A Multivariate Analysis of Forecast Disagreement: Confronting Models of Disagreement with Survey Data

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Motivation

Disagreement about Inflation Outlook (EA) as Measured from SPF Data (2015q2)

- Disagreement about future values of macroeconomic variables is high
 - So far, only univariate disagreement has been analyzed empirically
 - Models of heterogeneous expectation formation are calibrated/evaluated based on information about univariate/marginal disagreement
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 - Which implications does this have for theoretical models of heterogeneous expectation formation?

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A Stylized Scatter of Forecasts

Contribution of Paper

- Documentation of multivariate disagreement in macroeconomic expectations
- In particular, two new stylized facts:
 - ▶ Forecasters disagree about the relationship between different macroeconomic variables
 - ▶ Different forecasters tend to have different levels of disagreement
- Discussion of implications for models of heterogeneous expectation formation:
 - ▶ Along which dimensions do models fail?
 - ▶ Modifications of models to fit observed multivariate disagreement better
 - ▶ Suggestion how multivariate aspects can be taken into account during estimation of models
- What I do not:
 - ▶ Abstracting from behavioral issues (e. g., incentives or interaction between forecasters)
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- 2 Contribution of Paper
- 3 Related Literature**
- 4 Measurement of Disagreement
- 5 Data and Stylized Facts
- 6 Implications for Models of Disagreement
- 7 Conclusion

- **Empirical analyses of forecast disagreement** (Mankiw et al., 2003; Lahiri and Sheng, 2008; Patton and Timmermann, 2010; Rich and Tracy, 2010; Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013; Andrade et al., 2014; Doovern et al., 2012)
- **Empirical studies with multivariate aspects** (Banternghansa and McCracken, 2009; Dräger and Lamla, 2015)
- **Reduced-form models of heterogenous expectations:**
 - ▶ **Noisy information models** (Coibion and Gorodnichenko, 2012; Andrade and Le Bihan, 2013; Andrade et al., 2014)
 - ▶ **Models with agent-specific priors** (Lahiri and Sheng, 2008; Patton and Timmermann, 2010)
 - ▶ **Sticky information models** (Mankiw et al., 2003; Coibion and Gorodnichenko, 2012)
- **DSGE models with heterogenous expectation formation:**
 - ▶ **Models with constant levels of disagreement** (Nimark, 2008; Maćkowiak and Wiederholt, 2009; Lorenzoni, 2009)
 - ▶ **Sticky information DSGE models** (Mankiw and Reis, 2002)
 - ▶ **“Man-bites-dog” model** (Nimark, 2014)

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How to Measure Multivariate Disagreement

- Measures proposed by Banerghansa and McCracken (2009)
- Basic inputs: vector-valued forecasts ($y_{i,t+h|t}$) and their (cross-sectional) covariance

$$S_{t+h|t} = N_{t+h|t}^{-1} \sum_{i=1}^{N_{t+h|t}} (y_{i,t+h|t} - \bar{y}_{t+h|t})(y_{i,t+h|t} - \bar{y}_{t+h|t})'$$

- Overall/Multivariate disagreement (absolute):

$$D_{t+h|t} = \sqrt{\det(S_{t+h|t})}$$

- Individual disagreement (relative):

$$d_{i,t+h|t} = \sqrt{(y_{i,t+h|t} - \bar{y}_{t+h|t})' S_{t+h|t}^{-1} (y_{i,t+h|t} - \bar{y}_{t+h|t})}$$

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- **Survey of Professional Forecasters for Euro area:**

- ▶ Forecasts from research institutes, banks, and large corporations
- ▶ Sample: 1999q1–2015q1
- ▶ 1-year-ahead forecasts ($h = 4$) and long-term forecasts ($h = 20$)
- ▶ Forecasts for growth, inflation and the unemployment rate

- **Survey of Professional Forecasters for United States:**

- ▶ Longer sample: 1981q3–2014q4
- ▶ 1-quarter-ahead forecasts ($h = 1$) and 1-year-ahead forecasts ($h = 4$)
- ▶ Robustness check with additional forecasts for short- and long-term interest rates and industrial production

- **Consensus Economics survey data for twelve advanced economies:**

- ▶ Fixed-event forecasts (annual averages)
- ▶ Concentrate on forecasts with $h = 13$
- ▶ Sample: 1990m1–2015m4

