The Effect of Initial Conditions on Streamwise Vortices in the Plane Turbulent Mixing Layer

W. A. McMullan* and S. J. Garrett†

Department of Engineering, University of Leicester, LE1 7RH, United Kingdom

In this research we investigate the effect of inflow conditions on the streamwise vortices that exist in the plane turbulent mixing layer. Both initially-laminar and initially-turbulent mixing layers are considered here. The effect of the imposed fluctuations is assessed in the initially-laminar mixing layer simulations through the use of a white noise disturbance environment, and a physically-correlated fluctuation environment produced by an inflow generation method. The initially-turbulent mixing layer has its inflow condition produced by the inflow generation method. Each simulation produces good statistical agreement with the experimental data. The initially-laminar simulation with correlated fluctuations predicts the presence of statistically stationary streamwise vortices, whilst the other two simulations do not.

I. Introduction

The plane mixing layer is viewed as a canonical flow type and has been the subject of extensive research over the past 65 years. Classical turbulence theory suggests that the turbulent mixing layer should consist of an essentially random flow-field, yet flow visualisation of the flow revealed the presence of organised coherent structures in the flow. These structures have been observed in both gaseous and aqueous flows,^{1–4} over an extremely wide range of Reynolds numbers.^{5,6} These structures occupy the entire visual thickness of the mixing layer, and therefore must be of dynamical significance in the flow. At moderate Reynolds numbers, neighbouring vortex structures rotate laterally around each other and pair, resulting in a step-wise growth of the mixing layer.⁵ At higher Reynolds numbers, however, the structures in the flow appear to grow in a continuous manner.⁷ Recent experimental work by D'Ovidio & Coats⁸ has shown that in the pre-transition region of the flow, the growth-by-pairing mechanism is active, whilst beyond in the turbulent region of the flow, the coherent structures grow in a continuous linear manner, with the interactions between structures contributing nothing to the overall growth of the mixing layer. The distinction between the laminar and turbulent regions of the mixing layer is marked by the mixing transition,⁹ where small scales appear in the flow, and the mixing layer becomes three-dimensional. For initially-turbulent mixing layers, structures have been observed to emerge in the flow at significant downstream distances.¹⁰

Subsequent to the discovery of spanwise-orientated coherent structures a secondary, streamwise vortex structure was also discovered.⁹ These streamwise vortices manifested themselves as streaks in plan-view schlieren and shadowgraph images. In quantitative flow measurements, they are apparent through undulations in the mean velocity measurements across the span of the mixing layer. The origin and evolution of this streamwise vorticity remains an open research question. Experimental evidence suggests that the streaks may form owing to residual vorticity present in the flow upstream of the mixing layer,¹¹ and changes in upstream conditions have been noted to produce changes in the spanwise locations of the streaks.¹² Studies have shown that the spanwise wavelength of the streamwise vortices increases with interactions between primary spanwise rollers,^{9,13,14} or it can persist at a reasonably constant wavelength over long distances.^{2,15} The influence that this streamwise vorticity has on the flow evolution has also yet to be fully understood, and may be facility dependent. Experimental studies into the initially-turbulent mixing layer have not found any evidence for a stationary streamwise structure.¹⁶

*Lecturer, AIAA Senior Member.

†Professor, AIAA Member.

Modern supercomputing power is now capable of performing fully time-dependent simulation of the mixing layer flow. The controlled environment that numerical techniques offer permits careful simulation to provide further information on the topography of the vortex structures in the flow. The most common form of mixing layer simulation performed to date is the *temporal* mixing layer, in which the freestreams are set in opposing directions and the flow develops with time in the computational domain. This type of mixing layer simulation has met with considerable success, with Direct Numerical Simulation (DNS) of the flow capturing the roll-up,¹⁷ pairing of primary vortices,¹⁸ transition process,¹⁹ and streamwise vorticity in the flow. However, temporal simulations are limited as they predict the symmetric entrainment of fluid from each freestream, in contradiction to experimental data.⁴ Published simulations of the laboratory-frame, spatially-developing flow are less prevalent, with the majority being confined to either two-dimensional domains,^{20,21} or to the pre-transition region of the flow.²² High-resolution DNS studies have recently been published and have shown evidence of streamwise vorticity in the mixing layer.^{23,24} The capability of Large Eddy Simulation (LES) to capture the large-scale structures of the flow has been demonstrated,²⁵ yet very few LES studies of the fully three-dimensional flow have compared simulation data directly to experiment.^{26,27,28} Recent work by the present authors has demonstrated that the use of a ‘white-noise’ type fluctuation superposed onto the inflow condition does not result in a simulation which predicts stationary streamwise vortices,²⁹ a feature that has also been observed in previous simulations of the pre-transition mixing layer.²² Further, an initially-turbulent mixing layer simulation also did not produce statistically stationary streamwise structures.

