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On Dipole Source Representation for Laser-Generated Sound

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Abstract — It is normally assumed that at the thermo-optical regime of laser-generated ultrasound in solids the equivalent thermo-optical source can be modelled as a mechanical dipole comprising two horizontal forces applied to the surface in opposite directions. In the present paper this assumption is re-examined on the basis of the rigorous theory that takes into account all acoustical, optical and thermal parameters of a solid material and all geometrical and physical parameters of a laser beam. It is shown that the dipole-source representation is rather limited, especially in respect of generated longitudinal waves. A general criterion has been established under which the dipole-source representation gives predictions with acceptable errors.

I. INTRODUCTION

During the last two decades laser generation of surface and bulk acoustic waves in solids has been widely used in non-destructive testing [1,2] and experimental physical acoustics [3]. To interpret the behaviour of laser-generated sound it is normally assumed that at the thermo-optical regime of generation the equivalent source of acoustic waves can be modelled as a dipole comprising two horizontal forces applied to the surface in opposite directions (see, e.g. monographs [1,2] and the recent paper [4]). This very simplified model has been introduced on the intuitive basis and is used widely. However, a number of rigorous theories of laser generation of sound in solids have been developed in the eighties and early nineties that clearly show that there is much more in the phenomenon than just a response of the medium to a pair of forces.

In the present paper the assumption of dipole-source representation is re-examined on the basis of the rigorous theory first developed by the present author and his co-workers [5-7]. By analysing the expressions for laser-generated longitudinal, shear

and Rayleigh waves the applicability of dipole-source representation is discussed for all these cases.

II. THEORETICAL BACKGROUND

In this section we briefly describe the main features of the rigorous theory, following the paper [6] (unfortunately, there are many text distortions in [6]

Figure 1. Geometrical parameters of the problem

caused by its poor publisher's translation into English).

Let us assume that a two-dimensional laser beam (a line source) is incident in normal direction onto the surface of a metal which for simplicity is considered as elastically isotropic (see Figure 1). The components of the displacement vector of laser-generated elastic waves u_i must satisfy the following equations of thermo-elasticity: the equation of mechanical motion

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \sigma_{ij,j} , \quad (1)$$

where σ_{ij} are components of the elastic stress tensor and ρ is the mass density of the medium, and the linearised constitutive equation which takes into account temperature effects

$$\sigma_{ij} = 2\mu u_{ij} + [\lambda u_{kk} - \gamma K(T - T_0)]\delta_{ij}. \quad (2)$$

Here $u_{ij} = (1/2)(u_{i,j} + u_{j,i})$ are components of the linearised strain tensor, λ and μ are the elastic Lamé constants, $K = \lambda + 2\mu/3$ is the elastic compression modulus, γ is the thermal expansion coefficient and T_0 is the initial temperature. The equations (1) and (2) should be supplemented by the linearised equation of thermal balance in which we ignore the effects of viscosity:

$$\rho c_v \frac{\partial T}{\partial t} - \kappa T_{,ii} + \gamma K T_0 \frac{\partial u_{ii}}{\partial t} = - \frac{\partial}{\partial z} [\beta I(x) f(t) \exp(-az)]. \quad (3)$$

Here c_v is the material's specific heat at a constant volume, κ is the coefficient of thermal conductivity, β is the coefficient of laser light energy transmission into the medium, α is the light energy absorption coefficient, $I(x)$ is the spatial distribution of the intensity of laser radiation and $f(t)$ describes the intensity modulation law. In what follows we assume for simplicity that modulation is time-harmonic, i.e. $f(t) = 1 + \cos \omega t$.

The total field of generated acoustic waves must also satisfy the stress-free boundary conditions on the solid surface (at $z = 0$):

$$\sigma_{ij} n_j = 0, \quad (4)$$

where n_j are components of the normal unit vector to the surface. The generated temperature field must satisfy the condition of continuity of the thermal flow on the solid surface (at $z = 0$):

$$\partial T / \partial z = 0. \quad (5)$$

The traditional simplification of the system of simultaneous equations (1)-(3) is to ignore the small dilatational term $\gamma K T_0 \partial u_{ii} / \partial t$ in (3). In this case equation (3) does not contain mechanical displacements u_i and can be solved independently. Such an approximation is equivalent to ignoring the difference between the isothermal and adiabatic values of the elastic constants λ and μ .

