, u]$. We can then state the following.

LEMMA 4: *Consider any sales equilibrium with a given set of tiebreak probabilities, x^* . Then, if firm i uses sales, its equilibrium profits are uniquely defined as*

$$(2) \quad \Pi_i = \theta_i \pi_i^m + \frac{x_i^*}{1 - x_i^*} A_i.$$

Hence, the equilibrium profits of any firm i that uses sales will derive from its share of nonshoppers, θ_i , its advertising costs, A_i , and its equilibrium tiebreak probability, x_i^* . To begin to understand more about the equilibrium tiebreak probabilities, it is useful to now define the following expressions, where $\theta_{-i} = \theta - \theta_i$ refers to the total share of nonshoppers that are not designated to firm i :

$$(3) \quad \chi_i(u) \equiv 1 - \frac{A_i}{\pi_i(u)(1 - \theta_{-i}) - \theta_i \pi_i^m},$$

$$(4) \quad \tilde{u}_i \equiv \pi_i^{-1} \left(\frac{\theta_i \pi_i^m + A_i}{1 - \theta_{-i}} \right).$$

Intuitively, $\chi_i(u)$ is the level of x_i^* at which firm i 's equilibrium profits, (2), are equal to the maximum profits that firm i could obtain from advertising an offer, u , and successfully attracting all the shoppers, $\pi_i(u)(1 - \theta_{-i}) - A_i$. Further, $\tilde{u}_i$ can then be understood as the level of utility at which $\chi_i(u) = 0$, where firm i 's maximum profits from advertising are equal to its lowest possible profits from not advertising, $\theta_i \pi_i^m > 0$. Hence, firm i will never advertise $u_i > \tilde{u}_i$. Formally, we define $\chi_i(u)$ on $[u^m, \tilde{u}_i]$ with $\chi_i'(u) < 0$. Using Assumption A, we then know that $\chi_i(u^m) = 1 - (A_i / ((1 - \theta) \pi_i^m)) \in [0, 1)$ is weakly larger than $\chi_i(\tilde{u}_i) = 0$, or equivalently, $\tilde{u}_i \geq u^m$ for all i .

LEMMA 5: *Consider any sales equilibrium with a given upper utility bound $u > u^m$. Then, if firm i uses sales, (i) the upper bound must satisfy $u \leq \tilde{u}_i$, and (ii) firm i 's equilibrium tiebreak probability is uniquely defined as*

$$(5) \quad x_i^* = \chi_i(u).$$

If firm i uses sales on $u_i \in (u^m, u]$, then we know from above that $\tilde{u}_i$ must be weakly larger than u in order for firm i to be willing to set offers up to u . Moreover, if firm i uses sales, then its equilibrium profits, Π_i , must equal its expected profits from advertising any $u_i \in (u^m, u]$. Hence, by setting Π_i equal to its expected profits from advertising u , where it would attract the shoppers for sure, $(1 - \theta_{-i}) \pi_i(u) - A_i$, we know that firm i 's equilibrium tiebreak probability, x_i^* , must equal $\chi_i(u)$; any other $x_i \neq \chi_i(u)$ is incompatible with a sales equilibrium under our assumptions. Thus, the equilibrium tiebreak probabilities for any firms using sales must ensure that all such firms have exactly the same incentive to employ the common upper utility bound, u .

To help understand this further, first consider a fully symmetric setting. Here, the firms already have identical incentives and so (5) implies that any firms that engage in sales will share a common equilibrium tiebreak probability. More importantly, now consider an example asymmetric setting where only firms 1 and 2 engage in sales, and where firm 1 is relatively more willing to advertise higher utilities, $\tilde{u}_1 > \tilde{u}_2$.²⁰ In equilibrium, to ensure that the firms have the same incentive to adopt a common upper bound, (5) implies that firm 1 must be assigned a larger equilibrium tiebreak probability, $x_1^* > x_2^*$. This acts to make firm 1 (firm 2, respectively) relatively less (more, resp.) aggressive by enhancing (reducing, resp.) its expected payoffs from not advertising. However, as later shown, it is not the case that x_1^* and x_2^* prompt the firms to play symmetric strategies. Thus, the equilibrium tiebreak probabilities do not fully neutralize the firms' heterogeneities, they just partially offset them.²¹

C. Duopoly ($n = 2$)

As now formalized, the equilibrium under duopoly is unique and takes one of two forms. When advertising costs are sufficiently low, there is a sales equilibrium where both firms engage in sales, otherwise, there is a nonsales equilibrium.

LEMMA 6: *When $n = 2$, any sales equilibrium has a unique upper utility bound, u , (implicitly) defined by (6), and each firm's advertising probability and offer distribution are uniquely defined by (7) and (8):*

$$(6) \quad \chi_1(u) + \chi_2(u) = 1,$$

$$(7) \quad \alpha_i = 1 - \frac{A_j}{x_i^*(1 - \theta) \pi_j^m},$$

$$(8) \quad F_i(u) = \frac{\theta_j(\pi_j^m - \pi_j(u)) + (A_j/x_i^*)}{(1 - \theta) \pi_j(u)}.$$

When $n = 2$, previous results have shown that any sales equilibrium must involve both firms and that $x_i^* = \chi_i(u)$. Hence, as the tiebreak probabilities must sum to one, the unique equilibrium upper bound must satisfy (6). One can then derive each firm's advertising probability and offer distribution from the fact that each firm must earn its equilibrium profits over $u = u^m$ and $u \in (u^m, u]$.²²

²⁰From inspection of (4), this could arise because firm 1 has a relatively lower advertising cost, A_1 , a relatively lower share of nonshoppers, θ_1 , a relatively lower level of per-consumer profits at u^m , π_1^m , and/or a relatively higher level of per-consumer profits at u , $\pi_1(u)$.

²¹One may ask why the shoppers should behave in accordance with (5). Similar questions arise within the wider concept of endogenous sharing rules. As noted by Simon and Zame (1990, 863), "The answer is, as always, that equilibrium theory never explains why any agents would act in any particular way. Equilibrium theory is intended to explain how agents behave, not why."

²²In an extreme case where the firms are asymmetric but $A_i = A_j \rightarrow 0$, the only way for the firms to share a common upper utility bound is for $x_i^* \rightarrow 1$ and $x_j^* \rightarrow 0$. This limit equilibrium converges to the equilibrium of a

Proposition 1 now shows that the characterized sales equilibrium exists uniquely if advertising costs are sufficiently low, $(A_1/\pi_1^m) + (A_2/\pi_2^m) < 1 - \theta$. In contrast, if advertising costs are higher, the firms are deterred from competing against each other – instead, there exists a unique nonsales equilibrium where both firms select u^m and refrain from advertising.

PROPOSITION 1: *Given Assumptions X, U, and A, the game has the following unique equilibrium (where consumers always expect nonadvertising firms to offer u^m):*

- (i) *If advertising costs are low, $(A_1/\pi_1^m) + (A_2/\pi_2^m) < 1 - \theta$, each firm i offers $u_i = u^m$ and does not advertise with probability $(1 - \alpha_i) \in (0, 1)$ according to (7), and advertises a sale offer $u_i \in (u^m, \bar{u}]$ according to (8) with probability α_i , where the upper bound, $\bar{u}$, solves (6), and firm i 's equilibrium tiebreak probability, $x_i^* = 1 - x_j^* \in (0, 1)$, is given by (5).*
- (ii) *If advertising costs are high, $(A_1/\pi_1^m) + (A_2/\pi_2^m) \geq 1 - \theta$, both firms offer $u_i = u^m$ and never advertise, $\alpha_i = 0$, and firm i 's equilibrium tiebreak probability equals $x_i^* = 1 - x_j^* \in [\chi_i(u^m), 1 - \chi_j(u^m)]$.*

One can demonstrate the implications of Proposition 1 by specifying the exact source of utility and profit function. As a very simple example, consider a symmetric setting with unit demand. Following footnote 12, let $u_i = V - p_i$ and $\pi(u_i) = V - c - u_i$, where $u^m = 0$ and $\pi^m = V - c$. Proposition 1 then implies a clearinghouse equilibrium with $x_i^* = 0.5$, $\Pi_i = (\theta(V - c)/2) + A$, $\alpha_i = 1 - (2A/((1 - \theta)(V - c)))$, and $\bar{u} = (2(1 - \theta)(V - c) - 4A)/(2 - \theta)$. By using $F_i(p) = 1 - F_i(u)$, one can then further derive $F_i(p) = 1 - (\theta(V - p) + 4A)/(2(1 - \theta)(p - c))$, with $p^m = V - u^m = V$ and $\underline{p} = V - \bar{u} = c + (\theta(V - c) + 4A)/(2 - \theta)$. Further details and additional example settings involving downward-sloping demand, and multiproduct firms are included in Appendix A.

