Online Supplement for

A Deterministic Sequential Maximin Latin Hypercube Design Method Using

Successive Local Enumeration for Metamodel-based Optimization

Engineering Optimization

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Appendix

A. Formulations of Benchmark Problems

Six-hump camel-back (SC) function

$$f(\mathbf{x}) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$$
$$x_{1,2} \in [-2, 2]$$

Branin (BR) function

$$f(\mathbf{x}) = \left(x_2 - 5.1 \left(\frac{x_1}{2\pi} \right)^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left(1 - \frac{1}{8\pi} \right) \cos(x_2) + 10$$
$$x_1 \in [-5, 10], x_2 \in [0, 15]$$

Generalized polynomial function (GF)

$$f(\mathbf{x}) = u_1^2 + u_2^2 + u_3^2$$
$$u_i = c_i - x_1(1 - x_2^i), i = 1, 2, 3$$
$$c_1 = 1.5, c_2 = 2.25, c_3 = 2.625$$
$$x_{1,2} \in [-5, 5]$$

Goldstein and Price function (GP)

$$f(\mathbf{x}) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)]$$
$$\times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$$
$$x_{1,2} \in [-2, 2]$$

Griewank function(GN)

$$f(\mathbf{x}) = \sum_{i=1}^2 \frac{x_i^2}{200} + \prod_{i=1}^2 \cos(x_i / \sqrt{i}) + 1$$

$$x_{1,2} \in [-100, 100]$$

Rastrigin(RS) function

$$f(\mathbf{x}) = x_1^2 + x_2^2 - \cos 18x_1 - \cos 18x_2$$

$$x_{1,2} \in [-1, 1]$$

Hartman function (HN)

$$f(\mathbf{x}) = -\sum_{i=1}^4 c_i \exp[-\sum_j^{n_v} \alpha_{ij} (x_j - p_{ij})^2]$$

$$x_i \in [0, 1], \quad i = 1, \dots, n_v$$

where, $n_v = 6$, the value of $\alpha_{i,j}$, c_i and p_{ij} as follow.

i	$\alpha_{ij}, \quad j = 1, 2, \dots, 6$						c_i
1	10	3	17	3.5	1.7	8	1
2	0.05	10	17	8	8	14	12
3	3	3.5	1.7	17	17	8	3
4	17	8	0.05	10	0.1	14	3.2

i	$p_{ij}, \quad j = 1, 2, \dots, 6$					
1	0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2	0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3	0.2348	0.1451	0.3522	0.2883	0.3047	0.6650
4	0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

SUR-T1-14 (HD1)

$$f(\mathbf{x}) = (x_1 - 1)^2 + (x_{10} - 1)^2 + 10 \sum_{i=1}^9 (10 - i)(x_i^2 - x_{i+1})^2$$

$$x_i \in [-3, 2]$$

Internal combustion engine design problem (ICED) (Wanger and Papalambors, 1991)

$$\max f = K_0((\rho Q / A_f)\eta_t\eta_v - FMEP)w$$

where

$$\eta_v = \eta_{vb}[1 + 5.96 \times 10^{-3} w^2] / [1 + [(9.428 \times 10^{-5}) \times (4V / \pi N_c C_s)(w / d_I^2)]^2]$$

$$\eta_{vb} = \begin{cases} 1.067 - 0.038e^{w-5.25} & (w \geq 5.25) \\ 0.637 + 0.13w - 0.014w^2 + 0.00066w^3 & (w \leq 5.25) \end{cases}$$

$$S_v = (0.83)[(8 + 4c_r) + 1.5(c_r - 1)(\pi N_c / V)b^3] / [(2 + c_r)b]$$

$$FMEP = (4.826)(c_r - 9.2) + (7.97 + 0.253V_p + 9.7(10^{-6})V_p^2)$$

$$\eta_t = \eta_{iad} - S_v(1.5 / w)^{0.5}; \quad V_p = (8V / \pi N_c)wb^{-2}$$

$$s.t. \quad g_1 = K_1 N_c b - L_1 \leq 0; \quad g_2 = (4K_2 V / \pi N_c L_2)^{1/2} - b \leq 0$$

$$g_3 = d_I + d_E - K_3 b \leq 0; \quad g_4 = K_4 d_I - d_E \leq 0$$

$$g_5 = d_E - K_5 d_I \leq 0; \quad g_6 = (9.428)(10^{-5})(4V / \pi N_c)(w / d_I^2) - K_6 C_s \leq 0$$

$$g_7 = c_r - 13.2 + 0.045b \leq 0; \quad g_8 = w - K_7 \leq 0$$

$$g_9 = 3.6(10^6) - K_8 Q \eta_{tw} \leq 0; \quad \eta_{tw} = 0.8595(1 - c_r^{-0.33}) - S_v$$

B. Box plots of the optimization results for the benchmark problems

(a) SC function

(b) BR function

(c) PK function

(d) RS function

(e) GN function

(f) HN function

(g) HD1 function

(h) ICED

Figure B1. Box plots of optimal solutions for the benchmark problems

(a) SC function

(b) BR function

(c) Peaks function

(d) RS function

(e) GN function

(f) HN function

(g) HD1 function

(h) ICED

Figure B2. Box plots of number of function evaluations for the benchmark problems