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Post-local buckling-driven delamination in bilayer composite beams

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Post-Local Buckling Driven Delamination in Bilayer Composite Beams

Simon Wang, Christopher Harvey, Bin Wang, Andrew Watson

s.wang@lboro.ac.uk, c.m.harvey@lboro.ac.uk

Department of Aeronautical & Automotive Engineering
Loughborough University, LE11 3TU UK

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Motivation and Aim

- Problem definition

Fig. 1. A post-local buckled bilayer composite beam due to delamination under compression.

Motivation and Aim

- Problem definition

Fig. 2. Delamination tip internal forces and moments.

Motivation and Aim

- Delamination energy release rates (ERRs)

$$G = \frac{1}{2b^2} \left(\frac{M_{1B}^2}{D_1^*} + \frac{M_{2B}^2}{D_2^*} - \frac{M_B^2}{D^*} + \frac{N_{1B}^2}{A_1^*} + \frac{N_{2B}^2}{A_2^*} - \frac{N_B^2}{A^*} - \frac{2B_1 M_{1B} N_{1B}}{B_1^*} - \frac{2B_2 M_{2B} N_{2B}}{B_2^*} + \frac{2B M_B N_B}{B^*} \right)$$

$$= \{M_{1B}, M_{2B}, N_{1B}, N_{2B}\} [C] \{M_{1B}, M_{2B}, N_{1B}, N_{2B}\}$$

- ERR partitions GI and GII

Motivation and Aim

- Euler beam partitions:

$$G_{IE} = c_{IE} \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_1}{\beta_2} - \frac{N_2}{\beta_3} \right) \left(M_{1B} - \frac{M_{2B}}{\beta_{1'}} - \frac{N_1}{\beta_{2'}} - \frac{N_2}{\beta_{3'}} \right)$$

$$G_{IIE} = c_{IIE} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_1}{\theta_2} - \frac{N_2}{\theta_3} \right) \left(M_{1B} - \frac{M_{2B}}{\theta_{1'}} - \frac{N_1}{\theta_{2'}} - \frac{N_2}{\theta_{3'}} \right)$$

- Timoshenko beam partitions:

$$G_{IT} = c_{IT} \left(M_{1B} - \frac{M_{2B}}{\beta_1} - \frac{N_1}{\beta_2} - \frac{N_2}{\beta_3} \right)^2 \quad G_{IIT} = c_{IIT} \left(M_{1B} - \frac{M_{2B}}{\theta_1} - \frac{N_1}{\theta_2} - \frac{N_2}{\theta_3} \right)^2$$

- 2D elasticity partitions:

$$G_{I2D} = c_{I2D} \left(M_{1B} - \frac{M_{2B}}{\beta_{12D}} - \frac{N_1}{\beta_{22D}} - \frac{N_2}{\beta_{32D}} \right)^2 \quad G_{II2D} = c_{II2D} \left(M_{1B} - \frac{M_{2B}}{\theta_{12D}} - \frac{N_1}{\theta_{22D}} - \frac{N_2}{\theta_{32D}} \right)^2$$

Motivation and Aim

- Motivation and Aim:

Extensive experimental tests [1,2] have validated that Euler beam partitions give very accurate predictions of fracture toughness. The present work aims to validate if it still gives accurate predictions for post-local buckling driven delamination.

[1] Harvey CM, Wang S. Experimental assessments of mixed-mode partition theories. *Composite Structures* 94 (2012), 2057-67.

[2] Harvey CM, Eplett MR, Wang S. Experimental assessments of mixed-mode partition theories for generally laminated composite beams. *Composite Structures* 124 (2015), 10-18.

Analytical Development

- Post-local buckling mode shape

$$V_1(x_1) = A \frac{1}{2} \left(\cos \left(\frac{2\alpha\pi x_1}{a} \right) - \cos(\alpha\pi) \right)$$

