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Generating Training Data For Identifying Neurofuzzy Models Of Non-Linear Dynamic Systems

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Abstract—This paper presents a methodology for generating data for training a fuzzy relational model, one neuro-fuzzy modeling technique. Neuro-fuzzy modeling is a popular “grey-box” modeling technique used to model complex, non-linear plants utilizing input-output data, i.e. as an alternative to physical-based modeling. The controllable input variables of each of the generated training data set, are positioned at the centres of the fuzzy sets, so that the steady-state and dynamic performance of the model should be satisfactory whenever the control signal is stepped between the centres of its fuzzy sets. The rule confidences of the fuzzy rules are identified via the Global Least-Square (GLS) identification algorithm. The model performance is validated by using a simulated water level control system.

I. INTRODUCTION

The combination of neural networks and fuzzy logic, i.e., neuro-fuzzy technique, brings in the advantages of both approaches leading to a model with learning capability and adjustable parameters. Although the selection of model structure and parameters has an effect on the performance of the model [Zhou, 2008], the overall prediction ability of the model is greatly dependent on the completeness of input-output training data. The most important issue during off-line training is to obtain enough information to describe the plant over all operational conditions. The training data is obtained in the most common ways, i.e., experimental data, random data or normal operating data extracted from the actual plant.

Complete training data for all working conditions can never be accomplished in practice. One method of generating high quality training data [Wu and Dexter, 2003], is based on the idea that each combination of the input sets are 100% activated when all input data are at the apexes of the fuzzy sets. In second-order models, however, it is unlikely to guarantee that the outputs at two consecutive sampling instants are both at the apexes of the fuzzy sets of all of the inputs at the same time.

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A method called ‘uniform distribution’ [Laukonen and Passino, 1995], is used to generate the training data assuming that all of the inputs are evenly distributed in the input reference spaces. However, this method becomes rather complicated when previous system outputs are used as inputs to the model, as it is difficult to guarantee that the combination of all of the input variables are evenly distributed in the whole input space. An alternative method to identify the fuzzy relational model with unevenly distributed training data is proposed in [Sing and Postlethwaite, 2000]. In this method, the input spaces are partitioned into several regions, whose boundaries are determined by the local minimum mean squared error at the positions of the training data. However, the selection of the objective criteria to optimize the region distribution and the influence of the training data on the modelling remain open questions.

A different method to generate training data with multi-level pseudo-random (MLPR) step input signals is described in [Braun *et al.*, 2001]. The excitation signal attempts to fully occupy the universe of discourse for all operational conditions of the plant. However, it is a long procedure to generate enough data so as to have sufficient information to describe the dynamic and steady-state system behaviour. Other methods includes optimal use of the data with factorial design [Buragohain and Mahanta, 2008], as well as sine functions with different amplitudes and frequencies [Tan, 2004]. However, the experiment needs to be repeated several times to identify appropriate frequencies and amplitudes for the signals.

This paper proposes a method to generate input-output training data, based on the idea in which excitement control signals are at the centers of fuzzy sets, such that there are no iterated computation errors. This approach was used to identify a second-order model of a non-linear dynamic system in a fault identification scheme [Zhou, 2006].

The remainder of the paper is organized as follows. Section II presents the fuzzy modeling technique, model structure and identification scheme. The example used to assess the technique, a water-level control system, is discussed in Section III. Section IV presents simulation results and discusses the effectiveness of the proposed technique. Conclusions are drawn in Section V.

II. IDENTIFICATION OF A FUZZY RELATIONAL MODEL

A fuzzy relational model can be identified directly from input-output numerical data. The set of rule confidences are stored in an array (θ). Each of the rule confidences

represents the fuzzy relationship between fuzzy sets used in the antecedent and conclusion of the associated rule.

Consider a multi-input and single-output relational model consisting of n inputs ($x_1, x_2, \dots, x_n$) and one output y , where the input and output spaces are characterized by $r_1, r_2, \dots, r_n$ and j fuzzy reference sets respectively. The fuzzy relational model (FRM) can be described by the following fuzzy relational equation:

$$Y(k) = X_1(k) \bullet X_2(k) \bullet \dots \bullet X_n(k) \bullet R \quad (1)$$

where $Y(k)$ and $X_i(k)$ are fuzzy representations of the output $y(k)$ and inputs $x_i(k)$ at the sampling instant k .

