Supplemental Material for A Bayesian Approach for Estimating Multilevel Latent Contextual
Models (published in Structural Equation Modeling)

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This material provides additional results from the simulation studies as well as from the illustrative example. It is organized as follows.

A. Additional Results from Study 1

- Results for the Within-Group Effect of the Latent Predictor Variable (Tables A1 and A2)
- Comparing WinBUGS with the MCMC Sampler in *Mplus* (Table A3)
- Evaluating *Mplus*' Default Convergence Criterion (Table A4)

B. Additional Results from Study 2

- Convergence (Table B1)
- Results for the Between-Group Effect of the Latent Predictor Variable (Tables B2 and B3)

C. Additional Results from the Third International Mathematics and Science Study

- Comparing WinBUGS with the MCMC Sampler in *Mplus* (Table C1)

D. Program Codes

E. Example Trace Plots in Study 1

A. Additional Results from Study 1

Table A1 compares the maximum likelihood (ML) with the Bayesian approach with respect to their performance in recovering the within-group effect. In order to render results comparable between estimation approaches, we used the same subset of the simulated data sets to compute the results for the Bayesian approach and ML. Table A2 compares the different analysis models under the Bayesian approach. Because the comparison was based only on the Bayesian results, and the Bayesian approach did not lead to any estimation problems, we were able to use all of the simulated data sets. Table A3 compares WinBUGS with the MCMC sampler in *Mplus*. We specified *Mplus* in the same way as WinBUGS (i.e., single MCMC chain with a fixed length and the first tenth of iterations as burn-in iterations). As can be seen, *Mplus* provided almost the same results as WinBUGS did. Table A4 evaluates the *Mplus* default convergence criterion with respect to the statistical properties that resulted from the use of that criterion. As a benchmark, we ran the MCMC chain 100 times longer. As can be seen, this longer chain outperformed the *Mplus* default in terms of the RMSE in conditions with small sample sizes.

Table A1
Additional Results from Study 1: Relative Bias, Relative Root Mean Squared Error (RMSE), and Coverage Rate from Estimating the Within-Group Effect of the Latent Predictor Variable with the Maximum Likelihood Estimation Approach (ML) and the Bayesian Estimation Approach (Bayes)

Number of Groups	Group Size	Intraclass Correlation	Admissible Converged	Relative Bias		Relative RMSE		Coverage Rate		
				ML	Bayes	ML	Bayes	ML	Bayes	
$J = 25$	$n = 10$	$ICC_{\eta} = .1$	Data-Generating Model: PI							
		80.9	0.02	0.03	0.38	0.38	92.8	94.8		
		$ICC_{\eta} = .3$	92.5	0.02	0.03	0.43	0.44	93.5	94.8	
$J = 100$	$n = 30$	$ICC_{\eta} = .1$	93.7	0.01	0.01	0.21	0.21	93.9	95.3	
		$ICC_{\eta} = .3$	95.9	0.00	0.00	0.23	0.23	92.1	94.9	
	$n = 10$	$ICC_{\eta} = .1$	99.0	0.00	0.00	0.18	0.18	94.2	94.9	
		$ICC_{\eta} = .3$	99.9	0.01	0.01	0.21	0.21	95.8	96.0	
$J = 25$	$n = 30$	$ICC_{\eta} = .1$	100.0	0.01	0.01	0.10	0.10	95.7	96.3	
		$ICC_{\eta} = .3$	100.0	0.00	0.00	0.11	0.11	96.4	96.6	
	Data-Generating Model: CI									
	$n = 10$	$ICC_{\eta} = .1$	59.7	0.02	0.03	0.40	0.41	92.6	95.3	
		$ICC_{\eta} = .3$	88.8	0.03	0.07	0.44	0.46	94.6	95.6	
$J = 100$	$n = 30$	$ICC_{\eta} = .1$	87.7	0.01	0.01	0.22	0.23	93.5	94.1	
		$ICC_{\eta} = .3$	93.2	0.00	0.01	0.25	0.26	91.3	93.7	
	$n = 10$	$ICC_{\eta} = .1$	98.3	0.01	0.01	0.20	0.20	93.5	93.5	
		$ICC_{\eta} = .3$	99.9	0.00	0.01	0.22	0.23	95.2	95.7	
$n = 30$	$ICC_{\eta} = .1$	99.3	0.00	0.00	0.11	0.11	94.7	95.4		
	$ICC_{\eta} = .3$	99.9	0.00	0.01	0.12	0.12	94.0	94.4		

Note. Results are based on only those data sets for which ML provided an admissible converged solution. PI = multilevel latent contextual model with parallel indicators; CI = multilevel latent contextual model with congeneric indicators; J = number of groups; n = group size; ICC_{η} = intraclass correlation of the latent predictor.

