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A Comparative Study of Three New Conjugate Gradient Methods with Exact Line Search

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Abstract. Conjugate Gradient methods play an important role in solving unconstrained optimization, especially for large scale problems. In this paper, we compared the performance profile of the classical conjugate gradient coefficients *FR*, *PRP* with three new β_k . These three new β_k possess global convergence properties using the exact line search. Preliminary numerical results show that the three new β_k are very promising and efficient when compared to CG coefficients *FR*, *PRP*.

Keywords: Conjugate gradient method, exact line search, global convergence, large scale, unconstrained optimization.

PACS: 02.60.Pn, 02.60.Cb, 02.70.C

INTRODUCTION

Consider the unconstrained optimization problem:

$$\min f(x), x \in R^n \quad (1)$$

where $f: R^n \rightarrow R$ be bounded from below and a continuously differentiable function, and R^n denotes an n -dimensional Euclidean space. We use $g(x)$ to denote the gradient of f at x . Conjugate gradient methods are efficient for solving (1), especially when the dimension n is large. The conjugate gradient method for solving (1) generates a sequence of iterates by letting,

$$x_{k+1} = x_k + \alpha_k d_k, \quad k = 0, 1, 2, \dots \quad (2)$$

where x_k is current iterate point and α_k is a step length, which is computed by carrying out some line search, and d_k is the search direction defined by

$$d_k = \begin{cases} -g_k & \text{if } k = 0, \\ -g_k + \beta_k d_{k-1} & \text{if } k \geq 1, \end{cases} \quad (3)$$

where $g_k = g(x_k)$ and β_k is a scalar. If the line search is exact, the step length α_k is obtained in the direction d_k by the rule

$$\alpha_k = \arg \min_{\alpha \geq 0} f(x_k + \alpha d_k) \quad (4)$$

Well-known conjugate gradient methods include the Hestenes-Stiefel (*HS*) [1], Fletcher – Reeves (*FR*) [2], Polak-Ribiere-Polyak (*PRP*) [3,4], conjugate descent (*CD*) [5], Liu – Storey (*LS*) [6], and Dai – Yuan (*DY*) [7]. The parameters β_k of these methods are specified as follows:

$$\beta_k^{HS} = \frac{g_k^T (g_k - g_{k-1})}{(g_k - g_{k-1})^T d_{k-1}}, \quad \beta_k^{FR} = \frac{g_k^T g_k}{g_{k-1}^T g_{k-1}}, \quad \beta_k^{PRP} = \frac{g_k^T (g_k - g_{k-1})}{g_{k-1}^T g_{k-1}}, \quad \beta_k^{CD} = -\frac{g_k^T g_k}{d_{k-1}^T g_{k-1}}, \quad \beta_k^{LS} = -\frac{g_k^T (g_k - g_{k-1})}{d_{k-1}^T g_{k-1}},$$

$$\beta_k^{DY} = \frac{g_k^T g_k}{(g_k - g_{k-1})^T d_{k-1}}$$

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