In the present research, a series of Large Eddy Simulations are performed against experimental data of Browand & Latigo.³⁰ The aim of this research is to quantify the effects of the initial conditions on the streamwise vortex structure in the plane turbulent mixing layer. The recycling and rescaling method of Xiao *et al.*³¹ is used to generate the physically correlated inflow conditions for both initially-laminar and initially-turbulent simulations. A further initially-laminar simulation with white noise fluctuations providing the background disturbances is also performed. Mean flow statistics provide evidence for any changes in the bulk flow evolution owing to the changes in initial conditions, and cross-plane statistics will provide quantitative information on the streamwise vortex structure in the flow. Flow visualisation images will also provide qualitative information on the effect of the initial conditions on the flow structure.

II. Numerical Methods

The spatially filtered equations for conservation of momentum and mass for a uniform density fluid are

$$\frac{\partial \bar{u}_i}{\partial t} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} (-\bar{u}_i \bar{u}_j + 2\nu \bar{S}_{ij}) \quad (1)$$

$$\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) \quad (2)$$

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0 \quad (3)$$

These equations are discretised on a staggered mesh. The viscosity ν can consist of both a molecular and a subgrid component, $\nu = \nu_m + \nu_{sg}$, if a subgrid-scale model is used. In this study the Wale Adapting Local Eddy-viscosity (WALE) model³⁵ is utilised, with the eddy viscosity, ν_{sg} , calculated by

$$\nu_{sg} = (C_w \Delta)^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}} \quad (4)$$

where $S_{ij}^d = \frac{1}{2}(\bar{g}_{ij}^2 + \bar{g}_{ji}^2) - \frac{1}{3}\delta_{ij}\bar{g}_{kk}^2$, $\bar{g}_{ij} = \partial \bar{u}/\partial x$, and C_w is a model constant specified *a priori*. The WALE model is attractive for the simulation of free shear flows with initially laminar conditions, as it predicts zero eddy viscosity in the presence of pure shear. It has been shown in other work by the author that this model produces improved plane mixing layer predictions when compared to the standard Smagorinsky model.³⁶

Temporal advancement of the governing equations is performed by the Adams-Bashforth method. In this method, a provisional velocity field is obtained through

$$\bar{u}_i^* = \bar{u}_i^n + \Delta t \left(\frac{3}{2} H_i^n - \frac{1}{2} H_i^{n-1} \right) \quad (5)$$

with

$$H_i = \frac{\partial}{\partial x_j} (-\bar{u}_i \bar{u}_j + 2\nu \bar{S}_{ij}) \quad (6)$$

The provisional velocity $\bar{u}_i^*$ does not obey continuity, and is updated to the actual velocity at the next time step, $\bar{u}_i^{n+1}$, by the pressure solver. The pressure field is solved implicitly by the use of the continuity equation. The provisional velocity field of $\bar{u}_i^*$ is used to derive the actual velocity by including the gradient of an unknown pressure field $\bar{p}^{n+\frac{1}{2}}$, such that

$$\bar{u}_i^{n+1} = \bar{u}_i^* - \Delta t \frac{\partial \bar{p}^{n+\frac{1}{2}}}{\partial x_i} \quad (7)$$

As the new velocity field must have zero divergence, a Poisson equation can be found for the pressure field between the present and next time step

$$\nabla^2 \bar{p}^{n+\frac{1}{2}} = \frac{1}{\Delta t} \frac{\partial \bar{u}_i^*}{\partial x_i} \equiv R \quad (8)$$

This equation is then solved using a multi-grid method to increase the speed of convergence of the solution. A standard convective outflow condition is applied at the outflow plane of the computational domain.