Table A2
Additional Results from Study 1: Relative Bias, Relative Root Mean Squared Error (RMSE), and Coverage Rate from Estimating the Within-Group Effect of the Latent Predictor Variable with the Bayesian Estimation Approach

Number of Groups	Group Size	Intraclass Correlation	Relative Bias			Relative RMSE			Coverage Rate		
			DM	PI	CI	DM	PI	CI	DM	PI	CI
$J = 25$	$n = 10$	$ICC_{\eta} = .1$	-0.37	0.03	0.03	0.44	0.38	0.42	63.7	94.7	94.6
		$ICC_{\eta} = .3$	-0.40	0.03	0.06	0.48	0.44	0.50	69.7	94.8	94.5
$J = 100$	$n = 30$	$ICC_{\eta} = .1$	-0.36	0.01	0.01	0.38	0.21	0.22	17.1	95.5	95.0
		$ICC_{\eta} = .3$	-0.37	0.00	0.02	0.39	0.23	0.25	28.3	94.7	94.1
	$n = 10$	$ICC_{\eta} = .1$	-0.39	0.00	0.00	0.40	0.18	0.19	9.4	94.9	94.9
		$ICC_{\eta} = .3$	-0.37	0.01	0.02	0.40	0.21	0.22	20.2	96.0	95.6
$n = 30$	$ICC_{\eta} = .1$	-0.37	0.01	0.01	0.37	0.10	0.11	0.0	96.3	95.7	
	$ICC_{\eta} = .3$	-0.37	0.00	0.01	0.38	0.11	0.12	0.0	96.6	96.6	
$J = 25$	$n = 10$	$ICC_{\eta} = .1$	-0.43	0.15	0.01	0.49	0.49	0.41	50.1	93.0	95.2
		$ICC_{\eta} = .3$	-0.45	0.19	0.06	0.51	0.53	0.46	62.2	94.5	95.4
	$n = 30$	$ICC_{\eta} = .1$	-0.41	0.15	0.01	0.43	0.29	0.22	8.8	89.7	94.5
		$ICC_{\eta} = .3$	-0.42	0.13	0.00	0.45	0.32	0.25	15.3	92.2	94.0
$J = 100$	$n = 10$	$ICC_{\eta} = .1$	-0.43	0.15	0.01	0.45	0.27	0.21	4.0	88.9	93.4
		$ICC_{\eta} = .3$	-0.43	0.14	0.01	0.45	0.29	0.23	7.8	91.1	95.7
	$n = 30$	$ICC_{\eta} = .1$	-0.42	0.14	0.00	0.43	0.18	0.11	0.0	81.1	95.3
		$ICC_{\eta} = .3$	-0.42	0.14	0.01	0.43	0.20	0.12	0.0	84.5	94.4

Note. DM = doubly manifest model; PI = multilevel latent contextual model with parallel indicators; CI = multilevel latent contextual model with congeneric indicators; J = number of groups; n = group size; ICC_{η} = intraclass correlation of the latent predictor.

Table A3
Additional Results from Study 1: Relative Bias, Relative Root Mean Squared Error (RMSE), and Coverage Rate from Estimating the Between-Group Effect of the Latent Predictor Variable with the Bayesian Estimation Approach

