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Robust Cooperative Adaptive Cruise Control of Vehicles on Banked and Curved Roads with Sensor Bias

Jianglin Lan¹, Dezong Zhao¹ and Daxin Tian²

Abstract—This paper considers cooperative adaptive cruise control (CACC) for automated vehicles on banked and curved roads. In such context, the existing longitudinal control alone cannot guarantee robustly stable CACC, because the vehicle longitudinal dynamics are coupled with the lateral dynamics. This gives rise to the necessity of incorporating the lane keeping (LK) control with CACC for the following vehicle to guarantee lateral stability and adaptive cruise. A robustly independent design strategy is proposed for determining a linear parameter varying (LPV) LK controller and a constant gain CACC controller. The influence of sensor bias and noise is also considered and a linear time-varying generalized augmented state observer (LTV-GASO) is developed to obtain optimal state estimation of the CACC and LK systems. Designs of the observers and controllers for CACC and LK systems follow a similar scheme with an easily solved linear optimization problem formulation. This facilitates the overall vehicle control system design and implementation. Simulation of a platoon in actual traffic environment illustrates efficacy of the proposed designs in achieving robustly stable vehicle following and lateral stability.

I. INTRODUCTION

Cooperative adaptive cruise control (CACC) based on vehicle-to-vehicle communication has gained its popularity in improving traffic throughput, safety and energy efficiency [1]. The goal of CACC is to control the following vehicles (followers) to track the velocity of the leading vehicle (leader) whilst maintaining a safe relative position. When the vehicles run on straight and flat lanes, CACC can be realized via controlling only the longitudinal dynamics of the followers, which has been studied extensively in the literature using various approaches as reviewed in [2], [3]. This paper is dedicated to CACC under more realistic road conditions with unknown bank angle and curvature.

Nonlinear coupling exists between longitudinal and lateral dynamics of vehicles [4]–[6]. To handle this, the lane keeping (LK) control is combined with CACC for the follower to ensure lateral stability and vehicle following [7]. The lateral stability is affected by road conditions, so is the longitudinal control. Hence, the combination of LK and CACC becomes more important when considering the influence of road bank and curvature. The combined design of LK and CACC for

vehicles on curved roads is studied in [8] without considering road bank. This paper deals with the integrated design of LK and CACC for vehicles on banked and curved roads.

The simultaneous design of CACC and LK controllers is difficult due to their nonlinear coupling. Therefore, this paper proposes an independent design strategy where the two controllers are determined separately and robust against their coupling effects. A further challenge in designing LK controller is that the lateral dynamics are nonlinear functions of the longitudinal velocity. To facilitate the design, a linear lateral model is obtained by using feedback linearization and regarding the longitudinal velocity as a disturbance [8]. However, this LK controller is designed to be stable under the maximum longitudinal velocity, which introduces design conservativeness. To reduce the conservativeness, the lateral dynamics can be approximated by a convex combination of the linear models associated with the minimum and maximum longitudinal velocities. In this way, LK can be realized using a linear parameter varying (LPV) controller [9] or a fuzzy static output feedback controller [10]. However, road bank is not considered in [9], [10].

The existing controllers usually use full vehicle state to improve the CACC and LK performance. Although all the longitudinal state (relative position and velocity) are usually measured, only partial lateral state are available in practice. Moreover, the sensor measurements may have bias and noise. Therefore, it is necessary to use an observer to estimate the vehicle state. In [11], a nonlinear high-gain sliding mode observer is used to estimate the lateral state and road curvature. In [12], a Kalman observer is adopted to estimate the lateral state, and a Luenberger observer is proposed to estimate the road bank and grade angles. In [13], a Kalman filter is used to estimate the lateral state, road bank angle and sensor bias. However, the above observers are designed under a constant longitudinal velocity. To deal with time-varying longitudinal velocity, LPV observer [9] and fuzzy observer [14] are developed by using the approximated lateral models. However, the influence of road bank, sensor bias and noise are not considered in their designs.

Contributions of this paper are summarized as follows:

(i) A robust LPV controller is developed to ensure stable LK against variations in longitudinal velocity and road conditions based on the H_∞ optimization theory. Physical constraints of the steering angle are also considered. The controller gains are obtained from solving an optimization problem with linear matrix inequality (LMI) constraints.

