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<https://doi.org/10.1109/SAI.2017.8252211>

PUBLISHER

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VERSION

AM (Accepted Manuscript)

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Ji, Chenlu, Anne Liret, Gilbert Owusu, Jiyin Liu, Raphael Dorne, and Rupal Rana. 2019. "Service Scheduling to Minimise the Risk of Missing Appointments". figshare. <https://hdl.handle.net/2134/33537>.

Service Scheduling to Minimise the Risk of Missing Appointments

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Abstract—This paper introduces the risk minimisation objective in the Stochastic Vehicle Routing Problem (SVRP). In the studied variant of SVRP, technicians drive to customer sites to provide service. The service times and travel times are stochastic, and a time window is required for the start of the service for each customer. Most previous research uses a chance-constrained approach to the problem. Some consider the probability of journey duration exceeding the threshold of the driver's workload while others set restrictions on the probability of individual time window constraints being violated. Their objectives are related to traditional routing costs whilst a different approach was taken in this paper. The risk of missing a task is defined as the probability that the technician assigned to the task arrives at the customer site later than the time window. The problem studied in this paper is to generate a schedule that minimises the maximum risk and sum of risks of the tasks. Each task duration may be considered as following a known normal distribution. However the distribution of the start time of the service at a customer site will not be normally distributed due to time window constraints. Therefore a multiple integral expression of the risk was derived, and this expression works whether task distribution is normal or not. Additionally a deterministic heuristic searching method was applied to solve the problem. Experiments are carried out to test the method. Results of this work have been applied to an industrial case of SVRP where field engineering individuals drive to customer sites to provide time-constrained services. This original approach allows organisations to pay more attention to increasing customer satisfaction and become more competitive in the market.

Keywords—vehicle routing with time windows; stochastic service time; risk minimisation

I. INTRODUCTION

The growing interest on customer satisfaction has motivated researchers and businesses in building more customer-oriented models, taking time windows into consideration as an intrinsic component of workforce scheduling and vehicle routing problems [1]. On one hand, service provision organisations have to increasingly focus on providing customer satisfaction and

reassurance in the delivery process on offering good value or items. On the other hand, in real-world scheduling problems, most activities tend to be uncertain. For instance, at arrival on site, a technician may realise the task not matching his skills or tools, or tasks take longer or shorter than expected. [2]. Hence the environment in which services need to be delivered is inherently dynamic and subject to disruption in the workstack estimates as well as in the execution of jobs by workforce [3]. Thus, the purpose of this paper is threefold to the risk of missing appointments, where the risk arises from these uncertainties, to model the risk of a technician not being able to successfully complete his daily tour and to build schedules minimising those risks. A better schedule could help improving the level of customer satisfaction. Consequently, the company may become more competitive and acquire more customers. This study relates to two main objectives:

- How to generate a realistic schedule that can be actually executed in the field and can absorb variations in task time;
- How to identify tasks and technicians that are at a certain risk level of failure and assess different levels of risk.

In a classic VRP problem, the task duration is given by a single value which is either estimated by field experts or corresponds to the mean of historical data per task type. In this study, the task and travel durations are subject to stochastic events. It is proposed to model the duration of each task using a duration distribution instead of a single value. Then two levels of risk are modelled: 1) the risk of failing time window constraint for task and 2) the risk of technician failing their daily tour or finishing beyond their estimated end of day. Finally some solution search methods are experimented to generate a predictive schedule minimising the risk on tasks and resources.

This research focuses on the Stochastic Vehicle Routing Problem (SVRP), in which technicians drive to customer sites to provide services. In the problem it is assumed that service times

and travel times are stochastic, and a time window is associated with the start time of the service.

Previous relevant research uses a chance-constrained approach to the problem. For instance, [5; 6; 7; 8; 9; 10] consider the probability of route duration exceeding the threshold of the driver's workload, while [1; 4; 11; 12] set extra restrictions on the probability of individual time window constraints to be violated. Note that in these approaches, the objectives of the problem are related to traditional routing costs. In this paper, a new risk model is introduced that can be incorporated into the set of objectives to be minimised during the optimisation process.