Appendix A also shows how Proposition 1 can be used to characterize two common forms of sales that have remained unstudied within the clearinghouse literature. First, it characterizes sales behavior when firms use two-part tariffs. Existing theoretical work is very limited on this – we only know of Hendel, Lizzeri, and Roketskiy (2014), which shows how sales with nonlinear prices can emerge in a dynamic context with storable goods. In contrast, our framework considers a simpler clearinghouse setting while allowing for full asymmetry. We show that equilibrium sales will involve marginal cost pricing and firms mixing between not advertising a high fixed fee and advertising a stochastic lower fixed fee. While there is little empirical analysis available, our predictions are consistent with several anecdotal examples and some wider forms of evidence.²³

model that allows for $A = 0$ explicitly without our tiebreak rule. See online Appendix C1 for full details.

²³In practice, the use of such tariffs may also be driven by heterogeneous consumer preferences. However, consistent with our predictions, most UK suppliers of broadband, landline, and TV packages, as well as many gym

Second, it characterizes sales when firms hold prices constant but compete with some nonprice variable. This setting covers a broad set of commonly observed marketing practices, including (i) temporary extensions to package size or quantity, such as *X% Free* offers and bonus packs, (ii) temporary increases in product quality or content, such as the inclusion of free items or premiums, and (iii) other temporary increases in product value, such as the use of consumer finance deals, prize draws, or charity donations (see the discussions in Palazon and Delgado-Ballester 2009, Chen et al. 2012). As consistent with these phenomena, we show that equilibrium sales will involve firms mixing between not advertising a minimum regular package size/product value and advertising a sale with an increased package size/product value.

D. $n > 2$ Firms

We now extend the analysis beyond the simpler case of duopoly to offer new results about the number and type of firms that use sales in markets with $n > 2$ firms. Here, the sales literature with heterogeneous firms is scant because existing models quickly become intractable. Most notably, as part of their analysis, Baye, Kovenock, and de Vries (1992, Lemmas 7'–14') establish that only two firms can ever engage in sales behavior in a unit-demand clearinghouse model with zero advertising costs when firms differ in their shares of nonshoppers. The remaining firms with relatively larger shares of nonshoppers are less willing to compete and prefer to always set high (nonsale) prices to their nonshoppers. This finding has been extended to allow firms to vary in their product values (Kocas and Kiyak 2006) or costs (Shelegia 2012).

However, this two-firm prediction contrasts to common empirical findings where multiple heterogeneous sellers exhibit sales behavior (e.g., Lach 2002, Lewis 2008, Chandra and Tappata 2011). Instead, within our more general framework, we now demonstrate how *any* number of heterogeneous firms, $k^* \in [2, n]$, can engage in equilibrium sales. In particular, we explain the factors that determine the number of firms that use sales in equilibrium, and provide a broad characterization of the types of firms that are likely to use sales, depending on their advertising costs, nonshopper shares, and profit functions.

To proceed, we use (4) to index the firms in (weakly) decreasing order of $\tilde{u}_i$ from 1 to n , such that firm n is the least willing to advertise high utilities. We then focus on characterizing sales equilibria in two settings: (i) a quasi-symmetric setting where $u^m < \tilde{u}_i = \tilde{u}$ for all i , and (ii) a strict asymmetric setting where $u^m < \tilde{u}_n < \dots < \tilde{u}_1$.²⁴

Unlike the duopoly case, the tiebreak probabilities can now generate potential sources of sales equilibrium multiplicity. This can arise for two reasons. First, sales equilibria are now possible where at least one firm does not advertise. Here, Lemma 5 is insufficient to pin down a unique equilibrium value of x_i^* for firms that never

facilities and sports clubs, offer sales with reduced monthly fees but unchanged prices for charged services. Our predictions are also consistent with a finding in Giulietti, Waterson, and Wildenbeest (2014), which suggests that firms play mixed strategies with the implied final bill for an average consumer in the British electricity market where suppliers often employ two-part tariffs.

²⁴A third setting where a subset of firms have the same $\tilde{u}$ but where some remaining firms differ in $\tilde{u}$ can also be analyzed but is omitted for brevity due to its unnecessary complications.

advertise, $\alpha_i = 0$. Second, equilibrium multiplicity can also exist at knife-edge cases where $\tilde{u}_i = u$ for some firm i . Here, Lemma 5 implies that $x_i^* = 0$ such that firm i is indifferent between using sales or not. To avoid both of these ambiguities which are largely uninteresting from an economic perspective, we focus on sales equilibria where (i) firms that *never* advertise receive a zero equilibrium tiebreak probability, $x_i^* = 0$ if $\alpha_i = 0$, and (ii) advertising firms receive a positive equilibrium tiebreak probability, $x_i^* > 0$ if $\alpha_i > 0$. Lemma 7 now provides a preliminary step, before Proposition 2 summarizes our main equilibrium result.

LEMMA 7: *Consider any sales equilibrium that satisfies our restrictions with a given upper utility bound, $u > u^m$. Firm i uses sales if and only if $\tilde{u}_i > u$. Hence, (i) if $k^* = n$ then $u \in (u^m, \tilde{u}_n)$, and (ii) if $k^* \in [2, n)$ then $u \in [\tilde{u}_{k^*+1}, \tilde{u}_{k^*})$ and $K^* = \{1, \dots, k^*\}$.*

The basic intuition is straightforward—firm i will only be willing to engage in sales within a given sales equilibrium if the upper bound, u , is lower than the maximum utility that firm i could possibly wish to advertise, $\tilde{u}_i$, from (4). From this logic, Lemma 7 then goes on to make two immediate statements about the number and identity of firms that will use sales for a given u . If all firms use sales, $k^* = n$, then it must be that the upper bound is sufficiently low such that firm n is willing to use sales, $\tilde{u}_n > u$. Alternatively, if only $k^* \in [2, n)$ firms use sales, then the firms using sales must be those with the highest values of $\tilde{u}_i$, $K^* = \{1, \dots, k^*\}$. In particular, it must be that $u \in [\tilde{u}_{k^*+1}, \tilde{u}_{k^*})$ such that $u < \tilde{u}_i$ for $i \in K^* = \{1, \dots, k^*\}$, but $u \geq \tilde{u}_j$ for the remaining firms $j = \{k^* + 1, \dots, n\}$.

By using this together with an approach similar to Section IIC, we now characterize a unique sales equilibrium under the assumption that a sales equilibrium exists.²⁵ To avoid undue repetition of technical details, Proposition 2 jumps to the main result (see the proof for full details).

PROPOSITION 2: *When a sales equilibrium exists under our restrictions, it is unique. In such an equilibrium, firms $i \leq k^*$ engage in sales with interior probabilities, $\alpha_i \in (0, 1)$, while any remaining firms, $j > k^*$, never advertise $\alpha_j = 0$; k^* , x^* , and u are uniquely defined as follows:*

$$(9) \quad k^* = \begin{cases} n & \text{if } \sum_{i=1}^n \chi_i(\tilde{u}_n) < 1 < \sum_{i=1}^n \chi_i(u^m) \\ k \in [2, n) & \text{if } \sum_{i=1}^k \chi_i(\tilde{u}_k) < 1 \leq \sum_{i=1}^k \chi_i(\tilde{u}_{k+1}), \end{cases}$$

$$(10) \quad x_i^* = \begin{cases} \chi_i(u) \in (0, 1) & \text{if } i \leq k^* \\ 0 & \text{if } i > k^*, \end{cases}$$

$$(11) \quad \sum_{i=1}^{k^*} \chi_i(u) = 1.$$

²⁵ While Proposition 2 demonstrates equilibrium uniqueness, we are unable to prove existence for the general case when $n > 2$ as it is difficult to verify that $F'_i(u) > 0$ over the relevant u for all $i \in K^*$. However, existence can be guaranteed by further specifying the model—e.g., if the firms (i) are sufficiently symmetric, (ii) differ only in their advertising costs, or (iii) differ only in their profit functions when $\pi_i(u) = t_i \pi(u)$ where $t_i > 0$. An explicit asymmetric example is provided later in the section.