(ii) A linear time-varying generalized augmented state observer (LTV-GASO) is developed for obtaining optimal es-

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timization of the lateral state. It directly uses the time-varying lateral model and can handle the influence of longitudinal velocity, road condition changes, sensor bias and noise.

(iii) Design of the observer and controller to achieve stable CACC is formulated as a special case of that of LK, which facilitates the overall vehicle control system design. The CACC controller combines feedforward of the leader acceleration and feedback of the relative position and velocity. It also considers physical constraints of the longitudinal force input. The LTV-GASO is applied to estimate the relative position and velocity under sensor bias and measurement noise.

The rest of this paper is organized as follows. Section II describes the problem of integrating CACC and LK on banked and curved roads with sensor bias. Section III presents the LK design. Section IV presents the CACC design. Section V provides simulation study of a two-vehicle CACC system. Section VI draws conclusions.

II. PROBLEM DESCRIPTION

For simplicity and clarity, the following two-vehicle CACC system is investigated [5]:

$$\begin{aligned}\dot{\chi} &= \mathcal{A}\chi + \mathcal{B}\varphi + \mathcal{E}\bar{\rho} \\ \Upsilon &= \mathcal{C}\chi + \mathcal{F}\bar{f} + \bar{w}\end{aligned}\quad (1)$$

with

$$\mathcal{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 0 \\ -\frac{1}{m} \end{bmatrix}, \mathcal{E} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathcal{C} = I_2, \mathcal{F} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

where $\chi = [e_p - d_{\text{safe}} \ v_l - v]^\top$ is the state, φ is the control input, $\bar{\rho} = a_l + \bar{d}$ is the disturbance, Υ is the measured state, $\bar{f}$ is the sensor bias, and $\bar{w}$ is the noise. The variable e_p is the relative position between two vehicles. d_{safe} is a given constant representing the safe relative position. v_l is the longitudinal velocity of the leader. v is longitudinal velocity of the follower. φ is the longitudinal force input of the follower. a_l is the leader acceleration transmitted to the follower through vehicle-to-vehicle communication. $\bar{d} = F_{\text{drag}}/m - \dot{y}\dot{\psi}$ is the unknown disturbance acting on the follower, where F_{drag} is the aerodynamic drag, m is the vehicle mass, $\dot{y}$ is the lateral velocity and $\dot{\psi}$ is the yaw rate. $\bar{w} = [\bar{w}_1 \ \bar{w}_2]^\top$, where $\bar{w}_1$ and $\bar{w}_2$ are zero mean white noises with known covariances $Q_{\bar{w}_1}$ and $Q_{\bar{w}_2}$, respectively. Hence, the covariance of $\bar{w}$ is given as $Q_{\bar{w}} = \text{diag}\{Q_{\bar{w}_1}, Q_{\bar{w}_2}\}$.

The LK system of the follower can be represented by [15]:

$$\begin{aligned}\dot{x} &= A(v)x + B_c u + E(v)\rho + D_c d \\ y &= Cx + Ff + w\end{aligned}\quad (2)$$

with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2(C_f + C_r)}{mv} & \frac{2(C_f l_r + C_r l_f)}{m} & \frac{2(C_r l_r - C_f l_f)}{mv} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2(C_r l_r - C_f l_f)}{I_z v} & \frac{2(C_f l_f - C_r l_r)}{I_z} & -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v} \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0 \\ \frac{2C_f}{m} \\ 0 \\ \frac{2C_f l_f}{I_z} \end{bmatrix}, E = \begin{bmatrix} 0 \\ \frac{2(C_r l_r - C_f l_f)}{mv} - v \\ 0 \\ -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v} \end{bmatrix}, D_c = \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & d_s & 0 \end{bmatrix}, F = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

where $x = [e_y \ \dot{e}_y \ e_\psi \ \dot{e}_\psi]^\top$ is the state, $u = \delta$ is the control input, $\rho = \psi_{\text{des}}$ and $d = \sin(\theta)$ are the unknown inputs, $y = [\dot{e}_\psi \ e_y + d_s e_\psi]^\top$ is the output measurement, f is the sensor bias, and w is the noise. The variables e_y and e_ψ are lateral displacement and yaw angle error with respect to the road, respectively. δ is the front wheel steering angle. $\dot{\psi}_{\text{des}} = v/R$ is the desired yaw rate calculated from v and the road radius R . θ is the road bank angle. $w = [w_1 \ w_2]^\top$, where w_1 and w_2 are zero mean white noises with known covariances Q_{w_1} and Q_{w_2} , respectively. Hence, the covariance of w is given as $Q_w = \text{diag}\{Q_{w_1}, Q_{w_2}\}$. C_f and C_r are the cornering stiffness of the front and rear tires, respectively. l_f and l_r are the distances from the center of gravity of the vehicle to the front and rear tires, respectively. I_z is the yaw moment of inertia. g is the gravity acceleration. d_s is the longitudinal distance between the vehicle front and the point where the sensor measurement is made.