R is the fuzzy relation and “ $\bullet$ ” denotes fuzzy compositional inference. The fuzzy relational model consists of $r_1 \times r_2 \times \dots \times r_n \times j$ fuzzy rules. The rule confidence, $R_{r_1 r_2 \dots r_n j}$ indicates the amount of confidence of the output $y(k)$ when inputs are $x_{1,s_1}, x_{2,s_2}, \dots, x_{n,s_n}$, ($1 \leq s_1 \leq r_1, 1 \leq s_2 \leq r_2, \dots, 1 \leq s_n \leq r_n$). Equation (1) can be rewritten in the following form

$$y(t) = \phi^T(t) \theta \quad (2)$$

where $\phi^T(t)$ is the combination of input variables firing level; $\theta \in R^{n \times 1}$ is a vector of the elements of the fuzzy relational matrix R which measures the possibility of obtaining an output y in the set q , $q \leq j$, from inputs ($x_1, x_2, \dots, x_n$) in the sets $s_{11}, \dots, s_{nk}$ respectively ($k \leq r_n$). Here, the *Global Recursive Least Squares* (GRLS) algorithm is employed to estimate the rule confidences θ [Ljung, 1983], i.e.,

$$\left. \begin{aligned} \hat{\theta}(t) &= \hat{\theta}(t-1) + L(t) \left[y(t) - \phi^T(t) \hat{\theta}(t-1) \right] \\ L(t) &= \frac{P(t-1) \phi(t)}{\lambda + \phi^T(t) P(t-1) \phi(t)} \end{aligned} \right\} \quad (3.1)$$

$$P(t) = \frac{1}{\lambda} \left[P(t-1) - \frac{P(t-1) \phi(t) \phi^T(t) P(t-1)}{\lambda + \phi^T(t) P(t-1) \phi(t)} \right] \quad (3.2)$$

The initial values are $P(0) = C \cdot I$ and $\hat{\theta}(0) = 0$, where C is chosen as a large constant. A satisfactory fuzzy relational model can only be obtained if the training data are complete and noise-free. Hence, the quality of the fuzzy relational model is dependent on the quality of the training data [Campello and Amaral, 2001]. Simulation results have illustrated that different sets of training data can result in fuzzy models with different prediction performances [Tan and Dexter, 1996]. Therefore, acquiring high quality of the training data is a key issue in generating an accurate fuzzy relational model.

III. MODELLING A WATER LEVEL CONTROL SYSTEM USING A FUZZY RELATIONAL MODEL

A. Description of a Water level control system

In general, it is necessary to develop low-order models from high-order systems to analyze system for ease in term of analysis and subsequently control design. With expert knowledge a suitable model structure can be determined. A second-order water level control system, seen in Fig 1, is used to describe the modeling procedure for a nonlinear plant.

Fig 1 Schematic diagram for the water-tank process system

On the assumption that the liquid density remains constant, the nonlinear equations for the heights of water, H_1, H_2 , in the tanks are given by

$$\begin{aligned} A \frac{dH_1}{dt} + \beta_1 \sqrt{H_1} &= F_{in} \\ A \frac{dH_2}{dt} + \beta_2 \sqrt{H_2} &= \beta_1 \sqrt{H_1} \\ F_{in} &= k \cdot u_c \end{aligned} \quad (4)$$

where F_{in} is the volumetric flow rate into Tank1, controlled by the valve V . The influx of water depends linearly on the input variable $u_c \in [0, 1]$, and k is the inlet flow rate; A is the cross-sectional area of the tank; β_1 and β_2 are the valve coefficients, respectively.

Table 1 The parameter values of the water-level system

Variable	Value	Variable	Value
H_1	4m	β_1	$0.5 \text{ m}^{5/2} / \text{min}$
H_2	2.25m	β_2	$2/3 \text{ m}^{5/2} / \text{min}$
A	1 m^2	k	$1 \text{ m}^3 / \text{min}$

The steady-state response of the nonlinear relationship between the height H_2 and control signal u_c is shown in Fig 2. The parameters of the two-tank process is listed in Table 1 [Hussain and Ho, 2004].

Fig 2 Relationship between valve position and height of the water in Tank 2

The selected second-order fuzzy relational model has three inputs $u(n-1)$, $H_2(n-2)$, $H_2(n-1)$ and one output $H_2(n)$ so that,

$$H_2(n) = f(u(n-1), H_2(n-2), H_2(n-1)), \quad (5)$$

A schematic diagram of the fuzzy relational model shown in Fig 3,

Fig 3 Structure of the second-order fuzzy relational model

From Fig 2, the output of the subsystem is more sensitive to the valve position when it is in the region from half to fully open. Therefore, there are more divisions in the first part of fuzzy space.

The membership function distribution of the control signal $u(n-1)$ and the other two inputs are shown in Fig 4(a). The output fuzzy space has two fuzzy sets, for simplicity, as shown in Fig 4(b). Details of the structure and parameter can be found in [Zhou, 2008]. The initial values of rule confidence and covariance matrix are $\theta_0 = 0.5$, $C_0 = 1000 \cdot I$, respectively.