Before discussing the economic intuition and implications of Proposition 2, it is useful to provide a sketch of the technical proof. First, for a given $k^* \in [2, n]$, we derive the unique set of equilibrium tiebreaking probabilities, x^* in (10), and the unique upper bound, u in (11). As the nonadvertising firms have $x_i^* = 0$ by assumption, the values of $x_i^* \in (0, 1)$ for the advertising firms follow from Lemma 5 and must sum to one. Second, using Lemma 7, we then specify the conditions for the equilibrium upper bound, u , to be consistent with the stated number of advertising firms, k^* . In particular, for a given k^* , we require (9) to ensure that $u \in (u^m, \tilde{u}_n)$ if $k^* = n$, and $u \in [\tilde{u}_{k^*+1}, \tilde{u}_{k^*})$ if $k^* \in [2, n)$. Third, we show that (9) always specifies a unique equilibrium value of $k^* \in [2, n]$ provided advertising costs are sufficiently low: $\sum_{i=1}^n \chi_i(u^m) > 1$ or equivalently, $\sum_{i=1}^n (A_i/\pi_i^m) < (n-1)(1-\theta)$.²⁶ Finally, given k^* , the proof derives the unique advertising probabilities, $\alpha_i \in (0, 1)$, utility distributions, $F_i(u)$, and profits, $\bar{\Pi}_i$.

To examine the economic intuition of Proposition 2, we now further discuss the number and type of firms that use equilibrium sales, in turn.

The Number of Firms That Use Sales. First, consider the quasi-symmetric case, where $\tilde{u}_i = \tilde{u} > u^m$ for all i . Here, from (4), $\chi_i(\tilde{u}) = 0$ for all i , and so the conditions in (9) can never be satisfied for $k^* \in [2, n)$. Instead, any sales equilibrium must have $k^* = n$. If two firms wish to use sales, then they all wish to use sales. This equilibrium then resembles that under duopoly; all firms engage in sales on $(u^m, u]$, and the unique set of equilibrium tiebreaking probabilities, x^* , ensures that each firm has an identical incentive to adopt the common upper bound. However, the resulting tiebreak probabilities, utility distributions, and advertising probabilities need not be symmetric in equilibrium—they are only symmetric if the firms also have identical advertising costs, nonshopper shares, and profit functions. Equilibrium existence can be further demonstrated with $F'_i(u) > 0 \forall i$ over the relevant range if the firms are sufficiently symmetric.

Now consider the strict asymmetric setting, where $u^m < \tilde{u}_n < \dots < \tilde{u}_1$. Here, the intuition regarding k^* is more complex. However, some broad insights can be gained by simplifying to symmetric advertising costs, $A_i = A \forall i$, where any changes in A do not affect the ranking of firms in terms of $\tilde{u}_i$.

COROLLARY 1: *Suppose firms are strictly asymmetric, $u^m < \tilde{u}_n < \dots < \tilde{u}_1$, but advertising costs are symmetric, $A_i = A \forall i$. When a sales equilibrium exists, (i) only two firms use sales when advertising costs are sufficiently small, $k^* = 2$ when $A \rightarrow 0$, but (ii) all firms use sales when advertising costs are moderate, $k^* = n$ when $A \rightarrow (n-1)(1-\theta)/(\sum_i 1/\pi_i^m)$.*

When $A \rightarrow 0$, our findings are in line with the existing literature's two-firm result and generalize it to a broad range of market settings. When $A \rightarrow 0$, competition for the shoppers is fierce. Hence, the only way for any firms to have exactly the same incentives to employ a common upper bound is to give the firm with the highest

²⁶In contrast, if $\sum_{i=1}^n \chi_i(u^m) \leq 1$, then there is no solution for $k^* \in [2, n]$. Instead, in parallel to the duopoly case, there is a nonsales equilibrium with $k^* = 0$ for appropriate values of x^* .

incentive to advertise, firm 1, almost all the shoppers in a tie, $x_1^* \rightarrow 1$. In equilibrium, only firms 1 and 2 then use sales with $u = \tilde{u}_2$.

However, once we allow for higher advertising costs, the two-firm result becomes a special case of a new and more general relationship. Indeed, from (9), any number of heterogeneous firms $k^* \in [2, n]$ may now use sales in equilibrium. At the extreme, Corollary 1 states that *all* firms can use sales provided advertising costs are moderate. This appears paradoxical at first because an increase in A reduces the direct incentives for each firm to use sales, as evidenced by the associated reduction in $\tilde{u}_i$. However, the increase in A also softens the competition for the shoppers and reduces u in a way that prompts firms with lower $\tilde{u}_i$ to start using sales. Indeed, for moderate A , u can fall below $\tilde{u}_n$ such that *all* firms use sales. Thus, if the costs of informative advertising fall in the digital era, then Corollary 1 suggests that fewer firms might opt to use sales.

To conclude, we provide the following explicit example. Assume unit demand such that $\pi_i(u) = \Psi_i - u$ and $\pi_i^m = \Psi_i > 0$, where $\Psi_i = V_i - c_i$ denotes the value-cost markup. Suppose $n = 3$ and let the firms be symmetric aside from $\Psi_1 > \Psi_2 > \Psi_3 > 0$ such that $u^m < \tilde{u}_3 < \tilde{u}_2 < \tilde{u}_1$. The conditions in (9) then imply that $k^* = 2$ if $A \leq \underline{A}$, and $k^* = 3$ if $A \in (\underline{A}, A)$.²⁷ Firm 1 has the largest equilibrium tiebreaking probability, but it still advertises with the highest probability and offers the highest average utility offers. Equilibrium existence can be demonstrated with $F_i'(u) > 0$ over the relevant range $\forall i \leq k^*$ provided the firms are not too asymmetric.²⁸

The Types of Firms That Use Sales. To further explore the intuition of Proposition 2, we now consider its implications for the types of firms that are most likely to use sales. The existing literature only considers some specific dimensions under unit demand and zero advertising costs (e.g., Baye, Kovenock, and de Vries 1992; Kocas and Kiyak 2006; Shelegia 2012). However, in our general setting, we can offer a broad characterization. In particular, when $k^* < n$, Proposition 2 implies that the firms using sales will be the firms with the highest values of $\tilde{u}_i$ in (4). Corollary 2 then follows immediately and generally because $\tilde{u}_i$ is strictly decreasing in A_i and θ_i .

COROLLARY 2: *Suppose a sales equilibrium exists with $k^* < n$. Ceteris paribus, the firms with the lowest advertising costs, A_i , and shares of nonshoppers, θ_i , will use sales.*

However, understanding how a firm's profit function, $\pi_i(u)$, will impact its use of sales is more difficult because variations in profit functions may affect firms' profits differently at different utility levels. To proceed, we focus on the following functional form, $\pi_i(u) = \pi(u, \rho_i)$, where $\pi(\cdot)$ is common across firms, and $\rho_i > 0$ is a parameter representing firm i 's profitability. We assume that $\pi(u, \rho_i)$ is twice

²⁷ In particular, $\underline{A} \equiv (1 - \theta) \sqrt{(\Psi_1 - \Psi_3)(\Psi_2 - \Psi_3)}$ and $A = 2(1 - \theta) \left[(1/\Psi_1) + (1/\Psi_2) + (1/\Psi_3) \right]^{-1}$.

²⁸ Specifically, existence can be demonstrated for $(1/\Psi_1) + (1/\Psi_2) > 1/\Psi_3$. This condition also ensures that $\underline{A} < A$ and that Assumption A is satisfied for all $A < A$.

continuously differentiable and increasing in ρ_i for all $u \geq u^m$, where u^m maximizes $\pi(u, \rho_i)$ for any ρ_i .

COROLLARY 3: *Suppose a sales equilibrium exists with $k^* < n$, and that firms have symmetric shares of nonshoppers, $\theta_i = (\theta/n) \forall i$. Ceteris paribus, the firms that use sales will be those with the highest profitability, ρ_i , if increases in ρ_i raise per-consumer profits relatively more at higher rather than lower utility levels, $\pi_{\rho u}(u, \rho) \geq 0 \forall u > u^m$.*

Corollary 3 suggests that the effects of a firm's profitability, ρ_i , on its use of sales are ambiguous. However, it predicts that more profitable firms are more likely to use sales if $\pi_{\rho u}(u, \rho) \geq 0 \forall u > u^m$. This condition is satisfied for several common situations. For instance, it applies under unit demand or two-part tariffs, where ρ_i captures an increase in per-consumer demand or a reduction in marginal cost, and where $\pi_{\rho u}(\cdot) = 0$ as $\pi_{\rho}(u, \rho)$ is independent of u . Thus, in these instances, firms with larger per-consumer demand or lower marginal costs should be more likely to use sales. Alternatively, it also applies under downward-sloping demand.²⁹

III. Applications

Although our main contribution is methodological, this section provides three applications to illustrate how our framework can be used to help future research. Section IIIA examines a common procedure used in empirical work on sales and price dispersion. Section IIIB considers a family of games where firms engage in a prior activity to influence their share of nonshoppers, and Section IIIC presents a number of comparative statics to further analyze the effects of firm heterogeneities on sales.