It is seen that the CACC system is affected by the lateral dynamics of the follower (i.e. the term $\dot{y}\dot{\psi}$), which are affected by the longitudinal velocity v . Hence, there are nonlinear coupling between the CACC and LK systems, which should be considered in the design of controllers. Moreover, the CACC system is perturbed by the leader acceleration and unknown aerodynamic drag, while the LK system is perturbed by unknown road bank and curvature. These perturbations should also be considered in the design.

This paper aims to propose a robustly independent design strategy for determining the CACC and LK controllers for the follower. The controller φ is designed to stabilize the CACC system with robustness against the influence from lateral dynamics, aerodynamic drag and leader acceleration. The controller u is designed to stabilize the LK system with robustness against the influence from longitudinal velocity, road bank and curvature. Due to the existing sensor bias and noise, observers will also be developed to estimate the state of CACC and LK systems.

III. LANE KEEPING DESIGN

A state feedback LPV controller for realizing LK is designed in Section III-A. As seen in (2), the state x are not completely measured and the output measurement y is corrupted by sensor bias f and noise w . Therefore, an observer is developed in Section III-B for obtaining accurate estimation of x , which is to be used by the controller.

A. LK Controller

To remove the steady-state offset of lateral displacement e_y , the error integration $\int e_y dt$ will be used in the proposed controller. Define the augmented state as $\tilde{x} = [x^\top \ \int e_y dt]^\top$,

then the system (2) is augmented as

$$\dot{\tilde{x}} = \tilde{A}(v)\tilde{x} + \tilde{B}_c u + \tilde{D}(v)\tilde{d} \quad (3)$$

where

$$\tilde{A}(v) = \begin{bmatrix} A(v) & 0 \\ \Lambda & 0 \end{bmatrix}, \quad \Lambda = [1 \ 0 \ 0 \ 0], \quad \tilde{B}_c = \begin{bmatrix} B_c \\ 0 \end{bmatrix},$$

$$\tilde{D}(v) = \begin{bmatrix} 0 & 0 \\ \frac{2(C_r l_r - C_f l_f)}{mv} & g \\ 0 & 0 \\ -\frac{2(C_f l_f^2 + C_r l_r^2)}{I_z v} & 0 \end{bmatrix}, \quad \tilde{d} = \begin{bmatrix} \rho \\ -v\rho/g + d \end{bmatrix}.$$

Remark 3.1: By defining the lumped disturbance $\tilde{d}$, the system matrices in (3) depend on a single time-varying parameter v . This will allow us to reformulate (3) as an LPV system (4) with a single scheduling parameter. Therefore, the proposed LK design is less complex than the existing designs with two [9] or three [10] scheduling parameters.

In practice, the longitudinal velocity satisfies $v \in [v_{\min}, v_{\max}]$, where $v_{\min}$ and $v_{\max}$ are positive constants representing the minimum and maximum velocities, respectively. Note that the case of $v = 0$ is not considered in this paper. Define $\xi = 1/v$, then $\xi \in [\xi_{\min}, \xi_{\max}]$, where $\xi_{\min} = 1/v_{\max}$ and $\xi_{\max} = 1/v_{\min}$. Hence, the system (3) can be discretized using Euler's forward method [16] with sampling time t_s and rewritten into the following LPV form:

$$\begin{aligned} \tilde{x}_{k+1} &= \tilde{A}_k \tilde{x}_k + \tilde{B} u_k + \tilde{D}_k \tilde{d}_k \\ z_k &= \tilde{C} \tilde{x}_k + \tilde{B}_2 u_k \end{aligned} \quad (4)$$

with

$$\begin{aligned} \tilde{A}_k &= \sum_{i=1}^2 \alpha_i \tilde{A}_i, \quad \tilde{B} = t_s \tilde{B}_c, \quad \tilde{D}_k = \sum_{i=1}^2 \alpha_i \tilde{D}_i, \\ \alpha_1 &= \frac{\xi_k - \xi_{\min}}{\xi_{\max} - \xi_{\min}}, \quad \alpha_2 = \frac{\xi_{\max} - \xi_k}{\xi_{\max} - \xi_{\min}}, \\ \tilde{A}_1 &= I_5 + t_s A(\xi_{\min}), \quad \tilde{A}_2 = I_5 + t_s A(\xi_{\max}), \\ \tilde{D}_1 &= t_s D(\xi_{\min}), \quad \tilde{D}_2 = t_s D(\xi_{\max}). \end{aligned}$$