In this paper, an application of the proposed model to a real scheduling problem in field engineering services world is presented. In that context, technicians offer services to customers, associating with a time window to each visit, and services are subject to disturbance in delivery causing the actual service time to be inherently stochastic. Section II proposes a mathematical formulation to assess the risk of missing appointments that can be applied to a sequence of tasks allocated to the technician, and the risk for a technician exceeding his workload. This paper considered the scheduling problem in a new perspective, where the objective is to minimise the risks of missing appointments and failing finishing daily work in time with a limited number of technicians. In order to solve the problem, mathematical expression of the risks is proposed. Section III describes the methods for calculating risk; Section IV presents the preliminary analysis, and Section V presents advanced local search methods for minimising those risks. Section VI reports the experiments and presents the results. Section VII concludes this paper.

II. PROBLEM DEFINITION

A. Problem statement

Let $G = (V_0, A)$ be a complete digraph, where $V_0 = \{0, \dots, N\}$ is a set of vertices which denote customer locations and $A = \{(i, j) : i, j \in V_0, i \neq j\}$ is a set of arcs. The vertex 0 represents the depot. The set of customers is $V = V_0 \setminus \{0\} = \{1, \dots, N\}$. Each customer $i \in V$ has a time window $[a_i, b_i]$. If the technician arrives at customer i before a_i , it is necessary for him to wait until a_i . The following notation is defined:

- C maximum number of customers served by each technician;
- T maximum work time for each technician;
- K the set of required technicians in a feasible solution $K = \{1, \dots, K\}$;
- x_{kij} binary variable equal to 1 if technician k travels directly through arc (i, j) and 0 otherwise;
- d_{kij} travel time of technician k between customers i and j ;
- δ_{ki} uncertain work time of technician k at the customer i , a random parameter with known and independent probability density;
- AT_{ki} arrival time of technician k at customer i , a continuous variable;
- ST_{ki} service start time of technician k at customer i , a continuous variable;
- R_{ki} risk of technician k arriving late at customer i ;
- R_{Tk} risk of technician k not finishing work within the maximum work time.

The model for the problem is formulated below:

$$\min w \cdot R_{\max} + R_{\text{sum}} \quad (1)$$

Subject to:

Fig. 1. Description of risks

- M large number;

$$\sum_{j \in V} x_{k0j} = \sum_{i \in V} x_{kio} \leq 1, \quad \forall k \in K \quad (2)$$

$$\sum_{j \in V_0} x_{kij} = \sum_{j \in V_0} x_{kji}, \quad \forall i \in V, \forall k \in K \quad (3)$$

$$\sum_{k \in K} \sum_{j \in V, j \neq i} x_{kij} = 1, \quad \forall i \in V \quad (4)$$

$$\sum_{i \in V} \sum_{j \in V} x_{kij} \leq C + 2, \quad \forall k \in K \quad (5)$$

$$ST_{ki} + \delta_{ki} + d_{kij} - M(1 - x_{kij}) \leq AT_{kj}, \quad \forall i, j \in V, \forall k \in K \quad (6)$$

$$AT_{ki} \leq ST_{ki}, \quad \forall i \in V, \forall k \in K \quad (7)$$

$$a_i \leq ST_{ki}, \quad \forall i \in V, \forall k \in K \quad (8)$$

$$R_{ki} = P(AT_{ki} > b_i) \sum_{j \in V} x_{kij}, \quad \forall i \in V, \forall k \in K \quad (9)$$

$$R_{Tk} = P\left(\sum_{i \in V} \sum_{j \in V} d_{kij} x_{kij} + \frac{1}{2} \sum_{i \in V} \sum_{j \in V} \delta_{ki} x_{kij} > T\right), \quad \forall k \in K \quad (10)$$

$$R_{\max} \geq R_{ki}, \quad \forall i \in V, \forall k \in K \quad (11)$$

$$R_{\max} \geq R_{Tk}, \quad \forall k \in K \quad (12)$$

$$R_{\text{sum}} = \sum_{k \in K} \sum_{i \in V} R_{ki}, \quad \forall i \in V, \forall k \in K \quad (13)$$

The objective function (1) is to minimise the maximum risk of a schedule and minimise the sum of risks for all tasks. The weight w can be chosen sufficiently large to ensure the maximum risk is minimised first. Constraints (2) and (3) make sure that each technician starts from and finishes to a depot and he goes along a tour. Constraints (4) indicate that each customer is served by one technician. Constraints (5) guarantee each technician serves no more than C customers. Constraints (6)-(8) make sure the technician k serves customer i and then customer j , and he should start in the time window. Equations (9) show the task risks while Equations (10) state the technician risk which is the probability of the working time of technician k more than T . Equations (11)-(13) show the risks in this problem.