A. Implications for Empirical Procedures

Within a given market, price dispersion is broadly divided into two forms. The first temporal form involves price differences that vary over time, such as those generated by sales. The second spatial form arises from persistent inter firm heterogeneities related to firms' characteristics. As listed in the introduction, many studies within the large empirical literature on sales attempt to focus on the temporal form by using a cleaning procedure. This procedure retrieves a set of price residuals from raw price data by using a regression involving observable firm characteristics or firm-level fixed effects. The price residuals are then interpreted as resulting from a homogeneous symmetric market and used to perform a reduced-form analysis or structural estimation. We now use our framework to better understand when this procedure is valid, and to suggest some modifications for it to be applied more widely.

²⁹ Under downward-sloping demand with a minimum utility constraint, $u^{\min} \geq \min\{u_1^m, \dots, u_n^m\}$, ρ_i is best interpreted as a (sufficiently small) reduction in firm i 's marginal cost. There, $\pi_{\rho u}(\cdot) > 0$, because a reduction in marginal cost profitably extends over a larger number of units for higher u .

Wildenbeest (2011) provides the only formal justification for the cleaning procedure under a specific set of market conditions. A version of his arguments can be derived in our framework, where in contrast, we generalize to positive advertising costs. Under unit demand and single products, suppose that firms vary in product quality and costs subject to a common value-cost markup, $V_i - c_i = \Psi \forall i$. This implies symmetric profit functions, $\pi_i(u) = \Psi - u \forall i$. Hence, if firms also have symmetric shares of nonshoppers and advertising costs, then any sales equilibrium will involve symmetric utility distributions, $F_i(u) = F(u) \forall i$. Importantly, firms' subsequent *price* distributions are then simple translations of each other as $p_i(u_i) = V_i - u_i$ under unit demand. Therefore, after observing a panel of price observations, one can obtain a measure of firms' utility offers (under the assumption of a stationary, infinitely repeated game). Specifically, one can regress the raw price data on a set of firm-level fixed effects, $p_{it} = \alpha + \delta_i + \varepsilon_{it}$, to soak up the effects of the firm heterogeneities and return a set of "cleaned" residuals that correctly proxy the utilities up to a positive constant.³⁰

Downward-Sloping Demand. For the procedure to be valid under downward-sloping demand, one first needs a revised condition to ensure that firms' profit functions are symmetric. While a more general condition can be provided, it is sufficient for our purposes to focus on the following simple case where each firm i has a marginal cost $c_i \geq 0$, and a linear per-consumer demand function that varies only in its intercept, $q_i(p_i) = a_i - bp_i$ where $a_i \geq 0$ and $b > 0$. From footnote 12, we know $u_i = (a_i - bp_i)^2/2b$ and $\pi_i(u) = (1/b)[a_i - bc_i - \sqrt{2bu}][\sqrt{2bu}]$. Hence, profit functions are symmetric if $a_i - bc_i = \Psi \forall i$. This restriction maintains some sense of Wildenbeest's constant value-cost assumption. Under this new condition, one would then aim to recover the firms' utility draws from the raw price data. However, unlike unit demand, the relationship between prices and utilities is nonlinear, $u_i = (a_i - bp_i)^2/2b$. Therefore, the cleaning procedure no longer provides correct estimates of utility up to a positive constant.³¹ Instead, to recover the utility draws, one would have to use a more complex, data-intensive procedure to estimate some of the demand parameters.

Asymmetric Utility Distributions. We now return to unit demand but depart from Wildenbeest's constant value-cost condition. Here, firms' utility distributions will be asymmetric, $F_i(u) \neq F(u)$, with different mean utilities, u_i^{ave} , and so firms' price distributions are no longer simple translations of each other. Hence, any fixed-effects regression cannot correctly proxy utilities up to a positive constant. Instead, one could use the following modification. Rather than using fixed effects, each firm's utilities, $u_i = V_i - p_i$, could be estimated more directly from the price data by inferring V_i . As firms will set their highest price equal to V_i with probability $(1 - \alpha_i) > 0$, V_i can be inferred from firm i 's maximum observed price, or by establishing its regular

³⁰The estimated residuals equal $\varepsilon_{it} \equiv p_{it} - p_i^{ave}$, where p_i^{ave} is the average price of firm i . We also know that $p_{it} = V_i - u_{it}$ and $p_i^{ave} = V_i - u^{ave}$, where u^{ave} is the average utility from the symmetric utility distribution. Hence, $\varepsilon_{it} \equiv -(u_{it} - u^{ave})$. For many applications, such as the estimation of search costs, this is sufficient as only the difference in utilities matters.

³¹Specifically, $p_{it} = (a_i/b) - \sqrt{(2u_{it}/b)}$ with residuals, $\varepsilon_{it} = p_{it} - p_i^{ave} \equiv -[\sqrt{(2u_{it}/b)} - \sqrt{(2u^{ave}/b)}]$.

price using a statistical procedure (e.g., Hosken and Reiffen 2004). Having recovered the utilities, one could then use our framework to analyze the observed price dispersion or estimate a structural model. For instance, by using our theoretical predictions, one could use data on prices and advertising frequencies to estimate each firm's share of nonshoppers, θ_i , marginal cost, c_i , and/or advertising cost, A_i .

B. Investment and Obfuscation Games

We now illustrate how our framework's capacity to allow for asymmetries can also help develop wider theoretical results. Specifically, we provide some results for a family of games where each firm's share of nonshoppers is determined endogenously as a function of the firms' actions prior to sales competition. Such actions can be interpreted as any (long-run) marketing activity that influences consumer loyalty, such as (i) investments in persuasive advertising or sales force methods, or (ii) some forms of obfuscation that influence the level of complexity or search costs within the market.

Starting with Chioveanu (2008), Carlin (2009), and Wilson (2010), some related streams of literature have become popular in recent years (see Grubb 2015 and Spiegel 2016 for reviews, and some wider related models). However, as explained in the introduction, only Baye and Morgan (2009) allow for positive costs of informative advertising, $A > 0$; yet, due to the consequent difficulties of analyzing asymmetries, they only show the existence of a continuum of symmetric Nash equilibria and a unique symmetric equilibrium in *secure strategies*. In contrast, by utilizing the flexibility of our framework, we can characterize the unique symmetric *subgame perfect* Nash equilibrium (SPNE) across a general market setting (when it exists). Moreover, while their equilibria imply that an increase in the cost of informative advertising can have a positive, negative, or zero effect on the prior actions, we offer a unique, empirically testable prediction.

To begin, consider a market with $n \geq 2$ otherwise symmetric firms. In stage 1, each firm i chooses its level of action, $e_i \geq 0$, with an associated unit cost equal to $\tau \geq 0$. Each firm i 's share of nonshoppers, $\theta_i(e_i, e_{-i}) \in (0, 1)$, is determined by its own action, e_i , and the actions of its rivals, given by the vector e_{-i} . Furthermore, we assume $\theta_i(e_i, e_{-i}) = \theta_j(e_i, e_{-j})$, $\forall i, j$, whenever e_{-j} is obtained from e_{-i} by permutation. Then, in stage 2, having observed the resulting shares, $\{\theta_1(\cdot), \dots, \theta_n(\cdot)\}$, the firms compete in utility with informative advertising costs, $A > 0$, in line with our main model and under our previously stated conditions.