For the purposes of flow visualisation a passive scalar is also introduced into the flow domain, which is governed by the equation

$$\frac{\partial \bar{\xi}}{\partial t} = \frac{\partial}{\partial x_i} \left(-\bar{u}_i \bar{\xi} + \alpha \frac{\partial \bar{\xi}}{\partial x_i} \right) \quad (9)$$

where α is the diffusivity, which contains both a molecular and a subgrid component, $\alpha = \alpha_m + \alpha_{sg}$, if a subgrid-scale model is used. With a subgrid scale model employed, the subgrid diffusivity is set to $\alpha_{sg} = \nu_{sg}/0.7$. The scalar is discretised on the staggered mesh at the cell centre, and third-order upwinding scheme is used to calculate the scalar flux between cell faces. The Adams-Bashforth method is used to integrate the scalar field forward in time, with a method of discretisation very similar to equations 5–6.

For the simulation requiring a physically correlated inflow fluctuations, the recycling and rescaling method of Xiao *et al.*³¹ is employed. This method requires virtual domains to be placed upstream of the main simulation domain, in which the fluctuations are generated. The flow in the virtual domain then provides the inflow condition for the main simulation domain. This method has been used extensively in recent research, being used to specify inflow conditions for plane mixing layers,^{31,32} axisymmetric jet flows,³³ and multiphase flow.³⁴ In all of these flow configurations, the turbulent inlet condition produced main simulation flow-fields with accurate flow statistics.

III. Simulation Set-up

The experiments conducted by Browand & Latigo provide the reference data for the present simulations.³⁰ The rig in which the experiments were conducted had a large test section, with very low level fluctuations measured in both the freestream and the laminar boundary layers. The lateral walls of the test section were fixed, resulting in the formation of a small adverse streamwise pressure gradient. This pressure gradient resulted in a noticeable reduction of the low-speed freestream velocity with streamwise distance in the test section. Measurements of the mean and root mean squared (r.m.s.) mixing layer velocity statistics, along with documentation of the initial conditions of the mixing layer, make this an attractive experiment for comparison with LES data.

Inflow State	U_1 (ms ⁻¹)	θ_1 (mm)	U_2 (ms ⁻¹)	θ_2 (mm)	R
Laminar	25.6	0.457	5.2	0.86	0.66
Turbulent	25.6	0.864	5.2	0.86	0.66

Table 1. Flow Parameters of the simulations, consistent with experiments of Browand & Latigo.³⁰

The flow parameters of the experiment performed with laminar separating boundary layers are given in Table 1. The velocity ratio parameter of the flow, R , defined as

$$R = \frac{U_1 - U_2}{U_1 + U_2} \quad (10)$$

where U_1 and U_2 are the high- and low-speed stream velocities respectively. The parameter takes a value of $R = 0.66$ at the inlet plane of the computational domain.

The simulations each comprise a mixing layer domain, which extends $0.75 \times 0.61 \times 0.18\text{m}$ in the stream-wise, lateral, and spanwise directions respectively. The vertical domain extent matches that found in the experimental facility, with the upper and lower guidewalls being modelled as slip walls to reduce the mesh resolution requirements in these regions. The spanwise domain extent is chosen such that the developing mixing layer will be free from any spanwise confinement effects.³⁷ The spanwise boundaries are periodic in nature. The domain is discretised into $768 \times 256 \times 256$ cells in each direction respectively. The grid is refined in the plane of the splitter plate, resulting in minimum grid spacings of $\Delta x_{min} = 0.0002\text{m}$, and $\Delta y_{min} = 0.00004\text{m}$. The laminar boundary layers are resolved with 36 points in the cross-stream direction. The WALE subgrid scale model has its model coefficient set to $C_w = 0.56$. A previous study has shown that the evolution of the mixing layer is insensitive to a range of WALE model coefficient on a grid of similar resolution to that used here.³⁶ Each simulation has a time step of $\Delta t = 6 \times 10^{-7}\text{s}$, which produces a CFL number of less than 0.35 throughout the computation. The passive scalar is given a value of unity in the high-speed stream and zero in the low speed, and its primary purpose in this study is to produce flow visualisation outputs. Flow statistics are obtained over a period corresponding to 15 convective flow-through times, based on the mean convection velocity of the flow $U_c = 0.5(U_1 + U_2)$. Details specific to each simulation are described below.