In what follows we consider a typical case of the laser light penetration depth into the metal, $2\pi/\alpha$, being much smaller than the length of the thermal wave, $\lambda_T = 2\pi/k_T = 2\pi/(\omega \rho c_v \sqrt{2\kappa})^{1/2}$. Then, the

solution of (3) subject to the boundary conditions (5) results in the following expression for $\partial T / \partial t$:

$$\frac{\partial T}{\partial t} = [(1-i)k_T \beta I(x) / \rho c_v] \exp[-(1-i)k_T z - i\omega t]. \quad (6)$$

Representing particle vibration velocity in the medium $v_i = \partial u_i / \partial t$ via the scalar potentials φ and ψ

$$v_x = \partial \varphi / \partial x - \partial \psi / \partial z, \quad (7)$$

$$v_z = \partial \varphi / \partial z + \partial \psi / \partial x, \quad (8)$$

substituting (7) and (8) into equations (1) and (2) and using (6), one can derive the following wave equations versus potentials (factor $\exp(-i\omega t)$ is omitted):

$$\Delta \varphi + k_l^2 \varphi = [p / (\lambda + 2\mu)] I(x) \exp[-(1-i)k_T z] \quad (9)$$

$$\Delta \psi + k_t^2 \psi = 0. \quad (10)$$

Here $p = (1-i)k_T \beta \gamma K / \rho c_v$, while $k_l = \omega / c_l$ and $k_t = \omega / c_t$ are wavenumbers of longitudinal and shear waves in the elastic medium, and $c_l = [(\lambda + 2\mu) / \rho]^{1/2}$ and $c_t = (\mu / \rho)^{1/2}$ are their phase velocities.

Substituting (7) and (8) into the boundary conditions (4) and using equations (9) and (10), one can obtain the following homogeneous boundary conditions for the potentials:

$$k_t^2 \varphi + 2(\partial^2 \varphi / \partial x^2 - \partial^2 \psi / \partial x \partial z) = 0, \quad (11)$$

$$k_t^2 \psi - 2(\partial^2 \varphi / \partial x \partial z - \partial^2 \psi / \partial z^2) = 0. \quad (12)$$

Thus, the initial problem has been reduced to the solution of the equations (9) and (10) for potentials subject to the homogeneous boundary conditions (11), (12). It follows from (9)-(12) that the equivalent sources responsible for laser-generated sound are located in the volume of the medium and not on its surface which is described by stress-free boundary conditions (11), (12).

The solution of the problem (9)-(12) can be obtained using the two-dimensional Green's function in Lapwood's representation (for more details see [6]). The resulting formal expressions for φ and ψ include integration over wavenumber k and over spatial coordinates x and z . The contribution of the poles in the contour integration on the complex k -

plain gives the potentials of laser-generated Rayleigh waves propagating in both directions:

$$\varphi_R = -\frac{ip\Phi(k_R)}{F'(k_R)} \frac{4k_R^2 s}{(\lambda+2\mu)[(1-i)k_T+q]} \exp(\pm ik_R x - qz) \quad (13)$$

$$\psi_R = -\frac{2k_R p\Phi(k_R)}{F'(k_R)} \frac{2k_R^2 - k_t^2}{(\lambda+2\mu)[(1-i)k_T+q]} \exp(\pm ik_R x - sz) \quad (14)$$

Here k_R is the wavenumber of Rayleigh waves, $q = (k_R^2 - k_t^2)^{1/2}$, $s = (k_R^2 - k_l^2)^{1/2}$, $F'(k_R)$ is the value of the derivative $dF(k)/dk$ of the Rayleigh determinant $F(k) = (2k^2 - k_t^2)^2 - 4k^2 v_l v_t$ taken at the point $k = k_R$, where $v_l = (k^2 - k_l^2)^{1/2}$, $v_t = (k^2 - k_t^2)^{1/2}$, and

$$\Phi(k_R) = \int_{-\infty}^{\infty} I(x) \exp(-ik_R x) dx \quad \text{is the Fourier}$$

transform of the spatial distribution of the laser light intensity $I(x)$.

The contributions of the saddle points give the potentials of laser-generated longitudinal and shear bulk waves propagating into the medium. At relatively large distances from the laser-illuminated surface particle displacements in these waves are predominantly longitudinal, u_r , and shear, u_θ , respectively. Using asymptotic relations between the displacements and potentials (7), (8) in a polar system of co-ordinates, $u_r = -(1/c_l)\varphi$, $u_\theta = (1/c_l)\psi$, the expressions for laser-generated longitudinal and shear bulk waves can be written in the form:

$$u_r(r, \Theta) = -\frac{p\Phi(k_l \sin \Theta)}{(\lambda+2\mu)\sqrt{2\pi k_l r c_l}} \exp(ik_l r - i\pi/4) \times \left\{ \frac{k_l \cos \Theta}{(1-i)^2 k_T^2 + k_l^2 \cos^2 \Theta} + \frac{i2k_l^4 \sin \Theta \sin 2\Theta [(k_t/k_l)^2 - \sin^2 \Theta]^{1/2}}{[(1-i)k_T - ik_l \cos \Theta]F(k_l \sin \Theta)} \right\} \quad (15)$$

$$u_\theta(r, \Theta) = \frac{ip\Phi(k_l \sin \Theta)}{(\lambda+2\mu)2\sqrt{2\pi k_l r c_l}} \exp(ik_l r - i\pi/4) \times \left\{ \frac{k_t^4 \sin 4\Theta}{[(1-i)k_T + k_l(\sin^2 \Theta - k_l^2/k_t^2)^{1/2}]F(k_l \sin \Theta)} \right\}$$