The exact form of the function, $\theta_i(e_i, e_{-i})$, is allowed to depend upon the type of action. For instance, under (own) loyalty-increasing actions, $\theta_i(e_i, e_{-i})$ is strictly increasing in firm i 's action, e_i , and (weakly) decreasing in a rival's action, $e_{j \neq i}$, as consistent with persuasive advertising. Alternatively, under (own) loyalty-decreasing actions, $\theta_i(e_i, e_{-i})$ is strictly decreasing in e_i and (weakly) increasing in $e_{j \neq i}$, as consistent with some forms of obfuscation. However, in either case, we assume that the total proportion of nonshoppers, $\theta(\cdot) = \sum_{i=1}^n \theta_i(\cdot)$, is increasing and concave in any firm's action, e_i , with $\theta(\cdot) \rightarrow 1$ as $\sum_{i=1}^n e_i(\cdot) \rightarrow \infty$.³²

³² Hence, under a persuasive advertising interpretation, an increase in e_i expands the total proportion of persuaded (nonshopper) consumers, $\theta(\cdot)$, as well as increasing firm i 's proportion of nonshoppers, $\theta_i(\cdot)$. Under the obfuscation

We now seek to characterize the SPNE of the two-stage game. In particular, we wish to characterize a symmetric SPNE where the firms select positive actions before actively competing in a sales equilibrium (with $e_i = e > 0$ and $\alpha_i = \alpha > 0 \forall i$). To proceed, note that firm i will select its action, e_i , to maximize its equilibrium profits (net of any direct action costs), $\Pi_i = \theta_i(\cdot) \pi^m + (x_i^*/(1 - x_i^*))A - \tau e_i$. However, for our current purposes, it is better to use an alternative expression based on the expected profits from advertising the upper utility bound and attracting all the shoppers, $\Pi_i = \pi(u(\cdot)) [1 - \sum_{j \neq i} \theta_j(\cdot)] - A - \tau e_i$, where $u(\cdot)$ is now a function of the firms' actions. We can then state the following.³³

PROPOSITION 3: (i) *When a symmetric SPNE exists with $e_i = e^* > 0$ and $\alpha_i = \alpha^* > 0 \forall i$, the unique level of action, e^* , satisfies the following FOC:*

$$(12) \quad -\pi(u(\cdot))(n-1) \frac{\partial \theta_j(\cdot)}{\partial e_i} + \left(1 - \frac{(n-1)}{n} \theta(\cdot)\right) \pi'(u(\cdot)) \frac{\partial u(\cdot)}{\partial e_i} - \tau = 0.$$

(ii) *When actions are own loyalty-increasing (or decreasing), each firm's equilibrium action, e^* , is strictly increasing (decreasing) in the costs of informative advertising, A .*

Aside from increasing firm i 's total action costs by τ , the FOC in (12) suggests that a marginal increase in e_i generates two effects on the profits of firm i . The first effect changes the sum of available consumers that firm i can attract with a given offer of u by influencing its $(n-1)$ rivals' shares of nonshoppers. Under own loyalty-increasing actions, this effect is (weakly) positive by reducing rivals' nonshoppers, but under own loyalty-decreasing actions, the effect is (weakly) negative. In contrast, the second effect is always positive for either form of action. For a given sum of available consumers, it increases firm i 's profits per consumer. Specifically, an increase in e_i softens competition by raising the total proportion of nonshoppers, $\theta(\cdot)$, which prompts a reduction in the equilibrium value of u , such that $\pi'(u)(\partial u / \partial e_i) > 0$.

Now consider the comparative statics. An increase in the costs of informative advertising, A , enhances both of the main effects in (12). Within the first effect, an increase in A lowers u by reducing the firms' incentives to use advertised sales. This enhances firm i 's reward from attracting available consumers, $\pi(u)$. Thus, to reduce rival nonshopper shares, firm i will wish to select a higher action under own loyalty-increasing investment, and a lower action under own loyalty-decreasing investment. Within the second effect, an increase in A always enhances the incentives for firm i to select a higher action by increasing the ability of actions to soften competition; a higher A makes u more sensitive to e_i .

interpretation, an increase in e_i reduces firm i 's proportion of nonshoppers, $\theta_i(\cdot)$. However, the increase in e_i also makes the firms harder to compare and so it also increases the total proportion of nonshoppers, $\theta(\cdot)$.

³³ While Proposition 3 demonstrates uniqueness, equilibrium existence remains difficult to verify in the general case. However, it is clear that the costs of informative advertising, A , must be moderate. If A is sufficiently small, no symmetric equilibrium exists because firm i 's stage-2 profits exhibit a saddle point in e_i . This mirrors the existing literature with $A = 0$, e.g., Chioveanu (2008). However, if A is too large, then no sales equilibrium will exist.

Thus, under own loyalty-increasing actions both positive effects become larger, and so increases in the costs of informative advertising prompt higher equilibrium actions, $\partial e / \partial A > 0$. Hence, reductions in the costs of informative advertising may be beneficial to markets for two reasons: a direct effect in increasing sales competition, but also an indirect effect in reducing brand loyalty and thereby further reducing prices. To our knowledge, this result offers the first theoretical prediction about how the costs of informative advertising affect equilibrium levels of loyalty-enhancing marketing activities such as persuasive advertising. It remains untested empirically.

Under own loyalty-decreasing actions, increases in A prompt the first effect to become more negative and the second effect to become more positive. However, the first effect dominates such that increases in A lower equilibrium actions, $\partial e / \partial A < 0$. Hence, under an obfuscation interpretation, our model predicts that obfuscation should increase in response to reductions in informative advertising costs. This complements several theories that explain how firms obfuscation levels should rise after a fall in search costs due to advances in search technologies (e.g., Ellison and Ellison 2009, Ellison and Wolitzky 2012).

C. Comparative Statics

This subsection provides some new comparative static results. For symmetric market cases, the findings extend standard clearinghouse results to a generalized market setting. More substantially, for asymmetric market cases where the existing literature has offered limited results, we offer several new insights by isolating the effects of individual firm characteristics on sales and market performance. In line with footnote 8, these predictions remain untested empirically, as the empirical literature has focused on other issues. For simplicity, we derive the statics under duopoly. However, related results can also be derived for $n > 2$.

Changes in a Firm's Share of Nonshoppers. Under symmetry, our framework produces a generalized form of the standard clearinghouse result—an increase in the proportion of nonshoppers, θ , (and associated reduction in shoppers, $1 - \theta$) leads to a lower sales probability, α , higher equilibrium profits, Π , and lower average offers, $E(u)$. More interestingly, we can analyze a change in an individual firm's share of nonshoppers, θ_i . As these effects are difficult to characterize, we focus on evaluating a small increase in θ_i at the point of symmetry. To proceed, one must also stipulate whether the increase is associated with a reduction in shoppers, $1 - (\theta_i + \theta_j)$, or rival nonshoppers, $\theta_j = \theta - \theta_i$. We first consider the latter.

PROPOSITION 4: *In an otherwise symmetric market, consider a small increase in firm i 's nonshoppers θ_i (and associated reduction in θ_j). Starting from a point of symmetry, $\theta_i = \theta_j$, this increases firm i 's equilibrium profits, $\bar{\Pi}_i$, decreases firm i 's sales probability, α_i , and average offer, $E(u_i)$, and generates the opposite effects on firm j .*

Ceteris paribus, an increase in θ_i reduces $\tilde{u}_i$ and makes firm i less willing to offer higher utilities. However, to maintain the firms' incentives to employ a common u

in equilibrium, this is partially offset by a reduction in firm i 's equilibrium tiebreak probability, x_i^* (and an associated increase in x_j^*). Hence, when combined, these effects lead firm i (firm j) to gain higher (lower) equilibrium profits, use sales with a lower (higher) probability, and set lower (higher) average utility offers. While intuitive, the last result about average utility offers differs from Arnold et al. (2011) which considers asymmetric θ_i with unit demand and $A > 0$ under the past literature's tiebreak rule. Instead, they suggest an increase in θ_i leads firm i to become *more* aggressive in its advertised prices and so offer *higher* average utility offers. Unlike our results, this finding conflicts with standard results under $A = 0$ such as Narasimhan (1988).³⁴

Changes in a Firm's Advertising Costs. Under symmetry, one can verify a generalized form of the standard result—an increase in advertising costs, A , leads to a lower sales probability, α , higher equilibrium profits, Π , and lower average offers, $E(u)$. More substantially, our framework can now isolate the effects from a change in an individual firm's advertising cost, A_i .

PROPOSITION 5: *In an otherwise symmetric market, a small increase in firm i 's advertising cost, A_i , leads to no change in firm i 's equilibrium profits, $\bar{\Pi}_i$, an increase in firm j 's equilibrium profits, $\bar{\Pi}_j$, and a decrease in both firms' sales probabilities and average offers, α_k and $E(u_k)$ for $k = i, j$.*

Ceteris paribus, an increase in A_i reduces $\tilde{u}_i$ and makes firm i less willing to offer higher utilities. However, to maintain a common u , this is also accompanied by a reduction in firm i 's equilibrium tiebreak probability, x_i^* , (and associated increase in x_j^*). This leads to no aggregate effect on Π_i because the direct effect from A_i is exactly offset by the indirect effect from x_i^* . However, an increase in A_i raises firm j 's profits, Π_j , because the indirect effect raises x_j^* . Hence, given our assumptions, the standard profit result under symmetry is driven by the rise in rival, rather than own, advertising costs. Finally, the increase in A_i reduces both firms' use of sales and prompts a subsequent reduction in their average offers.