Number of Groups	Group Size	Intraclass Correlation	Relative Bias				Relative RMSE				Coverage Rate			
			PI		CI		PI		CI		PI		CI	
			WB	Mplus	WB	Mplus	WB	Mplus	WB	Mplus	WB	Mplus	WB	Mplus
$J = 25$	$n = 10$	Data-Generating Model: PI ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	-0.18	-0.17	-0.12	-0.11	1.22	1.23	1.29	1.30	96.4	96.5	96.6	96.7
			-0.05	-0.02	-0.03	-0.02	0.64	0.64	0.68	0.68	97.4	97.2	97.3	97.3
	$n = 30$	ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	-0.07	-0.07	-0.08	-0.07	1.08	1.06	1.08	1.08	96.2	96.1	96.1	96.2
			0.04	0.02	-0.01	-0.02	0.60	0.58	0.59	0.58	97.1	97.3	96.8	96.7
$J = 100$	$n = 10$	ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	-0.02	-0.03	-0.04	-0.04	0.72	0.70	0.69	0.70	93.6	94.2	94.4	94.4
			0.03	0.03	0.01	0.02	0.38	0.37	0.37	0.38	93.7	93.7	93.7	93.8
	$n = 30$	ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	0.01	-0.01	-0.02	-0.02	0.56	0.56	0.55	0.56	95.3	95.4	95.6	95.5
			0.07	0.03	0.03	0.03	0.32	0.30	0.31	0.30	93.4	94.8	96.1	95.4
$J = 25$	$n = 10$	Data-Generating Model: CI ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	-0.25	-0.25	-0.15	-0.14	1.23	1.23	1.28	1.29	97.8	97.7	96.5	96.8
			0.00	0.03	0.03	0.04	0.67	0.67	0.71	0.72	99.2	99.1	97.2	97.1
	$n = 30$	ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	-0.04	-0.04	-0.07	-0.04	1.16	1.15	1.15	1.15	96.9	96.8	96.0	96.0
			0.12	0.10	0.04	0.02	0.61	0.59	0.58	0.56	98.6	98.7	97.0	96.9
$J = 100$	$n = 10$	ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	0.04	0.02	-0.01	-0.01	0.75	0.72	0.70	0.70	94.8	95.5	95.8	95.9
			0.18	0.17	0.05	0.06	0.46	0.45	0.38	0.38	93.1	93.3	93.6	93.9
	$n = 30$	ICC $_{\eta} = .1$ ICC $_{\eta} = .3$	0.05	0.04	-0.04	-0.03	0.64	0.63	0.57	0.57	94.1	94.7	94.8	94.4
			0.22	0.16	0.03	0.03	0.45	0.39	0.31	0.31	88.1	93.2	95.6	95.4

Note. PI = multilevel latent contextual model with parallel indicators; CI = multilevel latent contextual model with congenic indicators; J = number of groups; n = group size; ICC $_{\eta}$ = intraclass correlation of the latent predictor; WB = WinBUGS.

Table A4
Additional Results from Study 1: Relative Bias, Relative Root Mean Squared Error (RMSE), and Coverage Rate from Estimating the Between-Group Effect of the Latent Predictor Variable with the Bayesian Estimation Approach

Number of Groups	Group Size	Intraclass Correlation	Relative Bias				Relative RMSE				Coverage Rate				
			PI		CI		PI		CI		PI		CI		
			PSR	PSR100	PSR	PSR100	PSR	PSR100	PSR	PSR100	PSR	PSR100	PSR	PSR100	
$J = 25$	$n = 10$	ICC $_{\eta} = .1$	-0.14	-0.17	-0.13	-0.11	1.36	1.22	1.38	1.30	95.8	96.5	95.6	96.8	
		ICC $_{\eta} = .3$	0.05	-0.02	-0.03	-0.02	0.79	0.64	0.71	0.68	95.6	97.2	97.0	97.1	
	$n = 30$	ICC $_{\eta} = .1$	-0.17	-0.09	0.04	-0.08	1.19	1.06	1.07	1.08	95.3	95.9	95.8	96.1	
		ICC $_{\eta} = .3$	-0.01	0.01	0.01	-0.03	0.59	0.58	0.56	0.58	97.3	97.2	97.3	96.7	
$J = 100$	$n = 10$	ICC $_{\eta} = .1$	-0.09	-0.03	-0.06	-0.02	0.76	0.70	0.66	0.70	94.5	94.2	93.3	94.1	
		ICC $_{\eta} = .3$	0.00	0.03	0.00	0.02	0.38	0.37	0.37	0.38	93.9	93.9	92.1	93.8	
	$n = 30$	ICC $_{\eta} = .1$	-0.06	-0.01	-0.01	-0.02	0.65	0.56	0.59	0.56	91.5	95.3	92.3	95.4	
		ICC $_{\eta} = .3$	0.02	0.04	0.04	0.02	0.36	0.30	0.32	0.30	90.6	94.7	93.1	95.4	
$J = 25$	$n = 10$	Data-Generating Model: CI													
		ICC $_{\eta} = .1$	-0.26	-0.25	-0.19	-0.12	1.37	1.23	1.37	1.30	96.8	97.8	96.7	96.5	
		ICC $_{\eta} = .3$	0.10	0.04	0.04	0.04	0.80	0.67	0.75	0.72	97.6	99.1	96.1	97.2	
		ICC $_{\eta} = .1$	-0.17	-0.06	0.10	-0.06	1.25	1.14	1.15	1.15	96.6	96.7	96.3	96.1	
	$n = 30$	ICC $_{\eta} = .3$	0.06	0.09	0.05	0.01	0.58	0.59	0.57	0.56	98.6	98.7	97.0	96.9	
		ICC $_{\eta} = .1$	-0.10	0.02	-0.03	0.01	0.69	0.72	0.67	0.71	96.3	95.5	94.5	95.5	
		ICC $_{\eta} = .3$	0.11	0.18	0.03	0.06	0.46	0.46	0.38	0.38	94.4	93.0	93.6	93.6	
		$n = 30$	ICC $_{\eta} = .1$	-0.02	0.04	-0.02	-0.04	0.74	0.64	0.61	0.57	90.0	94.5	91.9	94.8
ICC $_{\eta} = .3$	0.17		0.17	0.03	0.02	0.46	0.40	0.33	0.31	85.8	92.3	91.8	95.6		