A. Case LWN

In Case LWN (Laminar White Noise), Blasius profiles with momentum thicknesses that match the experimental data are imposed at the inlet plane of the mixing layer domain. The high-speed side boundary layer profile is shown in Figure 1a. Superposed onto this profile at each time step are pseudo-random white noise fluctuations, the profiles of which are presented in Figure 1b. The white-noise nature of the imposed fluctuations is similar to that reported in many other mixing layer simulations,^{22,23,24,36} and can be considered to represent an idealised form of the experimental flow.

B. Case LRRM

In any real mixing layer, the fluctuations that exist in the flow upstream of the splitter trailing edge will of course be physically correlated. Case LRRM (Laminar Recycling Rescaling Method) is an attempt to produce a mixing layer simulation that contains low-level physically correlated fluctuations in the upstream boundary layers.

The recycling and rescaling method employed here requires upstream ‘virtual’ domains in which the physically correlated flow is generated. On the high-speed side, these domains extend 0.12m and 0.06m upstream of the trailing edge splitter plate on the high- and low-speed sides respectively. The recycling plane is placed 0.01m upstream of the trailing edge splitter plate. The grid resolution in these virtual domains matches that found at the inlet plane of the main mixing layer domain. The solid splitter plate is included in the computational domain in this calculation, and is modelled through no-slip boundary conditions where required. The splitter plate is assumed to be of infinitesimal thickness. The properties of the boundary layers at the trailing edge of the splitter plate match those of Case LWN, shown in Figures 1a and 1b for the high-speed stream.

C. Case TRRM

Case TRRM (Turbulent Recycling Rescaling Method) shares the same computational domain as Case LRRM. This simulation has a high-speed stream that contains a turbulent boundary layer, the flow statistics of which are presented in Figures 1c and 1d. The current mesh resolution produces *a posteriori* estimates of the non-dimensional near-wall grid spacing of $\Delta x^+ \approx 15$, $\Delta y^+ \approx 1.5$, and $\Delta z^+ \approx 50$. Apart from the change in state of the high-speed side boundary layer, all other simulation parameters match those of Case LRRM.

IV. Results

The horizontal fixing of the lower guidewall in both simulation and experiment leads to the development of an adverse pressure gradient in the flow. This produced a small decrease in the low-speed freestream velocity with increasing streamwise distance from the splitter plate trailing edge. The variation of freestream velocity with streamwise distance is shown in Figure 2, for Case TRRM and the reference experiment. The freestream velocity is recorded at the vertical mid-point between the splitter plate and the corresponding test section guidewall. The comparison between simulation and experiment is extremely good. Therefore, the simulation results are normalised in the same manner as the experiments. Where velocity statistics are normalised by $\Delta U = U_1 - U_2$, the local values of streamwise velocity are used.

A. Mean Flow Statistics

A measure of the integral thickness of the mixing layer can be obtained through the momentum thickness, θ . This quantity is defined as

$$\theta = \frac{1}{\Delta U^2} \int_{-\infty}^{\infty} (U_1 - \bar{u}_t)(\bar{u}_t - U_2) dy \quad (11)$$

where $\bar{u}_t$ is the mean streamwise velocity. The normalised momentum thickness variations for each simulation are shown in Figure 3. The data are normalised by the initial momentum thickness, θ_i of each simulation. The initial momentum thickness is assumed to be equal to the high-speed stream boundary layer momentum thickness.

Both Cases LWN and LRRM produce reasonable agreement with the reference initially-laminar data. The reduction in growth of the mixing layer towards the outflow boundary is commonly observed in numerical simulations,^{23,24} and is attributed to the altering of the entrainment characteristics of coherent structures as they pass through the outflow boundary.³⁶ The momentum thickness growth rate is estimated from a straight-line fit through the self-similar region of the mixing layer. For Case LWN $d\theta/dx = 0.0209$, and for Case LRRM $d\theta/dx = 0.0238$. There exists a $\sim 14\%$ difference in the growth rate of the two initially-laminar simulations, with Case LRRM producing better agreement with the experimental data.

Good agreement is also obtained between Case TRRM and the initially-turbulent experimental data. It is apparent that the reduction in growth rate caused by initially turbulent inflow conditions is captured by the LES. The experimental data observed a switch in the momentum thickness growth rate in the initially turbulent mixing layer, from $d\theta/dx = 0.0163$ in the range of $0 \leq x/\theta_i \leq 400$, to $d\theta/dx = 0.0208$ in the range of $x/\theta_i \geq 600$. In Case TRRM, best-fit lines in these two regions produce momentum thickness growth rates of $d\theta/dx = 0.0161, 0.019$ respectively. The momentum thickness growth rates in Case TRRM suggest that the switch in growth is also present in the simulation, but the computational domain used here is too short to verify this switch.