Changes in a Firm's Profit Function. As previously explained, studying variations in firms' profit functions is difficult at a general level. However, we now provide some comparative statics by using the form of profit function introduced in Section IID, $\pi(u, \rho)$. In particular, we focus on situations where an increase in firm i 's profitability, ρ_i , raises firm i 's per-consumer profits relatively more at higher rather than lower utility levels, $\pi_{\rho u}(u, \rho) \geq 0 \forall u > u^m$, which we argued was consistent with many market environments.

Under symmetry, one can derive a new result; an industry-wide increase in profitability, ρ , increases firms' equilibrium profits, while also inducing an increase in

³⁴ With two exceptions, our findings remain robust in the alternative case where the increase in θ_i comes from a reduction in shoppers. First, an increase in θ_i now raises Π_j because there is no reduction in θ_j . Second, an increase in θ_i can provide reversed effects on α_j and $E(u_j)$ if advertising costs are relatively high. This arises due to the opposing effects of a decrease in shoppers, and an increase in x_j^* (which varies in A). However, firm i still offers a lower average utility than firm j .

the probability of sales, α , and average utility offers, $E(u)$. In addition, we can also isolate the effects from a change in an individual firm's profitability, ρ_i . The existing literature has only been able to consider a few specific cases involving individual changes in marginal costs or product values under unit demand and zero advertising costs (e.g., Kocas and Kiyak 2006). Related technical difficulties also remain in our framework. However, by evaluating a small change at the point of symmetry, we can substantially improve upon past results.

PROPOSITION 6: *In an otherwise symmetric market, consider a small increase in firm i 's profitability, ρ_i . Starting from a point of symmetry, $\rho_i = \rho_j$, this increases firm i 's equilibrium profits, $\bar{\Pi}_i$, sales probability, α_i , and average offer, $E(u_i)$, but decreases firm j 's equilibrium profits, $\bar{\Pi}_j$.*

An increase in ρ_i unambiguously increases firm i 's equilibrium profits and overall industry profits. Further, in the common cases where $\pi_{\rho u}(u, \rho) \geq 0$, an increase in ρ_i also raises firm i 's incentive to advertise higher utilities. This prompts an increase in firm i 's equilibrium tiebreak probability, x_i^* , to maintain a common u . Nevertheless, firm i increases its sales probability, α_i , and its average offer, $E(u_i)$, while firm j receives lower equilibrium profits, $\bar{\Pi}_j$.³⁵

IV. Conclusion

Due to the associated technical complexities, existing clearinghouse sales models are unable to fully consider the effects of firm heterogeneity. This restricts theoretical understanding, empirical analysis, and policy guidance with regards to sales and price dispersion, and many other topics in related literatures. The current paper has tried to fill this gap by providing a generalized clearinghouse sales framework. In addition, the paper has (i) provided original insights into the number and types of firms that use sales, (ii) offered new results on how firm heterogeneity affects market outcomes, (iii) extended a cleaning procedure that is commonly used within the empirical literature, and (iv) analyzed a family of games to study persuasive advertising and obfuscation in sales markets. By opening up the analysis of sales with firm heterogeneity, we hope that our framework can enable future research to further address many theoretical and empirical issues that remain underexplored.

A A.E E S

This appendix contains a number of equilibrium specifications by further detailing the exact source of utility and profit function. To help exposition, we illustrate the main steps within a context of full symmetry. Here, using the results from Sections IIC and IID, this also permits us to easily include the equilibrium details for $n \geq 2$ firms rather than just duopoly. Section A summarizes the basic n -firm equilibrium under full symmetry. Sections B–D then detail the equilibrium under

³⁵In general, the effects on α_j and $E(u_j)$ are more nuanced. However, for u sufficiently close to u^m , an increase in ρ_i leads both firms to increase their sales probabilities and average offers.

unit demand, downward-sloping demand, and multiple products, while discussing how our framework can reproduce and substantially extend many of the past literature's key predictions for pricing, advertising, and purchasing behavior. Sections E–F demonstrate how Proposition 1 can be used to characterize some common forms of sales that have remained unstudied within the clearinghouse literature, including cases where firms use two-part tariffs or nonprice variables such as package size (e.g., *X% Free*).

A. Equilibrium under Full Symmetry

Using the results from Sections IIC and IID, we can now summarize the basic n -firm equilibrium under full symmetry. Here, all firms will use sales in equilibrium and the equilibrium tiebreak rule will be fully symmetric, $x_i^* = 1/n$. In addition, for all i , we know $\Pi_i = \frac{\theta \pi^m}{n} + \frac{A}{n-1}$, $\alpha_i = 1 - \left(\frac{\frac{n}{n-1}A}{(1-\theta)\pi^m} \right)^{\frac{1}{n-1}}$, $u = \pi^{-1} \left(\frac{\theta \pi^m + \frac{n^2}{n-1}A}{\theta + n(1-\theta)} \right)$ and $F_i(u) = \left(\frac{\theta[\pi^m - \pi(u)] + \frac{n^2}{n-1}A}{n(1-\theta)\pi(u)} \right)^{\frac{1}{n-1}}$.

B. Equilibrium with Unit Demand

Building on footnote 12, suppose $u_i = V - p_i$ and $\pi(u_i) = V - c - u_i$, where $u_i^m = 0$ and $\pi^m = V - c$. Under full symmetry, this produces a simple clearinghouse equilibrium with $x_i^* = 1/n$, $\Pi_i = \frac{\theta(V-c)}{n} + \frac{A}{n-1}$, $\alpha_i = 1 - \left(\frac{\frac{n}{n-1}A}{(1-\theta)(V-c)} \right)^{\frac{1}{n-1}}$, and $u = \frac{n(1-\theta)(V-c) - \frac{n^2}{n-1}A}{\theta + n(1-\theta)}$. By using $F_i(p) = 1 - F_i(u)$, one can further derive $F_i(p) = 1 - \left(\frac{\theta(V-p) + \frac{n^2}{n-1}A}{n(1-\theta)(p-c)} \right)^{\frac{1}{n-1}}$ with $p^m = V - u^m = V$ and $\underline{p} = V - u$. This collapses to the (popularized) equilibrium of Varian (1980) when $A \rightarrow 0$. Under firm heterogeneity, the past literature has considered various asymmetries in nonshopper shares, product values, and/or costs in the duopoly setting under the restriction, $A_i = A_j = 0$. As detailed in online Appendix C1, our framework can obtain these equilibria in the limit when $A_i = A_j \rightarrow 0$ while allowing for any θ_i , c_i , and V_i . Moreover, our framework can also extend them to allow for positive and asymmetric advertising costs.

C. Equilibrium with Downward-Sloping Demand

Suppose each consumer has a downward-sloping demand function for firm i 's good, $q_i(p_i)$, and that firm i has a constant marginal cost, $c_i \geq 0$. Firm i then has a per-consumer profit function equal to $\pi_i(p_i) = (p_i - c_i)q_i(p_i)$, and the utility at firm i can be given by its associated consumer surplus, $u_i = S(p_i, q_i(p_i))$. Under our sales equilibrium, each firm i then chooses its price to maximize its profit subject to supplying its required utility draw, u_i , with $p_i^*(u_i) = \arg \max_{p_i} \pi_i(p_i)$ subject to $S(p_i, q_i(p_i)) = u_i$. It also follows that $p_i^m = \arg \max_{p_i} \pi_i(p_i)$, with $u_i^m = S(p_i^m, q_i(p_i^m))$ and $\pi_i^m \equiv \pi_i(u_i^m) = \pi_i(p_i^m)$.

Under full symmetry, this produces a standard clearinghouse equilibrium (e.g., Baye, Morgan, and Scholten 2004a). Specifically, it follows that $x_i^* = 1/n$, $\Pi_i = \frac{\theta\pi(p^m)}{n} + \frac{A}{n-1}$, $\alpha_i = 1 - \left(\frac{\frac{n}{n-1}A}{(1-\theta)\pi(p^m)} \right)^{\frac{1}{n-1}}$, and $u = \pi^{-1} \left(\frac{\theta\pi(p^m) + \frac{n^2}{n-1}A}{\theta + n(1-\theta)} \right)$. Using $F_i(p) = 1 - F_i(u)$, it follows that $F_i(p) = 1 - \left(\frac{\theta[\pi(p^m) - \pi(p)] + \frac{n^2}{n-1}A}{n(1-\theta)\pi(p)} \right)^{\frac{1}{n-1}}$ with $\underline{p} = p^*(u)$ and $\bar{p} = p^*(u^m) = p^m$.³⁶ Our framework shows how this clearinghouse equilibrium can be generalized to allow for asymmetric advertising costs, nonshopper shares, and profit functions (provided there is some binding minimum utility constraint if needed).