B. Risk definition and its mathematical expression

The risk in the problem is defined following the notations above. Given a schedule, with a sequence of tasks $\{i_1, i_2, i_3, \dots\}$ allocated to technician k and the start point of technician k is ki_0 , then the risk of missing the appointment for task i is the probability of the arrival time AT_{ki} being later than the time window b_i as R_{ki} shown in Fig. 1. In other words, each technician has to execute the task following the task order in the schedule, hence any task execution is dependent on the sequence of tasks preceding it. And the risk for a task to fail (being executed by the technician after its latest valid start time) is directly impacted by the duration distributions of the preceding tasks.

At this stage of the study, the travel time is assumed to be certain. With this assumption, for a given technician k , let s_{k0} be the departure time from the depot, the arrival time to the 1st task AT_{ki_1} is simply $s_{k0} + d_{ki_0i_1}$, which is a constant, and

suppose it can be managed to fall within the time window. Thus the risk of the 1st task is surely 0, which is

$$R_{ki_1} = P(AT_{ki_1} > b_{i_1}) = 0. \quad (14)$$

Fig. 2. Arrival time

Fig. 3. Start time

Then the start work time for the 1st task ST_{ki_1} is just the arrival time AT_{ki_1} , then the arrival time at the 2nd task AT_{ki_2} will be

$$AT_{ki_2} = ST_{ki_1} + \delta_{ki_1} + d_{ki_1i_2}, \quad (15)$$

where δ_{ki_1} is the work duration at the 1st task and $d_{ki_1i_2}$ is the travel time from the 1st task to the 2nd task. Note that δ_{ki_1} is a random variable with known and independent probability density. Denote the distribution function of AT_{ki_2} as f_{ki_1} , the risk for the 2nd task will be the probability that the arrival time AT_{ki_1} greater than the time window b_{i_2} , which is

$$\begin{aligned} R_{ki_2} &= P(AT_{ki_2} > b_{i_2}) \\ &= 1 - P(AT_{ki_2} \leq b_{i_2}) \\ &= 1 - \int_{AT_{ki_2} \leq b_{i_2}} f_{ki_1}(AT_{ki_2}) dAT_{ki_2} \\ &= 1 - \int_{X_1 \leq b_{i_2}} f_{ki_1}(X_1) dX_1, \end{aligned} \quad (16)$$

where X_1 denotes AT_{ki_2} .

Then because the task duration is uncertain, the technician may finish the task and arrive at the next customer before the time window and so the technician has to wait. So the start time of the 2nd task will be $ST_{ki_2} = \max\{AT_{ki_2}, a_{i_2}\} = \max\{X_1, a_{i_2}\}$, here ST_{ki_2} is a random variable with a new distribution of the format shown in Fig. 3. More specifically, suppose the arrival time AT_{ki_2} follows the distribution in Fig. 2 and the lower bound of the task time window a_{i_2} is 9:00, then the distribution of the task start time ST_{ki_2} will turn out to be the distribution shown in Fig. 3, the probability at 9:00 will be the sum of the probability of that arrive before 9:00.

Denote $X_2 = \delta_{ki_2} + d_{ki_2i_3}$, which is the work duration time δ_{ki_2} at task ki_2 plus the travel time between ki_2 and ki_3 . Then the arrival time at the 3rd customer is

$$\begin{aligned} AT_{ki_3} &= ST_{ki_2} + \delta_{ki_2} + d_{ki_2i_3} \\ &= \max\{X_1, a_{i_2}\} + X_2, \end{aligned} \quad (17)$$

and the risk for 3rd task will be

$$\begin{aligned} R_{ki_3} &= P(AT_{ki_3} > b_{i_3}) \\ &= P(\max\{X_1, a_{i_2}\} + X_2 > b_{i_3}) \\ &= 1 - P(\max\{X_1, a_{i_2}\} + X_2 \leq b_{i_3}) \\ &= 1 - P(X_1 + X_2 \leq b_{i_3}, a_{i_2} + X_2 \leq b_{i_3}) \\ &= 1 - P(X_1 + X_2 \leq b_{i_3}, X_2 \leq b_{i_3} - a_{i_2}) \\ &= 1 - \iint_{\substack{X_1 + X_2 \leq b_{i_3} \\ X_2 \leq b_{i_3} - a_{i_2}}} f_{ki_1}(X_1) f_{ki_2}(X_2) dX_1 dX_2. \end{aligned} \quad (18)$$