The visual thickness growth rate is obtained by tracing the wedge swept out by the 1% thickness lines of many superimposed instantaneous, spanwise averaged scalar fields. For a uniform density mixing layer, the visual thickness growth rate should be a linear function of R , following the form of

$$\delta'_{viz} = \frac{d\delta_{viz}}{x - x_0} = kR \quad (12)$$

where k is a constant. The visual thickness growth rate constants of all three simulations are shown in Table 2. The growth rate obtained in Case LWN, $k \approx 0.32 \approx 1/\pi$ has been reported elsewhere in simulations originating from laminar boundary layers with a white noise disturbance environment, and is linked to the continuous linear growth of the post-transition coherent structures. Case LRRM has a growth rate that is some 14% larger than that of Case LWN, and is close to the obtained experimental value. The predicted visual thickness growth rate of Case TRRM also agrees well with the experimental data. These data show that the simulated mixing layer is not only sensitive to the state of the separating high-speed side boundary layer (laminar or turbulent), but it is also sensitive to the nature of the fluctuations imposed on the laminar boundary layer.

The mean streamwise velocity profiles of each simulation, recorded in the self-similar region, are shown in Figure 4a. All three simulations produce data that matches extremely well with the reference experiment. The streamwise velocity fluctuation profiles are shown in Figure 4b. Case LRRM slightly over-predicts the fluctuation magnitude in the outer regions of the mixing layer, but is a reasonable prediction of the

Case	k
LWN	0.32
LRRM	0.366
Expt. (lam)	0.36
TRRM	0.3
Expt. (turb)	0.31

Table 2. Visual thickness growth rate constants.

experimental profile. The profiles extracted from Cases LWN and TRRM follow the experimental trends very well indeed. The variation of the maximum streamwise fluctuation with streamwise distance is shown in Figure 4c. For Case LWN, the large overshoot in u' is further downstream than in the experiment. This streamwise location of the overshoot in Case LRRM, however, is well-predicted. The value of u' in the self-similar region is slightly different in both initially-laminar simulations. From statistical interrogation alone it is not clear if this represents different self-similar states, as the fixed guidewalls of the domain prevent the mixing layers from being truly self-preserving. For Case TRRM there is a substantial over-prediction of u' in the initial region of the mixing layer. The reason for this over-prediction is not currently clear; the near-wall grid resolution may be insufficient ($y^+ \approx 1.5$), or it may be the case that the inflow generation method requires a recovery length in order to produce realistic turbulence structure. It should be noted, however, the the distribution of u' predicted by Case TRRM is qualitatively similar to that obtained by Bell & Mehta for an initially-turbulent mixing layer.¹⁶ Further downstream, however, the predicted value of u' in Case TRRM converges to that of the experimental data.

B. Flow Visualisation

The uniform density nature of these simulations precludes the direct use of numerical schlieren and shadowgraph imagery. Instead, the transported passive scalar is spanwise-averaged to produce flow visualisation images which are analogous to optical techniques.³⁸ Representative instantaneous, spanwise-averaged scalar fields from each simulation are shown in Figures 5–7. Using these images as a guide, the pre-transition and post-transition regions of the simulated mixing layers are discussed below.

1. Pre-Transition Vortices

Both initially laminar simulations contain K-H rollers in the initial region of the flow. As these proceed downstream they undergo pairing interactions, and the transition to turbulence is precipitated by such an interaction once the local Reynolds number has attained a value greater than 10,000.³⁹ For the transition to occur, a secondary streamwise structure must also have developed in the flow.¹³ Qualitative inspection of the distribution of a passive scalar in a $y - z$ plane can infer the presence of the streamwise structure.¹² Figures 8a and 8b show typical pre-transition vortices, in which the centre of rotation lies at $x = 0.05\text{m}$. These vortices are captured from Cases LWN and LRRM respectively. The spanwise-averaged scalar distribution is similar in each vortex - the scalar is wrapped around a central vortex core. However, the associated $y - z$ scalar field through the vortex core, shown in Figures 8a and 8b for Cases LWN and LRRM respectively, are remarkably different. In Case LWN a long-wavelength undulation is visible across the span, and tongues of unmixed fluid penetrate far across the layer. Inspection of sequences of these $y - z$ images shows that the undulations do not occur at fixed spanwise locations; rather their spanwise location varies on each primary roller that passes through the sampling plane. For Case LRRM, there are two clear rows of mushroom-shaped eruptions, one on the upper side of the vortex, and one on the lower side. Sequences of $y - z$ images show that these structures are located in apparently fixed spanwise locations, and follow a pattern very similar to that postulated by Bernal & Roshko.¹²