- Critical local buckling strain

$$\varepsilon_c = \frac{(\alpha\pi)^2}{3} \left(\frac{h_1}{a} \right)^2$$

- Amplitude A

$$(1 - \varepsilon_c) a / 2 = \int ds = \int_0^{(1-\varepsilon_c)a/2} \left[1 + (dV_1 / dx_1)^2 \right]^{1/2} dx_1 \approx \int_0^{a/2} \left[1 + (dV_1 / dx_1)^2 \right]^{1/2} dx_1$$

$$A = 1.052 \varepsilon_a^{1/2} 2a / (\alpha \pi)$$

Analytical Development

Post-local buckling total ERR G

$$G = \frac{1}{2E_1 b} \left[\frac{M_{1B}^2}{I_1} + \frac{N_{1B}^2}{A_1} + \frac{N_{2B}^2}{A_{2e}} - \frac{N_{3B}^2}{A_{3e}} \right] \quad G = \frac{1}{2} E_1 h_1 \varepsilon_a (4c_\alpha^2 \varepsilon_c + \lambda \varepsilon_a)$$

Euler beam partitions

$$G_{IE} = E_1 h_1 c_\alpha \varepsilon_c^{1/2} \varepsilon_a (2c_\alpha \varepsilon_c^{1/2} - \sqrt{3} \lambda \varepsilon_a^{1/2}) \quad G_{IIE} = E_1 h_1 \lambda \varepsilon_a^{3/2} (\sqrt{3} c_\alpha \varepsilon_c^{1/2} + 1/2 \varepsilon_a^{1/2})$$

Timoshenko beam partitions

$$G_{IT} = \frac{1}{2(1+3\lambda)} E_1 h_1 \varepsilon_a (2c_\alpha \varepsilon_c^{1/2} - \sqrt{3} \lambda \varepsilon_a^{1/2})^2 \quad G_{IIT} = \frac{\lambda}{2(1+3\lambda)} E_1 h_1 \varepsilon_a (2\sqrt{3} c_\alpha \varepsilon_c^{1/2} + \varepsilon_a^{1/2})^2$$

2D elasticity partitions

$$G_{I2D} = \frac{12 + 2.697^2 \lambda}{6(4.450 + 2.697 \lambda)^2} E_1 h_1 \varepsilon_a (4.450 c_\alpha \varepsilon_c^{1/2} - \sqrt{3} \lambda \varepsilon_a^{1/2})^2$$

$$G_{II2D} = \frac{(12\lambda + 4.450^2) \lambda}{6(4.450 + 2.697 \lambda)^2} E_1 h_1 \varepsilon_a (2.697 c_\alpha \varepsilon_c^{1/2} + \sqrt{3} \varepsilon_a^{1/2})^2$$

Experimental Validation

- The experimental data from the work

[8] Kutlu Z, Zhang KK. Composite panels containing multiple through-the –width delaminations and subjected to compression Part 1: Analysis. Composite Structures 31 (1995), 273-96.

[9] Kutlu Z, Zhang KK. Composite panels containing multiple through-the –width delaminations and subjected to compression Part 2: Experiments and verification. Composite Structures 31 (1995), 297-314.

Experimental Validation

- Table 1. Configurations of two composite panels containing a single through-the-width delamination.

Cases	Lay-ups	a (mm)	h_1 (mm)	Thickness h (mm)
1	[0 ₄ /0 ₁₂ //0 ₄]	38.1	0.518	2.59
2	[0 ₄ /0 ₁₂ //0 ₄]	19.05	0.508	2.54

Correction factor: $\alpha^2 = 11.738 * \left(\frac{h_1}{a}\right)^2 - 3.6654 * \left(\frac{h_1}{a}\right) + 0.9755$

Failure criterion: $f(G_I, G_{II}, G_{Ic}, G_{IIc}) = \frac{G_I}{G_{Ic}} + \frac{G_{II}}{G_{IIc}} - 1 = 0$

$$E_1 = 139.3 \text{ MPa}, G_{Ic} = 0.0876 \text{ N / mm}, G_{IIc} = 0.3152 \text{ N / mm}$$

Experimental Validation

- Table 2. Verification of total ERR G for case 1

$10^3 \varepsilon_0$	G [N/mm]		
	Work [5]	Eq. (23)	Work [2,3]
1.00	0.0426	0.0445	0.0402
1.20	0.0662	0.0682	0.0646
1.40	0.0924	0.0945	0.0921
1.60	0.1208	0.1231	0.1225
1.80	0.1499	0.1540	0.1559
2.00	0.1830	0.1872	0.1921
2.20	0.2106	0.2227	0.2311
2.40	0.2454	0.2605	0.2731