D. Equilibrium with Downward-Sloping Demand and Multiple Products

An equilibrium with downward-sloping demand where firms sell multiple products can be derived as an extension of Section C above. In particular, now suppose firm i has $K_i \geq 1$ products, where $\mathbf{c}_i = \{c_{i1}, \dots, c_{iK_i}\}$, $\mathbf{p}_i = \{p_{i1}, \dots, p_{iK_i}\}$, and $\mathbf{q}_i(\mathbf{p}_i) = \{q_{i1}(\mathbf{p}_i), \dots, q_{iK_i}(\mathbf{p}_i)\}$ denote the associated vectors of (constant) marginal costs, prices, and product demand functions per consumer. Many of the steps from Section C, then, apply immediately. In particular, under suitable demand assumptions, there exists a unique price vector that maximizes a firm's profits subject to supplying a given utility draw u across its products, such that $\mathbf{p}_i^*(u) = \arg \max_{\mathbf{p}_i} \pi_i(\mathbf{p}_i) = \mathbf{q}_i^*(\mathbf{p}_i)'(\mathbf{p}_i - \mathbf{c}_i)$ subject to $S(\mathbf{p}_i, \mathbf{q}_i^*(\mathbf{p}_i)) = u$, with resulting profits per consumer, $\pi_i(u) = \pi_i(\mathbf{p}_i^*(u))$.³⁷ Under full symmetry, this reproduces versions of the equilibrium of Simester (1997) when marginal costs are zero, $K \geq 1$, and $A \rightarrow 0$. More substantially, for any marginal costs and any K , we can permit positive asymmetric advertising costs and asymmetric shares of nonshoppers.

E. Equilibrium with Two-Part Tariffs

In line with the associated discussion in Section IIC, suppose the firms employ two-part tariffs to better extract consumer surplus as consistent with a form of oligopolistic first-degree price discrimination. In particular, reconsider the analysis in Sections C and D above where firm i has $K_i \geq 1$ products, downward-sloping demand functions, $\mathbf{q}_i(\cdot)$, and marginal costs, $\mathbf{c}_i$. However, now let each firm i set a K_i -dimensional vector of marginal prices (per unit of consumption), $\mathbf{p}_i$, and a single fixed fee, $f_i \geq 0$. It then follows that $\pi_i(\mathbf{p}_i, f_i) = \mathbf{q}_i(\mathbf{p}_i)'(\mathbf{p}_i - \mathbf{c}_i) + f_i$ and $u_i = S(\mathbf{p}_i, \mathbf{q}_i(\mathbf{p}_i)) - f_i$ where $S(\mathbf{p}_i, \mathbf{q}_i(\mathbf{p}_i))$ denotes a consumer's surplus at i gross of its fixed fee. To generate any utility, u' , i will choose $\mathbf{p}_i$ and f_i to maximize $\pi_i(\mathbf{p}_i, f_i)$ subject to $S(\mathbf{p}_i, \mathbf{q}_i(\mathbf{p}_i)) - f_i = u'$. This implies marginal cost

³⁶For comparison, standard clearinghouse models express the price distribution conditional on advertising. In our model, this equates to $F_i^A(p) = (1 - F_i(u))/\alpha_i$.

³⁷This constrained pricing decision can be thought of as a Ramsey problem. Individual prices are hard to fully characterize, but with additional restrictions, firms can be shown to optimally use lower prices on products that are more price-elastic and complementary to other products. See Simester (1997) and Armstrong and Vickers (2001) for more discussion.

pricing across each product, $\mathbf{p}_i = \mathbf{c}_i$, together with a suitably adjusted fixed fee, $f_i = S(\mathbf{c}_i, \mathbf{q}_i(\mathbf{c}_i)) - u'$. The full equilibrium can then be derived using $\pi_i(u) = S(\mathbf{c}_i, \mathbf{q}_i(\mathbf{c}_i)) - u$, $u^m = 0$, and $\pi_i(u^m) = S(\mathbf{c}_i, \mathbf{q}_i(\mathbf{c}_i))$.

F. Equilibrium with Nonprice Sales

For ease of exposition, consider a fully symmetric market with single products and unit demand. In line with the associated discussion in Section IIC, suppose each firm's price is fixed at $p > 0$, and that each firm employs some other sales variable $z_i \in [\underline{z}, \bar{z}]$. To avoid any unnecessary complications, we make two assumptions to ensure unique correspondences between z_i , u_i , and $\pi(u_i)$. First, let both the consumer's willingness to pay for i 's product, $V(z_i)$, and i 's marginal (per unit) cost, $c(z_i)$, be strictly increasing in z_i , such that $u(z_i) = V(z_i) - p$ is strictly increasing in z_i , and $\pi(z_i) = p - c(z_i)$ is strictly decreasing in z_i . Second, let $u(\underline{z}) = V(\underline{z}) - p \geq 0$ and $\pi(\bar{z}) = p - c(\bar{z}) > 0$. Because profits and utilities are monotone in z , we have $z = V^{-1}(p + u)$. We can then derive $\pi(u) = p - c(V^{-1}(p + u))$ and $u^m = V(\underline{z}) - p$. To ensure the equilibrium exists, we can verify that $\pi(u^m) = p - c(\underline{z}) > 0$ and $\pi'(u) < 0$. The full equilibrium can then be explicitly derived and shown to exhibit the features listed in Section IIC.

Appendices B and C can be found in the supplementary online Appendix.

REFERENCES

- Allen, Jason, Robert Clark, and Jean-François Houde. 2014. The Effect of Mergers in Search Markets: Evidence from the Canadian Mortgage Industry. *American Economic Review* 104 (10): 3365–96.
- An, Yonghong, Michael R. Baye, Yingyao Hu, John Morgan, and Matt Shum. 2017. Identification and Estimation of Online Price Competition with an Unknown Number of Firms. *Journal of Applied Econometrics* 32 (1): 80–102.
- Anderson, Simon P., Alicia Baik, and Nathan Larson. 2015. Personalized Pricing and Advertising: An Asymmetric Equilibrium Analysis. *Games and Economic Behavior* 92: 53–73.
- Anderson, Simon P., and Régis Renault. 2018. Firm Pricing with Consumer Search. In *Handbook of Game Theory and Industrial Organization*, Vol. 2, edited by Luis C. Corchón and Marco A. Marini, 177–224. Northampton, MA: Edward Elgar.
- Armstrong, Mark. 2015. Search and Ripoff Externalities. *Review of Industrial Organization* 47 (3): 273–302.
- Armstrong, Mark, and John Vickers. 2001. Competitive Price Discrimination. *RAND Journal of Economics* 32 (4): 579–605.
- Armstrong, Mark, and John Vickers. 2019. Discriminating against Captive Customers. *American Economic Review: Insights* 1 (3): 257–72.
- Arnold, Michael, Chenguang Li, Christine Saliba, and Lan Zhang. 2011. Asymmetric Market Shares, Advertising and Pricing: Equilibrium with an Information Gatekeeper. *Journal of Industrial Economics* 59 (1): 63–84.
- Barron, John M., Beck A. Taylor, and John R. Umbeck. 2004. Number of Sellers, Average Prices, and Price Dispersion. *International Journal of Industrial Organization* 22 (8–9): 1041–66.
- Baye, Michael R., Dan Kovenock, and Casper G. de Vries. 1992. It Takes Two to Tango: Equilibria in a Model of Sales. *Games and Economic Behavior* 4 (4): 493–510.
- Baye, Michael R., and John Morgan. 2001. Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets. *American Economic Review* 91 (3): 454–74.
- Baye, Michael R., and John Morgan. 2009. Brand and Price Advertising in Online Markets. *Management Science* 55 (7): 1139–51.