For the initially-turbulent simulation there are no K-H rollers in the initial region, as can be seen in Figure 7. Rather, the turbulent structure in the upstream turbulent boundary layer is ingested into the mixing layer. A typical $y - z$ scalar distribution recorded at $x = 0.05\text{m}$ (Figure 8c) shows evidence for the presence streamwise vorticity in the nearfield of the mixing layer. Inspection of sequences of these images

shows that the structures are seemingly free to wander across the span, depending on the upstream evolution of the separating turbulent boundary layer.

2. *Post-Transition Coherent Structures*

As noted above, the initially-laminar simulations undergo a transition to turbulence. In Figures 5 and 6 the transition occurs at approximately $x = 0.14\text{m}$, downstream of which coherent structures are visible in the mixing layer. These structures occupy the full visual thickness of the layer, and are therefore of dynamical significance in the flow evolution. The coherent structures present in Cases LWN and LRRM are qualitatively very similar; the structure cores are visible as bumps on the upper and lower edges of the mixing layer, and are connected with braid regions where the scalar lines are angled to the horizontal. Closer inspection, however, shows some interesting differences in the distribution of the spanwise averaged scalar in the structures. In Case LRRM the cores are very clearly identifiable, with very thin braid regions connecting the structures. In Case LWN, however, the structure cores are much less pronounced, with the interconnecting braid regions being much thicker (the scalar lines in the braid region are less tightly packed). The dependence of the scalar distribution in the post-transition coherent structures has also been observed in temporal mixing layers,⁴⁰ where a highly two-dimensional disturbance environment resulted in structures with concentrated cores and thin interconnecting braid regions.

Instantaneous cross-stream cuts through the cores of turbulent coherent structures captured at $x = 0.3\text{m}$ in Cases LWN and LRRM are shown in Figures 9a and 9b respectively. In Case LWN a secondary streamwise structure is present, and has a somewhat irregular appearance. Two rows of mushroom-shaped structures are present, one on the upper side of the mixing layer, and the other on the underside. Sequences of images recorded at this plane show that this secondary structure meanders across the span of the mixing layer from as each primary coherent structure convects through it. A more clearly pronounced double row of structures is visible in Case LRRM, and sequences of outputs show that these mushroom shaped eruptions remain largely in fixed spanwise locations from one primary spanwise structure to the next. For Case LRRM, it is straightforward to infer a streamwise structure in the turbulent mixing layer similar to that proposed by Bernal & Roshko.¹² It should be noted that the local Reynolds number, based on the visual thickness of the layer and the velocity difference across it, is $\sim 85,000$ in Case LRRM at this streamwise location. This is significantly higher than the Reynolds number of 4,000 in the Bernal & Roshko experiment.

In Case TRRM the spanwise-averaged flow visualisation of Figure 7 shows that coherent structures do eventually emerge in the mixing layer. Qualitatively, the spanwise-averaged scalar distribution in these structures is similar to that observed in Case LWN; the bumps in the scalar signifying the structure cores are surrounded by thick interconnecting braid regions. The emergence of these structures coincides with the switch in momentum thickness growth rate observed in both the simulation and the reference experiment. Experimental studies have also noted the emergence of quasi-two-dimensional structures in a mixing layer originating from initially-turbulent conditions,¹⁰ but their presence in those experiments was attributed to the evolving aspect ratio of the flow within a test section of fixed spanwise extent. It is not currently clear if the structures emerge in this simulation owing to the finite extent of the spanwise computational domain, and further simulations are planned to test this possible dependence. For the structures that are present in Case TRRM, a $y - z$ cut through a structure at $x = 0.3\text{m}$ shown in Figure 9c reveals a double row of mushroom shaped eruptions, one on each side of the mixing layer. In all three cases presented here, a secondary structure exists far into the fully-developed region of the mixing layer. The statistical nature of this structure is analysed below.