The subscript k denotes the sampling step, and z_k is the performance output with the given weights $\tilde{C}$ and $\tilde{B}_2$.

To stabilize the system (4), the controller is designed as

$$u_k = \sum_{i=1}^2 \alpha_i K_i \tilde{x}_k \quad (5)$$

where $K_i \in \mathbb{R}^{1 \times 5}$, $i = 1, 2$, are the gains determined from Theorem 3.1 by solving the LMIs (6) and (7). The obtained controller can ensure that the LK system (4) is robustly stable against the influence of longitudinal velocity and road conditions.

Theorem 3.1: The LK system (4) under the LPV controller (5) is poly-quadratically stable and satisfies the H_∞ performance $\sum_{k=0}^{\infty} \|z_k\|^2 < \gamma^2 \sum_{k=0}^{\infty} \|\tilde{d}_k\|^2$ and the input constraint $|u_k| \leq \delta_{\max}$, if there exist symmetric positive definite matrices $Q_i \in \mathbb{R}^{5 \times 5}$, matrices $X_i \in \mathbb{R}^{5 \times 5}$ and

$M_i \in \mathbb{R}^{1 \times 5}$, and a positive scalar γ such that for all $i = 1, 2$ and $j = 1, 2$,

$$\begin{bmatrix} X_i + X_i^\top - Q_i & \star & \star & \star \\ 0 & \gamma I & \star & \star \\ \tilde{A}_i X_i + \tilde{B} M_i & \tilde{D}_i & Q_j & \star \\ \tilde{C} X_i + \tilde{B}_2 M_i & 0 & 0 & \gamma I \end{bmatrix} > 0 \quad (6)$$

$$\begin{bmatrix} \delta_{\max}^2 I & M_i \\ M_i^\top & X_i + X_i^\top - Q_i \end{bmatrix} \geq 0 \quad (7)$$

where $\delta_{\max}$ is the maximum steering angle and the symbol $\star$ induces symmetry. Then the controller gains are obtained as $K_i = M_i X_i^{-1}$, $i = 1, 2$.

Proof: By using the results in [17], the LPV controller (5) is designed to ensure robustly poly-quadratic stability of the LK system (4) if the LMI (6) is satisfied. By using the results in [18], the input constraint is satisfied if the LMI (7) holds. The detailed proof can be found in [17], [18]. ■

B. Observer

For the observer design purpose, the discrete-time model of (2) is obtained with sampling time t_s and is given as

$$\begin{aligned} x_{k+1} &= A_k x_k + B u_k + E_k \rho_k + D d_k \\ y_k &= C x_k + F f_k + w_k \end{aligned} \quad (8)$$

where $A_k = I_4 + t_s A(v_k)$, $B = t_s B_c$, $E_k = t_s E(v_k)$ and $D = t_s D_c$. The matrices C and F are the same as in (2).

Assuming that the bank angle and sensor bias change slowly, then d_k and f_k can be modelled as the integrators $d_{k+1} = d_k$ and $f_{k+1} = f_k$, respectively. Defining d_k and f_k as auxiliary state variables, then (8) is augmented as

$$\begin{aligned} \bar{x}_{k+1} &= \bar{A}_k \bar{x}_k + \bar{B} u_k + \bar{E}_k \rho_k \\ y_k &= \bar{C} \bar{x}_k + w_k \end{aligned} \quad (9)$$

where

$$\begin{aligned} \bar{x}_k &= \begin{bmatrix} x_k \\ d_k \\ f_k \end{bmatrix}, \quad \bar{A}_k = \begin{bmatrix} A_k & D & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \\ 0 \end{bmatrix}, \\ \bar{E}_k &= \begin{bmatrix} E_k \\ 0 \\ 0 \end{bmatrix}, \quad \bar{C} = [C \ 0 \ F]. \end{aligned}$$