Similarly, in terms of the time window, the start time at the 3rd customer will be

$$ST_{ki_3} = \max\{AT_{ki_3}, a_{i_3}\} = \max\{\max\{X_1, a_{i_2}\} + X_2, a_{i_3}\}, \quad (19)$$

denote $X_3 = \delta_{ki_3} + d_{ki_3i_4}$, then the arrival time at 4th customer will be

$$AT_{ki_4} = \max\{\max\{X_1, a_{i_2}\} + X_2, a_{i_3}\} + X_3. \quad (20)$$

Thus the risk for 4th task will be

$$\begin{aligned} R_{ki_4} &= P(AT_{ki_4} > b_{i_4}) \\ &= 1 - P(\max\{\max\{X_1, a_{i_2}\} + X_2, a_{i_3}\} + X_3 \leq b_{i_4}) \\ &= 1 - P(X_1 + X_2 + X_3 \leq b_{i_4}, X_2 + X_3 \leq b_{i_4} - a_{i_2}, X_3 \leq b_{i_4} - a_{i_3}) \\ &= 1 - \iiint_{\substack{X_1 + X_2 + X_3 \leq b_{i_4} \\ X_2 + X_3 \leq b_{i_4} - a_{i_2} \\ X_3 \leq b_{i_4} - a_{i_3}}} f_{ki_1}(X_1) f_{ki_2}(X_2) f_{ki_3}(X_3) dX_1 dX_2 dX_3. \end{aligned} \quad (21)$$

Therefore, by the method of induction, the risk of nth task will be

$$R_{ki_n} = 1 - \int \cdots \int_D \prod_{l=1}^{n-1} f_{ki_l}(X_l) dX_1 dX_2 \cdots dX_{n-1} \quad (22)$$

where $D = \{(X_1, \dots, X_{n-1}) : \sum_{l=1}^{n-1} X_l \leq b_{i_n}, \sum_{l=2}^{n-1} X_l \leq b_{i_n} - a_{i_2}, \sum_{l=3}^{n-1} X_l \leq b_{i_n} - a_{i_3}, \dots, X_{n-1} \leq b_{i_n} - a_{i_{n-1}}\}$.

In addition, the risk R_{Tk} for technicians not being able to finish their work within their working hours T is also considered. By the method of induction, the start time of the last task for technician k is

$$ST_{ki_n} = \max\{\cdots \max\{\max\{\max\{X_1, a_{i_2}\} + X_2, a_{i_3}\} + X_3, a_{i_4}\} \cdots + X_{n-1}, a_{i_n}\}, \quad (23)$$

Denote $X_n = \delta_{ki_n}$, then the finish work time is $ST_{ki_n} + X_n$, which is the start time plus the last task duration time, denote

Fig. 4. Double integral

Fig. 5. Simpson rule

Fig. 6. Basic idea of Monte Carlo method

the distribution function of X_n as f_{ki_n} hence the risk will be

$$\begin{aligned} R_{Tk} &= P(ST_{ki_n} + X_n > T) \\ &= 1 - \int \cdots \int_D \prod_{l=1}^n f_{ki_l}(X_l) dX_1 dX_2 \cdots dX_{n-1} dX_n, \end{aligned} \quad (24)$$

where $D = \{(X_1, \dots, X_n) : \sum_{l=1}^n X_l \leq T, \sum_{l=2}^n X_l \leq T - a_{i_2}, \sum_{l=3}^n X_l \leq T - a_{i_3}, \dots, X_n \leq T - a_{i_n}\}$.

III. RISK CALCULATION

A. Simpson rule and Monte Carlo method

The risk mathematical expressions described in Section II are of the multiple integral formats. Multiple integral of a Riemann integrable function in n variables: $f(x_1, x_2, \dots, x_n)$ over a domain D in n -dimensional space can be described as

$$\int \cdots \int_D f(x_1, x_2, \dots, x_n) dx_1 dx_2 \cdots dx_n. \quad (25)$$

More specifically, as is shown in Fig. 4, the double integral $I = \iint_R f(x, y) dx dy$ is the volume under the surface $f(x, y)$ over the region R at the bottom which is the domain of integration, while the surface is the graph of the two-variable function to be integrated.