(16)

III. ANALYSIS AND DISCUSSION

Let us analyse equations (15) and (16) for different relations between the characteristic width of the laser beam a , the wavelengths of generated longitudinal and shear waves $\lambda_{l,t} = 2\pi/k_{l,t}$ and the wavelengths of thermal waves $\lambda_T = 2\pi/k_T$.

For metals at most frequencies $\lambda_T \ll \lambda_{l,t}$, or $k_T \gg k_{l,t}$. Using these inequalities, let us first discuss the effect of thermal wave penetration into a solid on generation of shear bulk waves described by equation (16). Keeping in mind that $p = (1-i)k_T \beta \gamma K / \rho c_v$, one can see that in the main approximation the radiation pattern of shear waves does not depend on k_T and for relatively narrow beams (when the effect of $\Phi(k, \sin \Theta)$ is negligible) is defined largely by the factor $\sin 4\Theta$ typical for dipole-type radiation from a pair of horizontal forces (see, e.g. [4]). Note that shear waves are not generated in normal direction because of the symmetry of the problem. Thus, for generated shear waves a thermo-optical source can be adequately represented as a dipole. The same is true also for generated Rayleigh surface waves described by equations (13) and (14).

For generated longitudinal waves (see formula (15)) the situation is essentially different. While the second term in figure brackets of (15) does not depend on k_T and describes the contribution of a dipole-type source, the first term does depend on k_T , and this limits the applicability of dipole-source representation in respect of generated longitudinal waves. The main influence of the first term occurs for waves generated in the normal direction to the surface ($\Theta = 0$) since the dipole-type contribution given by the second term is zero at $\Theta = 0$.

Typical radiation patterns of laser-generated longitudinal waves in aluminium calculated according to equation (15) for different values of laser beam width a are shown on Figure 2. As one can see, for very narrow laser beams ($k_l a \ll 1$; this corresponds to $a = 0.1$ mm on Figure 2) the relative amplitudes of longitudinal waves generated in normal direction are negligibly small as compared to the waves generated at oblique angles. The reason for this is that laser-generated sound field is a superposition of waves directly radiated into the bulk by a heated area and waves reflected from the surface. In the normal direction the directly radiated and reflected waves nearly cancel each other, the

Figure 2. Radiation patterns of laser-generated sound in aluminium at frequency 5MHz: $a = 0.1$ mm (solid curve); $a = 1.5$ mm (dashed curve); $a = 5$ mm (dashed-dotted curve)

cancellation being almost complete for very small penetration lengths of thermal waves $l_T \approx \lambda_T = 2\pi/k_T$. For metals with higher $\kappa/\rho c_v$, the value of $\lambda_T = 2\pi/k_T$ is larger and radiation in normal direction increases.

In the case of wider laser beams (larger values of $k_{l,t}a$) the relative contribution of the first term in (15) shows an additional increase because of the second term in (15) and of the whole expression (16) being suppressed by the factor $\Phi(k_{l,t}\sin\Theta)$.

If to assume that, to be considered as dipole radiation, relative amplitudes of longitudinal waves in the normal direction should not exceed 10% of their maximum values at oblique angles, then for $a < \lambda_l = 2\pi/k_l$ the following criterion of dipole-source representation can be obtained from equation (15):

$$\frac{k_l}{k_T} \frac{k_l a}{\sin(0.43k_l a)} < 0.08. \quad (17)$$

For $a > \lambda_l = 2\pi/k_l$ the contribution of longitudinal waves radiated in the normal direction is always significant and can even be prevailing for $k_{l,t}a \gg 1$. Obviously, in this case no dipole-source representation is possible for laser-generated longitudinal waves. In the limit of $k_{l,t}a \rightarrow \infty$ corresponding to one-dimensional geometry this reflects the well known fact that under such circumstances only longitudinal waves can be generated. Needless to say that any modification of the surface affecting its acoustic reflection coefficients (e.g., the presence of liquid droplets, thin films, surface roughness, etc.) can result in drastic changes in radiation patterns [7,8] which are incompatible with the dipole-source representation.

IV. CONCLUSION

Although the dipole-source representation for laser-generated sound may be considered helpful for understanding the patterns of radiated waves, it is rather limited, especially in respect of generated longitudinal waves. For this reason, it is preferable to always follow the rigorous theory, which is simple enough to be used for all engineering applications.

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