It can be verified that the system (9) is observable. Hence, the following linear time-varying generalized augmented state observer (LTV-GASO) is proposed to estimate the state $\bar{x}_k$:

$$\begin{aligned} z_{k+1} &= N_{k+1} z_k + G_{k+1} u_k + L_{k+1} y_k \\ \hat{\bar{x}}_k &= z_k + H_k y_k \end{aligned} \quad (10)$$

where z_k is the observer state and $\hat{\bar{x}}_k$ is the estimate of $\bar{x}_k$. The matrices N_{k+1} , G_{k+1} , L_{k+1} and H_k are to be designed.

Define the estimation error as $e_k = \bar{x}_k - \hat{\bar{x}}_k$. By using (10), it can be derived that $z_k = \bar{x}_k - H_k y_k - e_k$ and $e_k = \Phi_k \bar{x}_k - z_k - H_k w_k$ where $\Phi_k = I - H_k \bar{C}$. Hence, it follows from (9) - (10) that

$$\begin{aligned} e_{k+1} &= N_{k+1} e_k + (\Phi_{k+1} \bar{A}_k - N_{k+1} - L_{k+1}^1 \bar{C}) \bar{x}_k \\ &\quad + (\Phi_{k+1} \bar{B} - G_{k+1}) u_k + (N_{k+1} H_k - L_{k+1}^2) y_k \\ &\quad + \Phi_{k+1} \bar{E}_k \rho_k - L_{k+1}^1 w_k - H_{k+1} w_{k+1} \end{aligned} \quad (11)$$

where $L_{k+1} = L_{k+1}^1 + L_{k+1}^2$.

Define the following matrix equations:

$$\Phi_{k+1}\bar{A}_k - N_{k+1} - L_{k+1}^1\bar{C} = 0, \quad (12)$$

$$\Phi_{k+1}\bar{B} - G_{k+1} = 0, \quad (13)$$

$$N_{k+1}H_k - L_{k+1}^2 = 0, \quad (14)$$

$$\Phi_{k+1}\bar{E}_k = 0. \quad (15)$$

Applying (12) - (15) to (11) gives

$$\begin{aligned} e_{k+1} &= (\Phi_{k+1}\bar{A}_k - L_{k+1}^1\bar{C})e_k \\ &\quad - L_{k+1}^1w_k - H_{k+1}w_{k+1}. \end{aligned} \quad (16)$$

Using (15) makes the error dynamics (16) free from the disturbance term $\bar{E}_k\rho_k$. There exists a matrix H_{k+1} satisfying (15) if and only if the following rank condition holds:

$$\text{rank}(\bar{C}\bar{E}_k) = \text{rank}(\bar{E}_k) = r. \quad (17)$$

According to the system matrices given in (9), it can be verified that (17) always holds with non-zero longitudinal velocity. Hence, (15) is feasible and has a solution

$$H_{k+1} = \bar{E}_k(\bar{C}\bar{E}_k)^\dagger \quad (18)$$

where the superscript $\dagger$ denotes pseudo inverse.

By applying (18), the only unknown matrix in (16) is L_{k+1}^1 . Once L_{k+1}^1 is designed, the matrices N_{k+1} , G_{k+1} and L_{k+1}^2 can be derived from the equations (12) - (14). In this case, all the observer gains are obtained. Therefore, the main task of the observer design is to determine L_{k+1}^1 such that the error dynamics (16) are stable with minimum error variance. This is accomplished using the following theorem.

Theorem 3.2: The estimation error has minimum variance if the matrix L_{k+1}^1 is designed as

$$L_{k+1}^1 = \Phi_{k+1}\bar{A}_kP_k\bar{C}^\top(\bar{C}P_k\bar{C}^\top + Q_w)^{-1}. \quad (19)$$

Proof: Define the error covariance matrix as $P_k = \mathcal{E}\{e_k e_k^\top\}$, where $\mathcal{E}(\cdot)$ denotes the expectation. By using (16), it can be derived that

$$\begin{aligned} P_{k+1} &= \mathcal{E}\{e_{k+1}e_{k+1}^\top\} \\ &= (\Phi_{k+1}\bar{A}_k - L_{k+1}^1\bar{C})P_k(\Phi_{k+1}\bar{A}_k - L_{k+1}^1\bar{C})^\top \\ &\quad + L_{k+1}^1Q_w(L_{k+1}^1)^\top + H_{k+1}Q_wH_{k+1}^\top. \end{aligned} \quad (20)$$