- Baye, Michael R., John Morgan, and Patrick Scholten. 2004a. Price Dispersion in the Small and in the Large: Evidence from an Internet Price Comparison Site. *Journal of Industrial Economics* 52 (4): 463–96.
- Baye, Michael R., John Morgan, and Patrick Scholten. 2004b. Temporal Price Dispersion: Evidence from an Online Consumer Electronics Market. *Journal of Interactive Marketing* 18 (4): 101–15.
- Baye, Michael R., John Morgan, and Patrick Scholten. 2006. Information, Search, and Price Dispersion. In *Handbook in Information Systems: Economics and Information Systems*, Vol. 1, edited by Terrence Hendershott, 323–75. Bingley, UK: Emerald Group Publishing Limited.
- Brown, Jeffrey R., and Austan Goolsbee. 2002. Does the Internet Make Markets More Competitive? Evidence from the Life Insurance Industry. *Journal of Political Economy* 110 (3): 481–507.
- Burdett, Kenneth, and Kenneth L. Judd. 1983. Equilibrium Price Dispersion. *Econometrica* 51 (4): 955–69.
- Burdett, Kenneth, and Guido Menzio. 2018. The (Q, S, s) Pricing Rule. *Review of Economic Studies* 85 (2): 892–928.
- Carlin, Bruce I. 2009. Strategic Price Complexity in Retail Financial Markets. *Journal of Financial Economics* 91 (3): 278–87.
- Chandra, Ambarish, and Mariano Tappata. 2011. Consumer Search and Dynamic Price Dispersion: An Application to Gasoline Markets. *RAND Journal of Economics* 42 (4): 681–704.
- Chen, Haipeng, Howard Marmorstein, Michael Tsiros, and Akshay R. Rao. 2012. When More Is Less: The Impact of Base Value Neglect on Consumer Preferences for Bonus Packs over Price Discounts. *Journal of Marketing* 76 (4): 64–77.
- Chioveanu, Ioana. 2008. Advertising, Brand Loyalty and Pricing. *Games and Economic Behavior* 64 (1): 68–80.
- Dubovik, Andrei, and Maarten C.W. Janssen. 2012. Oligopolistic Competition in Price and Quality. *Games and Economic Behavior* 75 (1): 120–38.
- Ellickson, Paul B., and Sanjog Misra. 2008. Supermarket Pricing Strategies. *Marketing Science* 27 (5): 811–28.
- Ellison, Glenn, and Sara Fisher Ellison. 2009. Search, Obfuscation, and Price Elasticities on the Internet. *Econometrica* 77 (2): 427–52.
- Ellison, Glenn, and Alexander Wolitzky. 2012. A Search Cost Model of Obfuscation. *RAND Journal of Economics* 43 (3): 417–41.
- Galeotti, Andrea, and José Luis Moraga-González. 2009. Platform Intermediation in a Market for Differentiated Products. *European Economic Review* 53 (4): 417–28.
- Giulietti, Monica, Michael Waterson, and Matthijs Wildenbeest. 2014. Estimation of Search Frictions in the British Electricity Market. *Journal of Industrial Economics* 62 (4): 555–90.
- Grubb, Michael D. 2015. Failing to Choose the Best Price: Theory, Evidence, and Policy. *Review of Industrial Organization* 47 (3): 303–40.
- Guimaraes, Bernardo, and Kevin D. Sheedy. 2011. Sales and Monetary Policy. *American Economic Review* 101 (2): 844–76.
- Hendel, Igal, Alessandro Lizzeri, and Nikita Roketskiy. 2014. Nonlinear Pricing of Storable Goods. *American Economic Journal: Microeconomics* 6 (3): 1–34.
- Hitsch, Günter J., Ali Hortacsu, and Xiliang Lin. 2019. Prices and Promotions in U.S. Retail Markets: Evidence from Big Data. Chicago Booth Working Paper 17-18.
- Hosken, Daniel, and David Reiffen. 2004. Patterns of Retail Price Variation. *RAND Journal of Economics* 35 (1): 128–46.
- Hosken, Daniel, and David Reiffen. 2007. Pricing Behavior of Multiproduct Retailers. *B.E. Journal of Theoretical Economics* 7 (1): 1354–54.
- Janssen, Maarten C.W., and José Luis Moraga-González. 2004. Strategic Pricing, Consumer Search and the Number of Firms. *Review of Economic Studies* 71 (4): 1089–1118.
- Jensen, Robert. 2007. The Digital Provide: Information (Technology), Market Performance, and Welfare in the South Indian Fisheries Sector. *Quarterly Journal of Economics* 122 (3): 879–924.
- Kaplan, Greg, and Guido Menzio. 2015. The Morphology of Price Dispersion. *International Economic Review* 56 (4): 1165–1206.
- Kaplan, Greg, and Guido Menzio. 2016. Shopping Externalities and Self-Fulfilling Unemployment Fluctuations. *Journal of Political Economy* 124 (3): 771–825.
- Kocas, Cenk, and Tunga Kiyak. 2006. Theory and Evidence on Pricing by Asymmetric Oligopolies. *International Journal of Industrial Organization* 24 (1): 83–105.
- Lach, Saul. 2002. Existence and Persistence of Price Dispersion: An Empirical Analysis. *Review of Economics and Statistics* 84 (3): 433–44.

- Lach, Saul, and José L. Moraga-González.** 2017. Asymmetric Price Effects of Competition. *Journal of Industrial Economics* 65 (4): 767–803.
- Lewis, Matthew.** 2008. Price Dispersion and Competition with Differentiated Sellers. *Journal of Industrial Economics* 56 (3): 654–78.
- Moraga-González, José Luis, Zsolt Sándor, and Matthijs R. Wildenbeest.** 2013. Semi-nonparametric Estimation of Consumer Search Costs. *Journal of Applied Econometrics* 28 (7): 1205–23.
- Moraga-González, José Luis, and Matthijs R. Wildenbeest.** 2012. Comparison Sites. In *The Oxford Handbook of the Digital Economy*, edited by Martin Peitz and Joel Waldfogel, 224–53. New York: Oxford University Press.
- Nakamura, Emi, and Jón Steinsson.** 2008. Five Facts about Prices: A Reevaluation of Menu Cost Models. *Quarterly Journal of Economics* 123 (4): 1415–64.
- Nakamura, Emi, Jón Steinsson, Patrick Sun, and Daniel Villar.** 2018. The Elusive Costs of Inflation: Price Dispersion during the U.S. Great Inflation. *Quarterly Journal of Economics* 133 (4): 1933–80.
- Narasimhan, Chakravarthi.** 1988. Competitive Promotional Strategies. *Journal of Business* 61 (4): 427–49.
- Palazon, Mariola, and Elena Delgado-Ballester.** 2009. Effectiveness of Price Discounts and Premium Promotions. *Psychology and Marketing* 26 (12): 1108–29.
- Pennerstorfer, Dieter, Philipp Schmidt-Dengler, Nicolas Schutz, Christoph Weiss, and Biliana Yontcheva.** 2020. Information and Price Dispersion: Theory and Evidence. *International Economic Review* 61 (2): 871–99.
- Potters, Jan, and Sigrid Suetens.** 2013. Oligopoly Experiments in the Current Millennium. *Journal of Economic Surveys* 27 (3): 439–60.
- Robert, Jacques, and Dale O. Stahl, II.** 1993. Informative Price Advertising in a Sequential Search Model. *Econometrica* 61 (3): 657–86.
- Ronayne, David.** 2020. Price Comparison Websites. Warwick Economics Research Papers 1056.
- Shankar, Venkatesh, and Ruth N. Bolton.** 2004. An Empirical Analysis of Determinants of Retailer Pricing Strategy. *Marketing Science* 23 (1): 28–49.
- Shelegia, Sandro.** 2012. Asymmetric Marginal Costs in Search Models. *Economics Letters* 116 (3): 551–53.
- Sherman, Joshua, and Avi Weiss.** 2017. On Fruitful and Futile Tests of the Relationship between Search and Price Dispersion. *Economic Inquiry* 55 (4): 1898–1918.
- Simester, Duncan.** 1997. Note. Optimal Promotion Strategies: A Demand-Sided Characterization. *Management Science* 43 (2): 251–56.
- Simon, Leo K., and William R. Zame.** 1990. Discontinuous Games and Endogenous Sharing Rules. *Econometrica* 58 (4): 861–72.
- Sorensen, Alan T.** 2000. Equilibrium Price Dispersion in Retail Markets for Prescription Drugs. *Journal of Political Economy* 108 (4): 833–50.
- Spiegler, Ran.** 2016. Choice Complexity and Market Competition. *Annual Review of Economics* 8: 1–25.
- Stahl, Dale O., II.** 1989. Oligopolistic Pricing with Sequential Consumer Search. *American Economic Review* 79 (4): 700–12.
- Varian, Hal R.** 1980. A Model of Sales. *American Economic Review* 70 (4): 651–59.
- Wildenbeest, Matthijs R.** 2011. An Empirical Model of Search with Vertically Differentiated Products. *RAND Journal of Economics* 42 (4): 729–57.
- Wilson, Chris M.** 2010. Ordered Search and Equilibrium Obfuscation. *International Journal of Industrial Organization* 28 (5): 496–506.
- Woodward, Susan E., and Robert E. Hall.** 2012. Diagnosing Consumer Confusion and Sub-optimal Shopping Effort: Theory and Mortgage-Market Evidence. *American Economic Review* 102 (7): 3249–76.
- Zhang, Xing, Tat Y. Chan, and Ying Xie.** 2018. Price Search and Periodic Price Discounts. *Management Science* 64 (2): 495–510.