Before the data are used for training, the amplitude of each variable is normalized within $[0,1]$ to make the variables comparable in magnitude. This eases the procedure as it results in variables having the same importance, hence the model should operate in a more reliable manner [Skogestad and Postlethwaite, 2005].

Fig 4(a) Fuzzy division in the input reference spaces

Fig 4(b) Fuzzy division in the output reference space

B. Method of generating the input sequence

In order to obtain accurate global model performance, the training data should cover the whole input space.

The data is collected by varying positions of the control valve. It is impossible to get complete training data so as to cover the whole input space of the operational process. However, the complete behaviour of the system when the control signal U is at the center of fuzzy sets can be acquired in the following way. The excitation signal (**DS2**) is stepped according to the values of apexes of the fuzzy sets of the control input, as shown in Fig 5.

As can be seen in Fig 5, there are five groups of control sequences. Each group is composed of two parts: an odd signal sequence—the apexes of all fuzzy sets; an even signal sequence—one apex of the fuzzy sets. There are 10 step signals in each group. Consider the fourth group shown in Fig 5(d) as an example. In this sequence, the values of odd signals are the apexes of the fuzzy sets in the control reference space. The value of the even signals equals to the apex of the fourth fuzzy set. For the control sequence in Fig 5(a), the value of the even sequence is the apex of the first fuzzy set and in Fig 5(b), Fig 5(c), Fig 5(e), the even sequences are the apexes of the second, third and fifth fuzzy sets, respectively.

It should be noted that the order in which the training data is presented has little effect on the performance of the resulting model. The models developed using the same training data with different data orders show little difference between each other. The order of the training data used for developing the FRM in these experiments is from group 1 to group 5, as shown in Fig 5 (a)-(e).

This method of generating training data should minimize the modeling errors for step changes in the control signal because the values of the input are at the centers of fuzzy sets, maximizing the degrees of satisfaction of the rule antecedents.

Fig5 Control sequence to generate training data

It should be noted that there are only five fuzzy reference sets to describe the control signal. Increasing the number of fuzzy sets, decreases the model identification errors but increases the number of fuzzy rules, thus increasing computational complexity. In practice, a compromise must be made between the number of fuzzy rules, the amount of training data and computational demands.

Although the model is not fully trained, simulation experiments will demonstrate that the prediction ability of the neurofuzzy model is encouraging. In addition, the model developed in this paper is more flexible and is less dependent on the conditions of the generating procedure.

IV. VALIDATION OF MODEL PERFORMANCE

Model validation is an important step in any identification scheme. Clearly, the same data which is used for identification should not be used for testing. It is, however, difficult to find an appropriate test data set to prove the effectiveness of a model of a non-linear dynamic system.

The Root Mean Squared Error (RMSE) is the main criterion used to evaluate the model performance,

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2} \quad (6)$$

where N is the number of the test data; y_i is the i^{th} desired output, $\hat{y}_i$ is the i^{th} predicted output from the model. The model performance can be tested in two different ways: recursive estimation and one-step-ahead prediction [Ghiaus, 2001]. Here a recursive estimator, which predicts $\hat{y}(k+1)$ based on previous predictions, is used to test the output of the second-order model. Thus,

$$\hat{y}(k+1) = f(u(k), \hat{y}(k-1), \hat{y}(k)) \quad (7)$$

This method of validation is more reliable than that based on a one-step ahead predictor, $\hat{y}(k+1) = f(u(k), y(k-1), y(k))$, which uses the current and previous measurements of the actual output, since most models will give satisfactory results when tested in this way [Ghiaus, 2001].

Fig 6 (a) Test data (VS1) for model performance

Fig 6 (b) Test data (VS2) for model performance validation

Three models of the tank control system, which have been identified with different training data sets, are validated. Model **M1** is trained with data set **DS1**, which is generated using sine functions with different frequencies [Tan, 2004]. Model **M2** is generated from data set **DS2** and model **M3** is trained with data set **DS3**, which is generated using a MLPR test signal [Braun *et al.*, 2001].

Two test data sets are used to validate the performance of these three models (see Fig 6). The first data set (**VS1**), which is generated using a low-pass filtered random test signal, is totally different from any of the data used for training. The second data set **VS2**, which is generated using a deterministic test signal, is similar to the training data and contains more steady-state values. The results of the model prediction using the two test data sets are shown in Fig 7.