If one wants to calculate the risks according to the multiple integral expressions, one method can be used is a combination of Simpson rule and Monte Carlo methods. Simpson rule is known as a widely used method for numerical integral computation, the core thought of Simpson rule is to use the quadratic interpolation

$$\int_a^b f(x) dx \approx \frac{b-a}{6} \left[f(a) + 4f\left(\frac{b+a}{2}\right) + f(b) \right], \quad (26)$$

and the estimation is the integral under the dash line in Fig. 5 while the solid line is the original function. For n -dimensional function, it is applied on each direction of the multiple integral.

Monte Carlo integration is a powerful method for computing complicated or higher-dimensional integrals. Monte Carlo method is a sampling method based on probability theory. More precisely, let us draw random numbers in the xy -plane (dots in the Fig. 6), then the integral of function f is approximately given by the total area times the fraction of points that fall under the curve $f(x)$ over the number of all points. It is clear that the greater the number of points the more accurate is the evaluation of this area.

Therefore, Monte Carlo integration is used for the integrals of more than 5-dimensions, while we use the Simpson rule method is used to calculate the integrals for 5-dimensions or lower to guarantee the accuracy.

B. Accumulation method

Consider that in a schedule, risks of all tasks for each technician and the risk for each technician working over his workload will be calculated. Therefore instead of calculating the complicated integrals in multi-dimensions for each task, one way is to calculate risks from the perspective of each technician, where the accumulation method is proposed. It uses discrete approximation to represent all distributions for the calculation.

As an example, let us assume that a technician has 3 tasks and travel times are ignored to simplify the problem. The technician will start work at 8:00, and the 1st task time window is from 8:00 to 10:00. Because there is no uncertainty before the 1st task, there is no chance to arrive at 1st customer later than 10:00 which means no risk for the 1st task. Suppose the average duration time of the 1st task is 2 hours and this duration time is a random variable. Then after he finishes the 1st task, the distribution of the arrival time at 2nd customer follows the distribution which is discretely described in TABLE I column 2 and Fig. 2. The 2nd customer time window is from 9:00 to 11:00, and then the risk for this task will be the total probability of the arrival time after 11:00, which is 0.2065. As it is illustrated in Section II B, the start work time at 2nd customer will be of the distribution in TABLE I column 3 and Fig. 3 (The probability of the arrival time before 9:00 are all added to the one of 9:00).

Then suppose the 2nd task duration time follows the discrete distribution in TABLE II and Fig. 7, the accumulation method is for each segment of the work time will add in each segment of the start time, and their probabilities will be multiplied to have the new distribution. Then if the summation of the time is the same, the probabilities will be added up. For example, in terms of the arrival time 11:00, the production of the arrival time 9:00 and task duration time 2 hours will contribute the probability, as well as 9:30 and 1:30 hours, 10:00 and 1 hour, 11:00 and 0 hour, so the probability of the arrival time will be the sum of the probabilities of these four segments. Therefore, the

Fig. 7. Work time distribution

Fig. 7. Arrival time at the 3rd task

accumulation method can get the distribution of arrival time at 3rd task which is described in TABLE III and Fig. 8.

TABLE I. ARRIVAL AND START TIME DISCRETE DISTRIBUTION AT THE 2ND CUSTOMER

Time	Probability(Arrival)	Probability(Start)
7	0.022984	0
8	0.053991	0
8.5	0.129518	0
9	0.241971	0.448464
9.5	0.352065	0.352065
10	0.398942	0.398942
10.5	0.352065	0.352065
11	0.241971	0.241971
11.5	0.129518	0.129518
12	0.053991	0.053991
13	0.022984	0.022984

TABLE II. WORK TIME DISTRIBUTION OF THE 2ND TASK

Time(Hour)	Probability	Time(Hour)	Probability
0	0.022984	3.5	0.352065
1	0.053991	4	0.241971
1.5	0.129518	4.5	0.129518
2	0.241971	5	0.053991
2.5	0.352065	6	0.022984
3	0.398942		

TABLE III. ARRIVAL TIME DISTRIBUTION OF THE 3RD TASK

Time	Probability	Time	Probability
9	0.005154	14	0.223365
9.5	0.004046	14.5	0.157356
10	0.016691	15	0.104347
10.5	0.042592	15.5	0.05844
11	0.090607	16	0.030621
11.5	0.158367	16.5	0.015085
12	0.229649	17	0.007019

TABLE V. From the results it can be seen that the risks given by the fast calculation method are not far away from those from the other two methods, and so it is reasonably accurate for use in a simulator or dynamic scheduling tool as the calculation is fast.