Note. A single MCMC chain was run until it reached a potential scale reduction factor of 1.1 (PSR; i.e., *Mplus*' default convergence criterion) or 100 times longer (PSR100). PI = multilevel latent contextual model with parallel indicators; CI = multilevel latent contextual model with congeneric indicators; J = number of groups; n = group size; ICC $_{\eta}$ = intraclass correlation of the latent predictor.

B. Additional Results from Study 2

Table B1 summarizes the estimation problems. Table B2 compares the ML approach with the Bayesian approach with respect to their performance in recovering the between-group effect of the latent predictor. In both tables, results are shown for the multilevel latent contextual model with parallel indicators. Table B3 compares the different analysis models under the Bayesian approach.

Table B1
Additional Results from Study 2: Rates of Admissible Converged (AC), Nonconverged Solutions (NC), and Inadmissible Converged Solutions (IAC) Solutions

Number of Groups	Group Size	Intraclass Correlation		AC	NC	IAC
Data-Generating Model: Uncorrelated Predictor Variables						
$J = 50$	$n = 5$	$ICC_{\eta} = .05$	70.5	20.8	8.7	
		$ICC_{\eta} = .20$	84.4	6.9	8.7	
	$n = 20$	$ICC_{\eta} = .05$	92.2	5.6	2.2	
		$ICC_{\eta} = .20$	99.0	0.2	0.8	
$J = 200$	$n = 5$	$ICC_{\eta} = .05$	87.8	7.7	4.5	
		$ICC_{\eta} = .20$	99.2	0.3	0.5	
	$n = 20$	$ICC_{\eta} = .05$	99.9	0.1	0.0	
		$ICC_{\eta} = .20$	100.0	0.0	0.0	
Data-Generating Model: Correlated Predictor Variables						
$J = 50$	$n = 5$	$ICC_{\eta} = .05$	69.6	21.7	8.7	
		$ICC_{\eta} = .20$	81.3	9.6	9.1	
	$n = 20$	$ICC_{\eta} = .05$	90.4	7.8	1.8	
		$ICC_{\eta} = .20$	98.4	0.3	1.3	
$J = 200$	$n = 5$	$ICC_{\eta} = .05$	87.2	9.1	3.7	
		$ICC_{\eta} = .20$	99.0	0.3	0.7	
	$n = 20$	$ICC_{\eta} = .05$	99.7	0.1	0.2	
		$ICC_{\eta} = .20$	100.0	0.0	0.0	

Note. J = number of groups; n = group size; ICC_η = intraclass correlation of the latent predictor.

Table B2
Additional Results from Study 2: Relative Bias, Relative Root Mean Squared Error (RMSE), and Coverage Rate from Estimating the Between-Group Effect of the Latent Predictor Variable with the Maximum Likelihood Estimation Approach (ML) and the Bayesian Estimation Approach (Bayes)