The equation (20) can be rewritten as

$$\begin{aligned} P_{k+1} &= \Phi_{k+1}\bar{A}_kP_k\bar{A}_k^\top\Phi_{k+1}^\top + H_{k+1}Q_wH_{k+1}^\top \\ &\quad - (L_{k+1}^1\bar{C}P_k\bar{A}_k^\top\Phi_{k+1}^\top)^\top - L_{k+1}^1\bar{C}P_k\bar{A}_k^\top\Phi_{k+1}^\top \\ &\quad + L_{k+1}^1(\bar{C}P_k\bar{C}^\top + Q_w)(L_{k+1}^1)^\top. \end{aligned} \quad (21)$$

The matrix Q_w is positive definite, so is $(\bar{C}P_k\bar{C}^\top + Q_w)$. Hence, there exists an invertible matrix S satisfying

$$SS^\top = \bar{C}P_k\bar{C}^\top + Q_w. \quad (22)$$

Let $W = \Phi_{k+1}\bar{A}_kP_k\bar{C}^\top(S^{-1})^\top$. By using (22), the equation (21) can be rearranged as

$$\begin{aligned} P_{k+1} &= \Phi_{k+1}\bar{A}_kP_k\bar{A}_k^\top\Phi_{k+1}^\top + H_{k+1}Q_wH_{k+1}^\top \\ &\quad + (L_{k+1}^1S - W)(L_{k+1}^1S - W)^\top - WW^\top. \end{aligned} \quad (23)$$

It can be observed from (23) that the estimation error variance $\text{var}\{e_{k+1}\} = \text{trace}\{P_{k+1}\}$ is minimized if L_{k+1}^1 satisfies $L_{k+1}^1S - W = 0$, which gives (19). By applying (19) and the Woodbury matrix identity to (23), it yields

$$\begin{aligned} P_{k+1} &= \Phi_{k+1}\bar{A}_k(P_k^{-1} + \bar{C}^\top Q_w^{-1}\bar{C})^{-1}\bar{A}_k^\top\Phi_{k+1}^\top \\ &\quad + H_{k+1}Q_wH_{k+1}^\top. \end{aligned} \quad (24)$$

This completes the proof. $\blacksquare$

Based on the above analysis, the observer design procedure is summarized below:

Algorithm 1 LTV-GASO design procedure

- 1: Set initial values: $P_0 = P_{\text{initial}}$, $z_0 = \bar{x}_0 - \bar{E}_0(\bar{C}\bar{E}_0)^\dagger y_0$, $H_0 = 0$ and $k = 0$.
 - 2: Compute H_{k+1} and Φ_{k+1} using (18) and $\Phi_{k+1} = I - H_{k+1}\bar{C}$, respectively.
 - 3: Compute L_{k+1}^1 using (19).
 - 4: Compute N_{k+1} , G_{k+1} and L_{k+1}^2 using (12) - (14), and compute L_{k+1} using $L_{k+1} = L_{k+1}^1 + L_{k+1}^2$.
 - 5: Compute P_{k+1} using (24).
 - 6: Set $k = k + 1$ and go to step 2.
-

IV. COOPERATIVE ADAPTIVE CRUISE CONTROL DESIGN

The goal of CACC is to stabilize the relative dynamics (1) in the presence of known disturbance a_l and unknown disturbance $\bar{d}$. To remove the steady-state offset of relative position, the integration $\int(e_p - d_{\text{safe}})dt$ is to be used in the CACC controller. Define the new state vector as $\bar{\chi} = [\chi^\top \int(e_p - d_{\text{safe}})dt]^\top$, then by using (1), the control-oriented discrete-time CACC model is derived as

$$\bar{\chi}_{k+1} = \bar{A}_d\bar{\chi}_k + \bar{B}_d\varphi_k + \bar{E}_d\bar{\rho}_k \quad (25)$$

where $\bar{A}_d = I_3 + t_s \begin{bmatrix} \mathcal{A} & 0 \\ \Gamma & 0 \end{bmatrix}$, $\Gamma = [1 \ 0]$, $\bar{B}_d = t_s \begin{bmatrix} \mathcal{B} \\ 0 \end{bmatrix}$, and $\bar{E}_d = t_s \begin{bmatrix} \mathcal{E} \\ 0 \end{bmatrix}$.