Fig. 9. Example of gamma fit

TABLE IV. TEST EXAMPLE

Task number	Mean	Standard deviation	Earliest time	Latest time	Travel time
1	60	40	450	632	14
2	31	40	567	717	93
3	46	40	608	758	90
4	31	40	674	824	33
5	40	40	744	894	22
6	25	40	816	966	40
7	30	40	923	1073	45

TABLE V. RESULT COMPARISON

Risk	Simpson & M.C.	Accumulation	Fast calculation
Task 1	0	0	0
Task 2	0.00215	0.00089	0.00216
Task 3	0.27833	0.27566	0.27388
Task 4	0.393	0.38839	0.38087
Task 5	0.32722	0.31616	0.31736
Task 6	0.37398	0.36193	0.36863
Task 7	0.23619	0.23934	0.24702
Maximum	0.393	0.38839	0.38087
Sum	1.61088	1.58237	1.58992
Running time (ms ^a)	3460	28	2

Fig. 8. Example of normal fit

C. Fast calculation method

For large sample data, it is common to assume work time follows normal distribution. The calculation will become easier and faster due to the property of normal distribution which can be summarised as: When X and Y are independent random variables that are normally distributed, their sum is also normally distributed.

Furthermore, if time windows are ignored, the mean and variance of a work at a given point are just the accumulation of means and variances of the task duration times and travel times of the same technician up to this point. As it was discussed in Section III B, in real world, the start time distribution will not be normally distributed for all the tasks, due to the time window. However, it is reasonable to approximate the impact of time window constraints without losing the benefits of fast computation. Therefore, different thresholds and corresponding behaviours are considered:

- If latest arrival time is smaller than the earliest start then ignore arrival time distribution;
- If probability to arrive after earliest start is less than 50% then use the earliest start time as the mean but use a small standard deviation;
- If probability to arrive after earliest start is greater than 50% then use the arrival time as the mean and use the current standard deviation.

In addition, in the example in Section III C, the arrival time distribution follows normal distribution $N(10,1)$, and the task distribution used here is a normal distribution $N(3,1)$, and the curve in Fig. 8 shows the normal distribution $N(13,2)$. Thus the arrival time to the next task follows a similar normal distribution even though start time was affected by a time window earliest time, especially the right tail of the distribution is usually used to calculate the risk.

D. Comparison

This section focuses on comparing the above methods against running time and quality of results. An example is used to test the above methods. There are 7 tasks allocated to a technician and suppose task duration follows different normal distributions, of which the mean and standard deviation are shown in the TABLE IV, as well as the task time windows and the earliest and latest time for the time window is the number of minutes starts from 0:00. Then the risk calculation results by these three methods and the corresponding running time are illustrated in

task. Clearly a gamma fit $\Gamma(4.49, 22.52)$ can describe its distribution well.

V. SEARCHING PROCESS

Given a schedule, the risks are calculated, then the searching process will try to minimise the risk component of the objective function. This risk component is composed of the maximum risk of the schedule (the highest risk value over all tasks and technicians) and the sum of all task risks at task level.

Before carrying out the searching process, it is important to start from a good initial schedule. One method is to sort all tasks according to their time window start times and then assign to all compatible technicians as their 1st tasks, then their 2nd tasks, etc. Therefore, each technician's task sequence is sorted in the order

IV. DATA ANALYSIS

Task and technician data were analysed over a 12 month period. The data contain 72114 samples, and each task sample contains a geographical region, a task type, a set of required technician skills, and the actual time spent on the task etc. The tasks were grouped per task type and skill in order to analyse the

Fig. 10. Swap operator process

Fig. 11. Insert operator process

task duration distribution.

The actual time spent for the most representative task groups that have a large number of samples was analysed. From the distributions of actual times, it was revealed that most of them satisfy gamma distributions while 34.6% follow normal distributions. Therefore, it is reasonable for us to use the fast calculation method to calculate the risks since most of the real data follow normal distributions.

Fig. 9 and Fig. 10 illustrate the analysis on two typical examples. Fig. 9 shows the distribution of a repair type of task. A normal fit $N(89.24, 39.82^2)$ can describe its distribution well. While Fig. 10 shows the distribution of a provision type of

of task time windows. For example, there are 50 technicians and 100 tasks, and tasks are compatible with all technicians, all tasks are sorted according to their time window start times, and the initial 50 tasks are allocated to all technicians as their 1st task. The 51st to 100th tasks are assigned to all technicians as their 2nd task. Thus for each technician the time window of the 1st task is before that of the 2nd task. Consequently, the initial schedule constructed by this method is better than the schedule built by assigning tasks to technicians randomly.