C. *Streamwise Vorticity*

In all three simulations, sequences of $y - z$ flow fields are recorded at $x = 0.02, 0.05, 0.1, 0.15, 0.3, 0.45$, and 0.6m , with up to 960 samples obtained at each station. From these sequences, mean flow statistics are computed in order to determine the mean streamwise vortex structure in the flow. As noted by Bell & Mehta, the secondary shear stress ($\overline{u'w'}$) distribution broadly matches that of the streamwise vorticity in a mixing layer containing statistically stationary streamwise structure.

The locus of the mean centreline velocity, U_c , across the mixing layer span is plotted at each measurement station in Figure 10 for each simulation. All three plots show that the centreline location does vary somewhat across the span at each streamwise location. For Cases LWN and TRRM, the variations in centreline location are rather small in magnitude at each measurement station. There is little evidence for a regular pattern in

the plots for Case LWN at all streamwise locations. Case TRRM shows some evidence of a regular pattern at $x = 0.1, 0.15\text{m}$, but beyond this point the distributions again become somewhat irregular. Case LRRM, in contrast, shows a very regular spanwise variation in the centreline location at each measurement station. These measurements provide evidence of a statistically stationary streamwise structure in this simulation. A peak in the centreline distribution corresponds to a pair of stationary streamwise vortices with a common upflow, and a trough corresponds to a pair of stationary streamwise vortices with a common downflow. The wavelength of the variation increases with increasing streamwise distance, implying that the number density of the stationary streamwise vortices decreases as the flow evolves downstream.

In an analogous manner to the visual thickness defined above, another measure of the mixing layer thickness can be defined as the vertical distance between the locations where $U_{0.01} = U_2 + 0.01(U_1 - U_2)$, and $U_{0.99} = U_2 + 0.99(U_1 - U_2)$. The variation in this mixing layer thickness across the span is plotted as a function of the ‘pairing parameter’ in Figure 11. This normalised streamwise distance, x_i^* is defined as

$$x_i^* = \frac{Rx}{30\theta_i} \quad (13)$$

and has been shown to provide a measure of a normalised streamwise distance at which vortex event occur.¹³ The simulations and comparable experimental data from Bell & Mehta¹¹ have been normalised into this form. In Case LWN, the thickness variation is extremely low, and reaches a maximum at $x \approx 0.1\text{m}$ (the second measuring station). The maximum variation is 3.8%, significantly lower than the value reported in the comparable experimental data. In the post-transition region the variability in mixing layer hovers at approximately 3%. In contrast, the thickness variation data obtained from Case LRRM is in extremely good agreement with the comparable experimental data. A large peak in the thickness variation occurs downstream of $x_i^* = 2$, implying that the secondary streamwise structure is amplified through the process of the flow rolling up into discrete spanwise vortices. The variation then converges to a value of $\sim 4\%$ in the fully-developed region.

Case TRRM produces a maximum variation of 2.8% at the second measurement station, and the variability relaxes to $\sim 2.5\%$ for the remainder of the computational domain. The large value of θ_i substantially reduces the normalised streamwise distance of this calculation, and it is not clear if θ_i is even the correct scaling parameter for an initially turbulent boundary layer.⁴¹

Contour maps of normalised secondary shear stress for each of the simulations at selected measurement stations are shown in Figures 12, 13, and 14 for Cases LWN, LRRM, and TRRM respectively. The shear stress is normalised by the square of the velocity difference across the mixing layer $U_0 = U_1 - U_2$. In Case LWN, none of the measurement stations record a secondary shear stress contour map that provides evidence of a statistically stationary streamwise vortex structure. There appears to be evidence for some organisation in the peaks and troughs of the secondary shear stress contours - alternating bands of positive and negative $\overline{u'w'}$ are evident, but their spanwise scale is significantly larger than that of comparable experimental data.¹¹ As noted above, the sequences of $y - z$ images show that the spanwise location of the streamwise vorticity at $x = 0.05\text{m}$ varies from primary roller to primary roller, and as such the mean representation here is a reflection of the apparent randomness. At subsequent downstream measurement stations there is no particular evidence for regular peaks and troughs in the secondary shear stress contours across the span of the mixing layer. Instead, clumps of positive and negative secondary shear stress are present at all measurement stations, with no obvious pattern present in their distribution. The scale of the clumps of secondary shear stress increases with streamwise distance, implying that the local scale of the streamwise structure increases with increasing streamwise distance, but it is not possible to draw any definitive conclusions regarding the spanwise wavelength of the streamwise structure from these images.