Number of Groups	Group Size	Intraclass Correlation	Admissible Converged	Relative Bias		Relative RMSE		Coverage Rate	
				ML	Bayes	ML	Bayes	ML	Bayes
$J = 50$	$n = 5$	Data-Generating Model: Uncorrelated Predictor Variables							
		ICC $_{\eta} = .05$	70.5	-0.39	-0.47	3.40	1.16	96.5	99.2
		ICC $_{\eta} = .20$	84.4	0.07	-0.10	1.18	0.70	95.0	98.1
	$n = 20$	ICC $_{\eta} = .05$	92.2	0.17	-0.20	2.51	1.06	97.6	95.3
		ICC $_{\eta} = .20$	99.0	0.18	0.12	0.73	0.58	95.5	95.6
$J = 200$	$n = 5$	ICC $_{\eta} = .05$	87.8	0.19	-0.26	2.73	0.83	97.3	96.7
		ICC $_{\eta} = .20$	99.2	0.05	0.04	0.49	0.44	95.0	94.5
	$n = 20$	ICC $_{\eta} = .05$	99.9	0.11	-0.02	0.92	0.63	97.1	95.3
		ICC $_{\eta} = .20$	100.0	0.03	0.03	0.25	0.25	95.7	94.1
$J = 50$	$n = 5$	Data-Generating Model: Correlated Predictor Variables							
		ICC $_{\eta} = .05$	69.6	-0.39	-0.52	3.48	1.16	96.3	99.0
		ICC $_{\eta} = .20$	81.3	0.04	-0.16	1.10	0.66	95.2	98.4
	$n = 20$	ICC $_{\eta} = .05$	90.4	0.17	-0.22	2.88	1.08	97.7	96.0
		ICC $_{\eta} = .20$	98.4	0.19	0.10	0.76	0.56	95.3	96.0
$J = 200$	$n = 5$	ICC $_{\eta} = .05$	87.2	0.13	-0.29	3.10	0.88	97.5	96.4
		ICC $_{\eta} = .20$	99.0	0.06	0.03	0.53	0.44	94.7	95.6
	$n = 20$	ICC $_{\eta} = .05$	99.7	0.14	-0.03	1.06	0.66	97.0	95.4
		ICC $_{\eta} = .20$	100.0	0.03	0.03	0.26	0.26	95.6	94.7

Note. Results are based on only those data sets for which ML provided an admissible converged solution. J = number of groups; n = group size; ICC $_{\eta}$ = intraclass correlation of the latent predictor.

Table B3
Additional Results from Study 2: Relative Bias, Relative Root Mean Squared Error (RMSE), and Coverage Rate from Estimating the Between-Group Effect of the Latent Predictor Variable with the Bayesian Estimation Approach

Number of Groups	Group Size	Intraclass Correlation	Relative Bias		Relative RMSE		Coverage Rate			
			DM	PI	DM	PI	DM	PI		
$J = 50$	$n = 5$	Data-Generating Model: Uncorrelated Predictor Variables								
		ICC $_{\eta} = .05$	-0.67	-0.41	0.73	1.39	38.5	99.1		
		ICC $_{\eta} = .20$	-0.53	0.06	0.58	0.80	42.8	97.3		
	$n = 20$	ICC $_{\eta} = .05$	-0.59	-0.15	0.72	1.14	72.0	94.9		
		ICC $_{\eta} = .20$	-0.44	0.14	0.50	0.59	62.2	95.0		
$J = 200$	$n = 5$	ICC $_{\eta} = .05$	-0.66	-0.14	0.68	0.94	0.2	96.3		
		ICC $_{\eta} = .20$	-0.54	0.05	0.55	0.46	0.9	93.8		
	$n = 20$	ICC $_{\eta} = .05$	-0.56	-0.02	0.60	0.63	21.2	95.3		
		ICC $_{\eta} = .20$	-0.43	0.03	0.44	0.25	7.4	94.1		
$J = 50$	$n = 5$	Data-Generating Model: Correlated Predictor Variables								
		ICC $_{\eta} = .05$	-0.68	-0.46	0.74	1.38	36.9	98.9		
		ICC $_{\eta} = .20$	-0.55	-0.01	0.60	0.77	39.5	98.0		
	$n = 20$	ICC $_{\eta} = .05$	-0.60	-0.17	0.72	1.18	69.5	95.8		
		ICC $_{\eta} = .20$	-0.45	0.12	0.51	0.58	55.6	95.7		
$J = 200$	$n = 5$	ICC $_{\eta} = .05$	-0.67	-0.16	0.68	0.98	0.2	96.2		
		ICC $_{\eta} = .20$	-0.56	0.04	0.57	0.47	0.5	94.8		
	$n = 20$	ICC $_{\eta} = .05$	-0.58	0.07	0.61	3.17	18.8	95.4		
		ICC $_{\eta} = .20$	-0.45	0.03	0.46	0.26	4.3	94.7		

Note. DM = doubly manifest model; PI = multilevel latent contextual model with parallel indicators; J = number of groups; n = group size; ICC_{η} = intraclass correlation of the latent predictor.