There are two operators used in the searching process: the swap operator and the insert operator. The swap operator is to exchange any two task positions in a schedule as long as the skill codes are matched with technicians' and their time windows are

overlapped. In contrast, the insert operator is to withdraw a task from a technician and allocate it to another technician. It takes out the task and assigns it to another technician who has the matched skill and put it in the right position in this technician's task sequence according to task time windows' earliest time. For example, the technician has 3 tasks and the time windows are 8:00 to 10:00, 10:00 to 12:00 and 15:00 to 17:00, while the time window of the task which will be assigned to this technician is 13:00 to 15:00. Then it would be the 3rd task for the technician. By now these two operators are used separately in the searching process, given a good initial schedule, the swap searching process which only use the swap operator is adapted and then the insert searching process which only use the insert operator is carried on. It is reasonable to repeatedly use these two operators to obtain a better final schedule, until no improvement is found in the schedule.

The swap searching process is shown in Fig. 11 and can be illustrated as follows:

- Given an initial schedule, after calculating the risks, choose two tasks of which time windows are overlapped and swap these two tasks and check the risks of this new schedule;
- If the maximum risk of all tasks and technicians in this schedule is smaller or the maximum risk is the same but the sum of all task risks is smaller, the new schedule can be regarded as a better solution and this swap is performed. Otherwise, if it is not a better schedule, keep using the previous schedule;
- Then the searching process keeps on running from the latest considered task position until all tasks are checked;
- Then search from the beginning task position of the schedule until no better schedules are obtained by swapping all pairs of tasks.

The insert process follows a similar pattern:

- Given an initial schedule, after calculating the risks, withdraw a task from an technician and allocate it to another technician;
- Then check the risks of this new schedule. If the maximum risk is smaller or the maximum risk is the same but the sum of all task risks is smaller, the new schedule can be regarded as a better solution and this move is performed. Otherwise, if it is not a better solution, keep using the previous schedule;
- Then the searching process keeps on from the latest considered task position until all tasks are checked;
- Then search from the beginning task position of the schedule until no better schedules are obtained by checking each task.

The process is shown in Fig. 12.

VI. COMPUTATIONAL EXPERIMENTS

A. Experiment settings

Our testbed is composed of 50 technicians and 200 tasks. The task information includes the task ID, the mean of task duration, the standard deviation of task duration, the time window earliest time and latest time, and the task location coordinates. The technician information includes the technician ID, start and end working times as well as technician start location.

The travel distance is the straight line distance between any two locations defined by their coordinates, combined with a correction factor 1.3, which means the real travel distance is 30% longer than the straight line distance. Then the travel time is calculated by the average speed of 40 km/h.

The searching process here is first the swap process, then the insert process, followed by swap process and insert process again. At the end the result schedule shows the allocation of tasks to technicians, the risk for each task, the maximum risk and the sum of the risks for all tasks.

All the computational experiments were carried out using java on a personal computer with a 64-bit windows 7 system, Intel Core i5-4210U 1.7GHz-2.4GHz CPU, 8G RAM.

B. Basic results

In the basic experiment, a specific time window is set for each task, and the same priority (or no priority) is assigned to all tasks. TABLE VI shows the initial schedule and the result schedule after the searching process. From the table, it is clear that the searching algorithm greatly improves the initial schedule. In the result schedule, the maximum risk is reasonably small and the sum of all risks is small which means the overall risk is small. Hence, we arrive at the conclusion that the searching method is effective.

TABLE VI. RISK COMPARISON OF THE INITIAL SCHEDULE AND THE RESULT SCHEDULE

	Initial schedule	Result schedule
Maximum risk	0.99991711	0.03039636
Sum of all risks	2.9903556	1.03314135
Running time (ms)	2233	

TABLE VII. TASK POSITION COMPARISON FOR TASKS WITH SPECIFIC TIME WINDOWS

Task priority	Final schedule	Final schedule (priority)
Maximum risk	0.03039636	0.03039636
Sum of risks	1.03314135	1.17532724
Running time (ms)	2233	3035

Task priority	Task position ^a	Task position (priority) ^b
1	2.45	2.93
5	2.642857143	2.314285714
10	2.433333333	1.8