Since the leader acceleration $a_{l,k}$ is known to the follower, the CACC controller can be designed as

$$\varphi_k = u_k^x + u_k^d \quad (26)$$

where $u_k^d = ma_{l,k}$ is the feedforward action to compensate the leader acceleration. $u_k^x = K_x\bar{\chi}_k$ is the state feedback action to ensure that the closed-loop system (25) is strictly stable and satisfies the H_∞ performance

$$\sum_{k=0}^{\infty} \|Z_k\|^2 < \bar{\gamma}^2 \sum_{k=0}^{\infty} \|\bar{d}_k\|^2 \quad (27)$$

where $Z_k = \mathcal{C}_z\bar{\chi}_k + \mathcal{B}_z u_k^x$ is the performance output with the given weights $\mathcal{C}_z$ and $\mathcal{B}_z$, and $\bar{\gamma}$ is a positive scalar. If (27) holds, then the system (25) is stable for any bounded disturbance $\bar{d}_k$, which also implies the input-to-state string stability of the vehicle platoon [19].

The gain K_x is determined using the following theorem.

Theorem 4.1: The CACC system (25) under the controller (26) is strictly stable and satisfies the H_∞ performance

(27) and the input constraint $|u_k^x| \leq \bar{\varphi}_{\max}$, if there exists a symmetric positive definite matrix $\mathcal{P} \in \mathbb{R}^{3 \times 3}$, matrices $\mathcal{X} \in \mathbb{R}^{3 \times 3}$ and $\mathcal{M} \in \mathbb{R}^{1 \times 3}$, and a positive scalar $\bar{\gamma}$ such that

$$\begin{bmatrix} \mathcal{X} + \mathcal{X}^\top - \mathcal{P} & \star & \star & \star \\ 0 & \bar{\gamma}I & \star & \star \\ \bar{\mathcal{A}}_d \mathcal{X} + \bar{\mathcal{B}}_d \mathcal{M} & \bar{\mathcal{E}}_d & \mathcal{P} & \star \\ \mathcal{C}_z \mathcal{X} + \mathcal{B}_z \mathcal{M} & 0 & 0 & \bar{\gamma}I \end{bmatrix} > 0 \quad (28)$$

$$\begin{bmatrix} \bar{\varphi}_{\max} I & \star \\ \mathcal{M}^\top & \mathcal{X} + \mathcal{X}^\top - \mathcal{P} \end{bmatrix} \geq 0 \quad (29)$$

where $\bar{\varphi}_{\max}$ is the maximum value of u_k^x and is defined as $\bar{\varphi}_{\max} = \varphi_{\max} - a_{\max}$, with the maximum longitudinal force $\varphi_{\max}$ and the maximum leader acceleration $a_{\max}$. Then the controller gain is obtained as $K_x = \mathcal{M}\mathcal{X}^{-1}$.

Proof: The presented result is the special case of Theorem 3.1 when the system is time-invariant. Hence, the proof follows directly from that of Theorem 3.1 by setting the scheduling number to one. ■

Due the existence of sensor bias and noise, an observer is designed to obtain accurate estimation of the state χ_k and used by the controller (26). To this end, the discrete-time model of (1) is derived as

$$\begin{aligned} \chi_{k+1} &= \mathcal{A}_d \chi_k + \mathcal{B}_d \varphi_k + \mathcal{E}_d \bar{\rho}_k \\ \Upsilon_k &= \mathcal{C} \chi_k + \mathcal{F} \bar{f}_k + \bar{w}_k \end{aligned} \quad (30)$$

where $\mathcal{A}_d = I_2 + t_s \mathcal{A}$, $\mathcal{B}_d = t_s \mathcal{B}$ and $\mathcal{E}_d = t_s \mathcal{E}$. The matrices $\mathcal{A}$, $\mathcal{B}$, $\mathcal{E}$, $\mathcal{C}$ and $\mathcal{F}$ are defined in (1).

Since the system (30) is in the same form as (8) by setting $Dd_k = 0$, the state χ_k (and thus the augmented state $\bar{\chi}_k$) can be estimated using the LTV-GASO in Section III-B.

V. CASE STUDY

A two-vehicle CACC system is simulated with the parameters same as in [5], which are listed in Table I.