Multivariate Disagreement ($D_{t+h|t}$) over Time

(a) US-SPF

(b) EA-SPF

... a Closer Look at Individual Components

United States: Short-Term ($h = 4$)

... a Closer Look at Individual Components

Unconditional Averages

	Euro area		US	
	$h = 4$	$h = 20$	$h = 1$	$h = 4$
$sd(g)$	0.36	0.27	1.07	0.75
$sd(\pi)$	0.27	0.22	0.88	0.62
$sd(u)$	0.31	0.65	0.12	0.31
$Corr(g, \pi)$	0.16	0.10	0.02	0.03
$Corr(g, u)$	-0.25	-0.17	-0.17	-0.42
$Corr(\pi, u)$	-0.10	-0.02	-0.07	-0.14

Notes: The values shown are based on the averages for each element of $S_{t+h|t}$ over the full sample. These averages are computed as $\bar{s}_{ij,h} = \sum_{t=1}^T s_{t+h|t}^{ij}$, where $s_{t+h|t}^{ij}$ denotes the element of $S_{t+h|t}$ corresponding to the i^{th} row and the j^{th} column. $sd(\bullet)$ is computed as the square root of the main diagonal elements. $Corr(\bullet, \bullet)$ is computed as the correlations that correspond to the off-diagonal elements.

Assessment of Rank Persistence (based on $d_{i,t+h|t}$)

Visualizing Persistence ...

Assessment of Rank Persistence

Measurement

- Kendall's coefficient of concordance: Formal assessment of rank persistence
- $W=0$: No persistence
- $W=1$: Perfectly stable ranking

Assessment of Rank Persistence

EA-SPF/US-SPF

	Euro area		US	
	$h = 4$	$h = 20$	$h = 1$	$h = 4$
Since 1981q3	-	-	0.37	0.43
Since 1999q1	0.14	0.21	0.25	0.34
1999q1–2008q2	0.16	0.31	0.27	0.39
Since 2008q3	0.22	0.37	0.33	0.45
<i>For comparison (since 1999q1):</i>				
Point forecasts				
g	0.13	0.30	0.18	0.28
π	0.19	0.44	0.13	0.39
u	0.14	0.27	0.13	0.28
Forecast uncertainty				
g	0.56	0.65	-	-
π	0.32	0.62	-	-
u	0.54	0.56	-	-

Notes: The table shows values of Kendall's coefficient of concordance. The statistics have been calculated by accounting for missing values as proposed in Boero et al. (2014).

Assessment of Rank Persistence

Consensus Forecast Data

	Full Sample			Since 1999		
	$h = 1$	$h = 13$	$h = 24$	$h = 1$	$h = 13$	$h = 24$
USA	0.21	0.24	0.25	0.27	0.22	0.25
Japan	0.14	0.26	0.31	0.19	0.32	0.35
Germany	0.16	0.17	0.19	0.22	0.20	0.20
France	0.29	0.29	0.33	0.33	0.32	0.24
UK	0.30	0.36	0.42	0.33	0.31	0.50
Italy	0.25	0.24	0.27	0.23	0.19	0.26
Canada	0.26	0.22	0.24	0.40	0.24	0.25
Netherlands	0.21	0.17	0.31	0.32	0.27	0.39
Norway	0.21	0.22	0.17	0.21	0.19	0.17
Spain	0.38	0.30	0.22	0.43	0.25	0.25
Sweden	0.26	0.20	0.30	0.25	0.21	0.21
Switzerland	0.24	0.24	0.25	0.25	0.24	0.25
Average	0.24	0.24	0.27	0.29	0.25	0.28

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Implications for Models of Disagreement

Andrade et al. (2014): Model Structure

- Model structure:

$$y_t = (I_k - \Phi)\mu_t + \Phi y_{t-1} + \nu_t^y \quad (1)$$

$$\mu_t = \mu_{t-1} + \nu_t^\mu \quad (2)$$

$$y_{it} = y_t + \eta_{it} \quad (3)$$

with $\nu_t^y \sim \mathcal{N}(0, \Sigma^y)$, $\nu_t^\mu \sim \mathcal{N}(0, \Sigma^\mu)$, and $\eta_{it} \sim \mathcal{N}(0, \Sigma^\eta)$