In contrast, the $\overline{u'w'}$ contours from Case LRRM do show a clear banding across the span of the domain. At $x = 0.05\text{m}$ there are two rows of structures, one on each side of the mixing layer. The sign of each band of $\overline{u'w'}$ alternates across the span of the mixing layer. Further downstream at $x = 0.1\text{m}$ the streamwise vortices have evolved into a single row of alternating sign structures. The scale of the structures increases with increasing downstream distance, with six structures clearly visible at the most downstream measurement station. The interfaces between the bands in these contour maps correspond to the peaks and troughs recorded in the centreline variation of Figure 10b, and demonstrate how the streamwise vortices produce common upflows and downflows between them.

In Case TRRM there is no particular evidence for organised streamwise vorticity at any of the streamwise measurement stations. This lack of stationary streamwise vorticity is in agreement with available experi-

mental data,¹⁶ and may be caused by the presence of strong streamwise vorticity in the upstream turbulent boundary layer.

V. Summary

Large Eddy Simulations of the plane turbulent mixing layer have been performed. This study has shown that not only does the mixing layer display a sensitivity to the state of the separating high-speed side boundary layer, but it also displays a sensitivity to the nature of the imposed fluctuations in an initially laminar flow. A simulated mixing layer originating from laminar boundary layers with a low-level white noise disturbance environment does not produce statistically stationary streamwise vortices. In contrast, a simulated mixing layer originating from laminar boundary layers with physically correlated fluctuations of the same magnitude does predict statistically stationary streamwise vortices. The properties of these stationary streamwise vortices are in general agreement with those observed in comparable experiments. The simulated initially-turbulent mixing layer does not contain statistically stationary streamwise vortices, again in agreement with experimental observations.

The findings presented here show that care must be taken to impose realistic inflow conditions for even initially-laminar mixing layers. The presence of statistically stationary streamwise vortices in the mixing layer appears to have a significant impact on the predicted growth rate of the flow. As no experiments provide a complete description of the flow conditions at the trailing edge of the splitter plate, it is not currently possible to accurately recreate the initial conditions of any reference experimental data. The sensitivity of the mixing layer to the magnitude of the imposed fluctuations will be the subject of a future study, given that the present study only deals with one particular set of initially-laminar inflow conditions.

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(a) Laminar inflow: mean streamwise velocity.

(b) Laminar inflow: velocity fluctuations.

(c) Turbulent inflow: mean streamwise velocity.

(d) Turbulent inflow: velocity fluctuations.

Figure 1. Initial conditions of the high-speed side boundary layers in the simulations.

Figure 2. Freestream velocity variation along the length of the computational domain in Case TRRM.

Figure 3. Momentum thickness variation of the simulations.

Figure 4. Mixing layer flow statistics. Experimental data recorded at $x/\theta_i = 1780$ in the initially-laminar case, and $x/\theta_i = 1000$ in the initially-turbulent case.

Figure 5. Spanwise-average visualisation of the instantaneous passive scalar distribution in Case LWN.

Figure 6. Spanwise-average visualisation of the instantaneous passive scalar distribution in Case LRRM.

Figure 7. Spanwise-average visualisation of the instantaneous passive scalar distribution in Case TRRM.

Figure 8. Instantaneous $y - z$ cuts through the cores of pre-transition vortices at $x = 0.05\text{m}$ in (a) and (b). An instantaneous $y - z$ plane through the flow at $x = 0.05\text{m}$ is shown in (c).

Figure 9. Instantaneous $y - z$ cuts through the cores of post-transition structures at $x = 0.3\text{m}$.

Figure 10. Mixing layer centreline locus at each measurement station.

Figure 11. Mixing layer thickness variation plots. Data of Bell & Mehta¹¹ shown for comparison.

Figure 12. Mean secondary shear stress contours in Case LWN. Note the changes in axes lengths and contour levels.

Figure 13. Mean secondary shear stress contours in Case LRRM. Note the changes in axes lengths and contour levels

Figure 14. Mean secondary shear stress contours in Case TRRM. Note the changes in axes lengths and contour levels