TABLE I: Vehicle parameters

Parameter	Value	Parameter	Value
m	1650 kg	$v_{\min}$	2 m/s
I_z	2315.3 kg · m ²	$v_{\max}$	30 m/s
l_f	1.11 m	$\delta_{\max}$	10 deg
l_r	1.59 m	d_{safe}	10 m
C_f	66500 N	$a_{\max}$	3 m/s ²
C_r	49400 N	$\varphi_{\max}$	3000 N
F_{drag}	$51 + 1.26v + 0.4342v^2$ N	g	9.8 m/s ²
d_s	2 m	t_s	0.01 s

The LK controller gains K_i , $i = 1, 2$, are determined by solving the LMIs in Theorem 3.1 with $\bar{C} = [0.1 \times I_5; 0_{1 \times 5}]$ and $\bar{B}_2 = [0_{5 \times 1}; 1]$ and are given as follows:

$$\begin{aligned} K_1 &= [-0.1706 \quad -0.0612 \quad -1.147 \quad -0.0535 \quad -0.063], \\ K_2 &= [-0.4077 \quad -0.0124 \quad -1.2041 \quad -0.0167 \quad -0.1398]. \end{aligned}$$

The CACC controller gain K_x is determined by solving the LMIs in Theorem 4.1 with $\mathcal{C}_z = [20000 \times I_3; 0_{1 \times 3}]$ and $\mathcal{B}_z = [0_{3 \times 1}; 1]$ and are given as $K_x = [45363 \quad 26004 \quad 25224]$.

The CACC system is simulated with the initial conditions $\chi(0) = 0$, $P_{\text{initial}} = 10^{-3} \times I_3$ and $Q_{\bar{w}} = 10^{-6} \times I_2$. The LK

Fig. 1: Profiles of leader velocity v_l , leader acceleration a_l , road curvature $1/R$, and road bank angle θ .

Fig. 2: LTV-GASO state estimation performance.

system is simulated with initial conditions $x(0) = [0.2 \ 0 \ 0 \ 0]$, $P_{\text{initial}} = 10^{-3} \times I_6$ and $Q_w = 10^{-6} \times I_2$. The sensor biases $\bar{f} = 0.1$ m/s and $f = 0.05$ deg/s are tested.

To simulate the platoon in actual traffic environment, the Highway Fuel Economy Test (HWFET), which is the standard highway operation of a vehicle, is adopted to generate the velocity v_l within [2 m/s, 30 m/s] and the acceleration a_l of the leader, as shown in Fig. 1. To validate the platoon on banked and curved roads, the profiles of time-varying road curvature $1/R$ and bank angle θ given in Fig. 1 are considered.

The simulation results are depicted in Figs. 2 - 4. It is seen from Fig. 2 that the proposed LTV-GASO achieves good estimation of the CACC and LK systems state with negligible errors, in the presence of time-varying road conditions, sensor biases and noise. The top plot in Fig. 3 shows the time response of the relative position error $e_p - d_{\text{safe}}$. It is seen that the relative position e_p is controlled to be the safe value d_{safe} with negligible error. It can also be observed from

Fig. 3: Time response of relative position error $e_p - d_{safe}$ and lateral displacement e_y .

Fig. 4: Longitudinal force input φ and steering angle δ .

the bottom plot in Fig. 3 that the lateral displacement e_y is controlled to zero on flat roads. When curvatures occur, there are small lateral displacements with magnitude within $(-0.2 \text{ m}, 0.2 \text{ m})$. Therefore, the proposed controllers realize robustly stable CACC and LK on banked and curved road. Moreover, the associated longitudinal force input φ and steering angle δ are both within the given limits, as can be seen from Fig. 4. The above results demonstrate efficacy of the proposed observer-based CACC and LK designs.

VI. CONCLUSION

This paper proposes a strategy for achieving robust CACC and LK of automated vehicles on banked and curved roads. The constant gain CACC and LPV LK controllers are designed independently but following a similar scheme with a linear optimization problem formulation. To handle sensor bias and noise, a LTV-GASO observer is further developed to obtain optimal state estimation of the CACC and LK systems. The case study shows that the proposed strategy can

achieve robustly stable vehicle following and lateral stability, in the presence of leader acceleration disturbance, banked and curved roads, sensor bias and noise. The basic idea of the proposed design strategy is illustrated using the two-vehicle CACC system, and it is directly applicable for platooning multiple vehicles.

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