C. Additional Results from the Third International Mathematics and Science Study

Table C1 compares WinBUGS with the MCMC sampler in *Mplus*. We specified *Mplus* in the same way as WinBUGS (for the program codes, see below); that is, we ran 50,000 burn-in iterations, saved every fifth of the subsequent iterations, and stopped when the chain reached a length of 90,000 (saved) iterations. As can be seen, the *Mplus* results did not differ much from the WinBUGS results.

Table C1
Additional Results from the Third International Mathematics and Science Study: Bayesian Estimates of the Effects of Perceived Teacher Distractibility on Mathematics Achievement

Parameter	DM Model			CI Model			CN Model		
	Mplus		WB	Mplus		WB	Mplus		WB
	Est.	SE		Est.	SE		Est.	SE	
Student Level									
Loadings									
Indicator 1	1.000	—	1.000	—	1.000	—	1.000	—	—
Indicator 2	1.153	0.059	1.143	0.059	1.229	0.069	1.235	0.069	0.069
Indicator 3	0.931	0.047	0.930	0.047	0.947	0.049	0.947	0.049	0.049
Error Variances									
Indicator 1	0.449	0.020	0.449	0.020	0.460	0.020	0.461	0.020	0.020
Indicator 2	0.265	0.021	0.268	0.021	0.248	0.023	0.247	0.023	0.023
Indicator 3	0.453	0.019	0.453	0.019	0.458	0.019	0.458	0.019	0.019
Within-Group Coefficient	−0.037	0.023	−0.038	−0.042	0.033	−0.046	0.033	−0.042	−0.038
Residual Variance	0.474	0.015	0.474	0.015	0.475	0.015	0.476	0.015	0.015
Class Level									
Loadings									
Indicator 1	1.000	—	1.000	—	1.000	—	1.000	—	—
Indicator 2	1.153	0.059	1.143	0.059	0.648	0.132	0.660	0.131	0.131
Indicator 3	0.931	0.047	0.930	0.047	0.802	0.161	0.819	0.161	0.161
Error Variances									
Indicator 1	0.032	0.009	0.032	0.009	0.027	0.009	0.027	0.009	0.009
Indicator 2	0.015	0.005	0.016	0.005	0.016	0.005	0.015	0.005	0.005
Indicator 3	0.016	0.005	0.016	0.005	0.015	0.005	0.016	0.005	0.005
Between-Group Coefficient	−1.191	0.268	−1.180	−2.190	0.663	−2.268	0.677	−1.776	−1.773
Residual Variance	0.456	0.070	0.457	0.397	0.083	0.390	0.084	0.382	0.083

Note. DM = doubly manifest model; CI = multilevel latent contextual model with congeneric indicators; CN1 = multilevel latent contextual model with congeneric noninvariant indicators; WB = WinBUGS; Est. = estimate; SE = standard error

D. Example Program Codes

```

model{
# -----
# Example WinBUGS Code
# -----

# N... total number of students
# group... indicator of class membership
# J... number of classes
# p... number of indicators
# eta1... perceived teacher distractibility
# eta2... mathematics achievement
# Y... indicators of eta1

# Likelihood
for (i in 1:N){
  eta2[i] ~ dnorm(expr_1[i], invVarZeta.w)
  expr_1[i] <- alpha + beta.w * eta1.w[i]
               + beta.b * eta1.b[group[i]] + zeta.b[group[i]]
}
for (j in 1:J){
  zeta.b[j] ~ dnorm(0, invVarZeta.b)
}
for (i in 1:N){
  for (k in 1:p){
    Y[i, k] ~ dnorm(expr_2[i, k], invVarEps.w[k])
    expr_2[i, k] <- ny[k] + lambda.w[k] * eta1.w[i]
                  + lambda.b[k] * eta1.b[group[i]]
                  + eps.b[group[i], k]
  }
}
for (j in 1:J){
  for (k in 1:p){
    eps.b[j, k] ~ dnorm(0, invVarEps.b[k])
  }
}
for (i in 1:N){
  eta1.w[i] ~ dnorm(0, invVarEta1.w)
}
for (j in 1:J){
  eta1.b[j] ~ dnorm(0, invVarEta1.b)
}

# Prior distributions
# intercept
alpha ~ dnorm(0, 0.001)