^a. The average task position in its corresponding technician's tour by considering risks (minimising risks)

^b. The average task position in its corresponding technician's tour by considering task priorities in risks (minimizing weighted risks by task priority)

TABLE VIII. SEARCHING RESULTS (VALUE OF OBJECTIVE FUNCTIONS AND RUNNING TIME)

C. Tasks with priority

In real-world, tasks appear to have different importance or priority according to the business objectives. If a technician fails to start a high priority task then the penalty should be higher. For instance, an emergency task should have high priority, a task for an important customer should have high priority, the task with great influence should have high priority etc. In this experiment, a priority value is assigned to each task. For the same test sample, 200 tasks are considered, 30 of them are with high priority (the importance score is 10), 70 of them are with medium priority (the importance score is 5), and the others are with no priority (the importance score is 1).

Consequently, it is beneficial for the cost model to take into consideration the task priority. In order to reach this objective the risk penalty is multiplied by the task priority. In other words, the higher the priority the higher the risk cost should be.

The tasks are still with specific time windows. To avoid the high penalty cost, the high priority tasks tend to be scheduled early. This can be seen from TABLE VII which shows the average positions of tasks with different priorities in the task sequences of the corresponding technicians. For example, a task with an importance score 10 is at position 1 means this task is assigned to its technician as his 1st task. From TABLE VII it can be seen that the high priority tasks (the importance score is 10) are moved to an early position significantly. As a result of minimising the penalty for high priority tasks, the risk of those with low priorities will increase. Therefore, the sum of risks are a bit higher than in the case where all tasks have the same priority, as shown in TABLE VIII.

D. Tasks with morning and afternoon time windows

It is very common in practice that the service company give only two time windows for customer to choose. In this case, the time window for any task is either morning or afternoon. With these two common time windows, the results for the tasks with and without priorities have similar relationship to the situation where tasks having specific time windows. The results of the positions of the morning and afternoon task are shown in TABLE IX. As shown in the table, the important tasks are all moved forward during the searching process when the priority scores for tasks are considered.

Comparing the results for the tasks with specific time windows and those for the tasks with two larger time windows (morning and afternoon), it can be seen that the larger time window makes the risk smaller.

TABLE IX. TASK POSITION COMPARISON FOR TASKS WITH MORNING AND AFTERNOON TIME WINDOWS

Task priority	Task position ^a	Task position (priority) ^b
1	2.42	2.73
Morning	1.5	1.88
Afternoon	3.34	3.58
5	2.628571429	2.257142857
Morning	1.828571429	1.342857143
Afternoon	3.428571429	3.171428571
10	2.5	2.4
Morning	1.466666667	1.4
Afternoon	3.533333333	3.4

^a. The average task position in its corresponding technician's tour by considering risks (minimising risks)

TABLE X. SEARCHING RESULTS FOR TASKS WITH MORNING AND AFTERNOON TIME WINDOWS

VII. CONCLUSION

This research discussed the stochastic vehicle routing problem with time windows from a new perspective, which is to minimise the risks given a certain amount of resources. In this paper, risks for tasks with time windows were successfully modelled and different risk models were proposed. This model was applied to a real world problem in the telecommunication sector. Results have shown that the schedule generated is more robust minimising the risk of failure in general and pushing important tasks earlier in the schedule to avoid their failure.

In a dynamic environment, risk assessment tool or risk based scheduling engine, the risk calculation needs to be fast but also realistic. This paper first focused on the accuracy of the risk distribution function but the resulting running time was not acceptable for the optimisation search process. Therefore, as a second step, it was investigated whether task duration could be modelled as normal distributions according to historical data. Fortunately, this was the case and it enabled us to reduce significantly the risk calculation processing time allowing the search process to generate schedules in an acceptable time.

As for the future work, the task duration which follows gamma or other distributions will be considered. Investigation may also be extended to task durations expressed as the

combination of different distributions. Furthermore, it would be interesting to apply the risk model in a dynamic environment, where the risk model will dynamically update the risks according to external events such as task progression or completion, task rejection, task actual delays etc. At the same time, risk model will be used to help simulation or scheduling engines to make better decision when new task arrives or when risk becomes too high to suggest reallocation of work to less

Task priority	Final schedule	Final schedule (priority)
Maximum risk	0.00080612	0.00086320
Sum of risks	0.01188102	0.01468723
Running time (ms)	3154	3729

risky position. In addition, heuristic methods will be studied to improve the searching process.

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