Fig 7 (a) Comparing model performances with test data set *VS1*

Fig 7 (b) Comparing model performances with test data set *VS2*

All three models are able to describe the dynamic behaviour of the system with test data *VS*, although the predictions of the model **M3** are much closer to the actual system output compared to **M1** and **M2**. However, **M3** has an obvious defect when tested using the steady-state test data *VS2*, as shown in Fig 7 (b). **M1** shows better performance than that of **M2** with the test data *VS2*.

Table 2 RMSEs of the models with test data *VS1* & *VS2*

Model	Training data		RMSE	
	Name	Number	<i>VS1</i>	<i>VS2</i>
M1	<i>DS1</i>	130768	0.0728	0.1306
M2	<i>DS2</i>	32460	0.1363	0.2074
M3	<i>DS3</i>	365994	0.0678	0.8017

Table 2 lists the RMSEs of the three models for each test data set. Model **M3** produces better dynamic performance for test data *VS1*. For test data *VS2*, **M1** has better performance compared to the rest; **M3** has the worst performance. This is because the training data used to identify model **M3** failed to capture the information about the plant at all operating conditions.

As can be seen in Table 2, when using test data set *VS1* for validation, **M2** has an RMSE almost twice as high as

those for **M1** and **M3**. However, **M2** is identified using training data set size approximately 4 times smaller than that of **M1** and 11 times smaller than that of **M3**. The difference in the size of the three data sets is due to the generation method. In fact, a much longer time period is necessary to generate the training data used to identify **M1** and **M3**. Moreover, trial and error must be used to obtain appropriate training data using these schemes.

Improvement of model **M2**

M2 developed with training data *DS2* characterizes the steady-state (low frequency) behaviour of the system. However, higher frequency behaviour of the system is not so well captured. Therefore, in order to alleviate this and improve the performance of **M2**, a chirp signal is introduced at the end of *DS2* to excite a wider range of frequencies, i.e., *DS2* is extended to *DS4*. The frequency content of the chirp signal is chosen according to the bandwidth of the system. A new model **M4** is identified from *DS4*.

Fig 8 (a) Comparison of the model performances with improved version: Test data *VS1*

Fig 8 (b) Comparison of the model performances with improved version: Test data *VS2*

The comparison of the model performances of **M1**, **M2**, **M3** and **M4** are shown in Fig 8. The Euclidian distances in the input reference space between the training data and test data sets are presented in Table 3. It can be seen that the average distance between the test data and the training data for *DS2* is much greater than those for *DS1*, *DS3* and *DS4*. However, the number of training data in *DS4* is still much smaller than that in *DS1* and *DS3*.

Table 3 Distance between the test data and training data used to identify the four models

Training data \ Test data	<i>DS1</i>	<i>DS2</i>	<i>DS3</i>	<i>DS4</i>
<i>VS1</i>	1.5507×10^{-5}	0.1054	3.7979×10^{-6}	4.2339×10^{-5}
<i>VS2</i>	4.5328×10^{-5}	0.1323	5.4602×10^{-6}	3.6349×10^{-5}

Table 4 RMSEs of the four models

Model	Training data		RMSE (m)	
	Name	Number	<i>VS1</i>	<i>VS2</i>
M1	<i>DS1</i>	130768	0.0728	0.1306
M2	<i>DS2</i>	32460	0.1363	0.2074
M3	<i>DS3</i>	365994	0.0678	0.8017
M4	<i>DS4</i>	65788	0.0659	0.0891

As can be seen in Table 4, **M1** and **M4** show better performance with both the test data sets *VS1* and *VS2*. However, the generation of data set **M1** involves more trial and error testing. **M3** shows better performance with dataset *VS1* but worse performance with dataset *VS2*, since it has obvious defects under specific input conditions. In addition, the training data needed for identifying **M3** is five times greater in size than that for **M4**. The amount of data needed to identify Model **M4** is relatively small and its performance is better, both in terms of predicting the steady-state and the dynamic behaviour of the system.

V. CONCLUSIONS AND DISCUSSION

Different ways of generating training data have been compared and a new method of generating training data has been proposed, which reduces the required time and computational effort in comparison with currently available methodologies. The procedure for generating the training data can be summarized as follows:

1. Specify the structure of the FRM;
2. Specify the membership functions for the reference fuzzy sets describing each of the input/output variables used in the FRM;
3. Vary the control signal so that it follows the sequence of step changes described in section III ;
4. Drive the control signal using a chirp excitation signal;

It has been shown that the use of the chirp signal is necessary to excite the system over a wider range of frequencies and improve the dynamic performance of the

model. Results obtained from a simulated water-level control system have demonstrated that the proposed training procedure can successfully generate a neurofuzzy model of a non-linear dynamic system.

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