Experimental Validation

- Table 3. Verification of total ERR G for case 2

$10^3 \varepsilon_0$	G [N/mm]		
	Work [5]	Eq. (23)	Work [2,3]
2.20	0.0345	0.0410	0
2.30	0.0667	0.0737	0.0002
2.40	0.0965	0.1056	0.0202
2.50	0.1355	0.1398	0.0545
2.60	0.1726	0.1732	0.0887
2.70	0.2130	0.2069	0.1231

Experimental Validation

- Table 4 Analytical prediction of propagation behaviour for case 1 panel

$10^3 \varepsilon_0$	Euler			Timoshenko			2D Elasticity		
	a [mm]	f	G_{II}/G (%)	a [mm]	f	G_{II}/G (%)	a [mm]	f	G_{II}/G (%)
0.6	38.1	<0	17.6	38.1	<0	79.8	38.1	<0	43.0
1.6	38.1	<0	92.4	38.1	<0	99.8	38.1	<0	82.1
2.0	38.1	<0	100	38.1	<0	100	38.1	<0	87.4
2.37	38.1	<0	100	38.1	<0	100	38.1	0	90.7
2.42	38.1	<0	100	38.1	<0	100	39.1	0	92.0
2.67	38.1	0	100	38.1	0	100	45.1	0	97.1
2.69	39.1	0	100	39.1	0	100	45.6	0	97.3
2.76	43.6	0	100	43.6	0	100	48.1	0	98.4
2.82	48.1	0	100	48.1	0	100	50.8	0	99.2
2.86	50.8	0	100	50.8	0	100	-	-	-

Experimental Validation

- Fig. 3. Analytical prediction of propagation behaviour for case 1 panel

Experimental Validation

- Fig. 4. Experimental validation of the analytical prediction for case 1 panel

Experimental Validation

- Fig. 5. Experimental validation of the analytical prediction for case 1 panel

Experimental Validation

- Table 5 Analytical prediction of propagation behaviour for case 2 panel

$10^3 \varepsilon_0$	Euler			Timoshenko			2D Elasticity		
	a [mm]	f	G_{II}/G (%)	a [mm]	f	G_{II}/G (%)	a [mm]	f	G_{II}/G (%)
2.12	19.05	<0	9.80	19.05	<0	76.0	19.05	<0	38.5
2.40	19.05	<0	28.7	19.05	<0	84.6	19.05	<0	49.3
2.42	19.05	≥ 0	29.5	19.05	<0	85.0	19.05	<0	49.8
2.43	30.70	0	94.6	19.05	<0	85.1	19.05	<0	50.0
2.48	31.00	0	96.5	19.05	<0	86.0	19.05	<0	51.2
2.50	31.10	0	97.3	19.05	<0	86.3	19.05	≥ 0	51.7
2.52	31.12	0	97.8	19.05	<0	86.6	40.05	0	94.0
2.60	31.30	0	99.9	19.05	<0	87.7	41.65	0	95.4
2.62	32.50	0	100	19.05	<0	87.9	42.00	0	95.7
2.68	36.05	0	100	19.05	<0	88.7	43.60	0	96.8
2.74	40.05	0	100	19.05	<0	89.9	45.20	0	97.7
2.80	43.30	0	100	19.05	<0	90.0	47.05	0	98.5
2.82	45.00	0	100	19.05	≥ 0	90.1	48.10	0	98.8
2.83	45.50	0	100	45.05	0	100	48.50	0	98.9
2.84	46.00	0	100	46.05	0	100	48.90	0	99.0
2.86	47.50	0	100	47.05	0	100	49.70	0	99.2
2.88	48.50	0	100	49.05	0	100	50.08	0	99.4
2.90	50.08	0	100	50.08	0	100	-	-	-

Experimental Validation

- Fig. 6. Analytical prediction of propagation behaviour for case 2 panel

Experimental Validation

- Fig. 7. Experimental validation of the analytical prediction for case 2 panel

Experimental Validation

- Fig. 8. Experimental validation of the analytical prediction for case 2 panel

Conclusions

- Present analytical formula gives accurate calculation of the total ERR.
- Wang-Harvey Euler beam mixed-mode partition theory also governs the propagation of post-local buckling driven delamination.
- Further validations are necessary.

Thank you very much and questions are
welcome!