```

```

# within-class coefficient
beta.w ~ dnorm(0, 0.001)
# between-class coefficient
beta.b ~ dnorm(0, 0.001)
invVarZeta.w ~ dgamma(0.001, 0.001)
# within-class residual variance
varZeta.w <- pow(invVarZeta.w, -1)
invVarZeta.b ~ dgamma(0.1, 0.1)
# between-class residual variance
varZeta.b <- pow(invVarZeta.b, -1)
for (k in 1:p){
  # intercepts
  ny[k] ~ dnorm(0, 0.001)
}
lambda.w[1] <- 1
lambda.b[1] <- 1
for (k in 2:p){
  # within-class loadings
  lambda.w[k] ~ dnorm(0, 0.001)
  # between-class loadings
  lambda.b[k] <- lambda.w[k]
}
for (k in 1:p){
  invVarEps.w[k] ~ dgamma(0.001, 0.001)
  # within-class error variances
  varEps.w[k] <- pow(invVarEps.w[k], -1)
  invVarEps.b[k] ~ dgamma(0.1, 0.1)
  # between-class error variances
  varEps.b[k] <- pow(invVarEps.b[k], -1)
}
invVarEta1.w ~ dgamma(0.001, 0.001)
# within-class variance of eta1
varEta1.w <- pow(invVarEta1.w, -1)
invVarEta1.b ~ dgamma(0.1, 0.1)
# between-class variance of eta1
varEta1.b <- pow(invVarEta1.b, -1)
}

```

MODEL:

```
! -----
! Example Mplus Code
! -----
```

```
! eta1... perceived teacher distractibility
! eta2... mathematics achievement
! Y... indicators of eta1
```

%WITHIN%

```
eta1.w by Y1 Y2 (lambda.w2) Y3 (lambda.w3);
Y1 (varEps.w1);
Y2 (varEps.w2);
Y3 (varEps.w3);
eta1.w (varEta1.w);
eta2 on eta1.w (beta.w);
eta2 (varZeta.w);
```

%BETWEEN%

```
eta1.b by Y1 Y2 (lambda.w2) Y3 (lambda.w3);
Y1 (varEps.b1);
Y2 (varEps.b2);
Y3 (varEps.b3);
[Y1] (ny1);
[Y2] (ny2);
[Y3] (ny3);
eta1.b (varEta1.b);
eta2 on eta1.b (beta.b);
[eta2] (alpha);
eta2 (varZeta.b);
```

MODEL PRIORS:

```
! within-class loading
lambda.w2 ~ N(0, 1000);
! within-class loading
lambda.w3 ~ N(0, 1000);
! within-class error variances
varEps.w1 ~ IG(0.001, 0.001);
varEps.w2 ~ IG(0.001, 0.001);
varEps.w3 ~ IG(0.001, 0.001);
! within-class variance of eta1
varEta1.w ~ IG(0.001, 0.001);
! within-class coefficient
beta.w ~ N(0, 1000);
! within-class residual variance
varZeta.w ~ IG(0.001, 0.001);
! between-class error variances
```



```
varEps.b1 ~ IG(0.1, 0.1);
varEps.b2 ~ IG(0.1, 0.1);
varEps.b3 ~ IG(0.1, 0.1);
! intercepts
ny1 ~ N(0, 1000);
ny2 ~ N(0, 1000);
ny3 ~ N(0, 1000);
! between-class variance of eta1
varEta1.b ~ IG(0.1, 0.1);
! between-class coefficient
beta.b ~ N(0, 1000);
! intercept
alpha ~ N(0, 1000);
! between-class residual variance
varZeta.b ~ IG(0.1, 0.1);
```

E. Example Trace Plots in Study 1

Figure E1 provides example trace plots of the between-group effect in the model with congeneric indicators, which illustrate the iterative samples generated by WinBUGS under the most challenging condition ($J = 25$, $n = 10$, $ICC_{\eta} = .1$).

Figure E1. Example trace plots of the between-group effect. PSR = potential scale reduction factor; $\frac{\text{MCMCSE}}{\text{PSD}}$ = Markov chain Monte Carlo standard error (MCMCSE) divided by the posterior standard deviation (PSD); MAP = mode of the posterior distribution; BCI = Bayesian confidence interval.