- Agents form optimal forecasts based on Kalman filtering
- Model features not in line with new stylized facts:
 - ▶ No persistence of disagreement ranking
 - ▶ Relatively high cross-sectional correlation of forecasts

Implications for Models of Disagreement

Andrade et al. (2014): Effects of Introducing True Heterogeneity

- New feature: **Heterogenous signal-to-noise ratios:**

$$\eta_{it} \sim \mathcal{N}(0, \delta_i \Sigma^\eta) \quad \text{with} \quad \delta_i \sim \mathcal{G}(1, \sigma_\delta^2) \quad (4)$$

Implications for Models of Disagreement

Nimark (2014): Structure of DSGE Model

- Model structure:
 - ▶ Island economy as in Lorenzoni (2009)
 - ▶ Agents observe island-specific productivity ($a_{j,t} = a_t + \varepsilon_{j,t}$), island-specific demand, and a sub-set of goods prices
 - ▶ Additional “man-bites-dog” signal ($z_t^a = a_t + \eta_t$) about common productivity shock
⇒ Probability of this signal rises with (absolute) size of productivity shock
- ▶ Fig. 1 of Nimark (2014)
- Main mechanism to generate variation in disagreement:
 - ▶ Different interpretation of “man-bites-dog” signals due to different priors across agents
 - ▶ Level of disagreement depends on frequency of occurrence of these signals (and dispersion of priors)
- Model is estimated and evaluated using information about univariate disagreement only

Implications for Models of Disagreement

Nimark (2014): Implied Cross-Sectional Correlation between Different Forecasts

► Fig. 6 of Nimark (2014)

- Interesting features of multivariate disagreement that have previously been ignored in the literature
- They suggest that agents are truly heterogeneous (beyond being subject to different information flows)
- Models of heterogeneous expectation formation have neglected this dimension so far
- How can models be adapted?
 - ▶ Heterogeneous signal-to-noise ratios
 - ▶ Learning models? Models with different types of forecasters?
 - ▶ At least, multivariate information should be used for estimation ([More](#))
- What can we learn in a broader context?
 - ▶ Observed correlations contain information about (perceived) relevance of different types of macroeconomic shocks
 - ▶ Heterogeneity of agents might not only be important in terms of the current state-vector (e. g., credit constraints) but also in terms of expectations

Background on Multivariate Measure

$D_{t+h,t}$ as a Function of One Variance

Background on Multivariate Measure

$D_{t+h,t}$ as a Function of Variances

Background on Multivariate Measure

$D_{t+h,t}$ as a Function of Correlations

► Back

Occurrence of “Man-Bites-Dog” Signal

Posterior $s_t = 1$ benchmark specification

Posterior $s_t = 1$ using mean of survey responses

Michigan survey news heard indices

Intuition behind “Man-Bites-Dog” Signal

FIGURE 1. UNCONDITIONAL DISTRIBUTION OF x , CONDITIONAL PROBABILITY OF OBSERVING THE SIGNAL y , AND THE IMPLIED CONDITIONAL DISTRIBUTION OF x

Implications for Models of Disagreement

Nimark (2014): Suggestion to Take Multivariate Aspects into Account

- Modification of model to take multivariate dimension of disagreement into account
- Nimark (2014) assumes that forecasts are drawn from univariate distributions:

$$f_{t,\pi}^j \sim \mathcal{N} \left(\int E[\pi_{t+1} | \Omega_{j,t}] dj, \sigma_{f\pi}^2(s^t) \right)$$

$$f_{t,\Delta y}^j \sim \mathcal{N} \left(\int E[\Delta y_{t+1} | \Omega_{j,t}] dj, \sigma_{\Delta y}^2(s^t) \right)$$

- I suggest to use a bivariate distribution with covariance matrix:

$$\Sigma_f(s^t) = \begin{bmatrix} \sigma_{f\pi}^2(s^t) & \sigma_{f\pi, f\Delta y}(s^t) \\ \sigma_{f\pi, f\Delta y}(s^t) & \sigma_{f\Delta y}^2(s^t) \end{bmatrix}$$

- Reflect this also in the model's LLF $\Rightarrow$ more effective use of sample information:
 - ▶ Keep track of forecast pairs
 - ▶ Need not even throw away univariate forecasts

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