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CONVERGENCE BEHAVIOURS OF AN ADAPTIVE STEP-SIZE CONSTANT MODULUS ALGORITHM FOR DS-CDMA RECEIVERS

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ABSTRACT

A convergence analysis of the blind adaptive step-size constant modulus algorithm (AS-CMA) for direct-sequence code division multiple access (DS-CDMA) receivers is presented. Analytical results show similar convergence behaviours of the AS-CMA receiver and the adaptive step-size stochastic approximation (AS-SA) receiver. Simulations show that the blind AS-CMA algorithm performs comparably with the non-blind AS-SA in a Markovian type of nonstationary environment. The robustness of the proposed algorithm to different settings of the initial step-size is also shown.

1. INTRODUCTION

An adaptive step-size constant-modulus algorithm (AS-CMA) has been proposed for a DS-CDMA receiver operating in nonstationary environments [1, 2]. The algorithm adaptively varies the step-size in order to minimise the constant modulus (CM) criterion. In contrast to the adaptive step-size minimum output energy (AS-MOE) receiver proposed in [3], AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalise the channel and tends to cancel the desired signal itself when the spreading waveform has a mismatch [4]. In reality, the multipath distortion, the cause of mismatch, must not be neglected especially when the transmission of high data rate is required.

In this paper, we study the convergence behaviours of the AS-CMA receiver in multipath fading nonstationary channels as compared to the adaptive step-size stochastic approximation (AS-SA) receiver [5]. Further analysis from [1] reveals similarity in convergence behaviours of the Y_n process, the derivative of the tap-weight of with respect to the step-size, of AS-CMA and AS-SA.

Simulation results show that the performance in terms of signal-to-interference plus noise ratio (SINR) of AS-CMA is comparable to that of the non-blind AS-SA receiver. Insensitivity of initial step-size settings is also confirmed. The trajectories of the step-sizes of AS-CMA are shown to behave similarly to those of AS-SA while AS-CMA enjoys the advantage of not requiring a training sequence for the adaptation process.

2. SIGNAL MODEL

Consider the real signal model of an additive white Gaussian noise (AWGN) K -user synchronous DS-CDMA channel, the baseband

received signal is defined as

$$r(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^K A_k b_k(i) c_k(t - iT) + w(t), \quad (1)$$

where A_k represents the received amplitude of the k th user. The data bits $b_k(i)$ are independent identically distributed (i.i.d.) and $b_k(i) \in \{-1, +1\}$. The symbol period is denoted by T . The spreading waveform of the k th user $c_k(t)$ is N -dimensional and has unit energy property, i.e., $\|c_k\|_2^2 = 1$. The AWGN $w(t)$ has power spectral density σ_w^2 . The spreading codes $c_k(t)$ can be modified to take into account the effect of the channel and the pulse shaping waveform as $\tilde{c}_k(t) = c_k(t) * \psi(t) * h_k(t)$, where $*$ denotes convolution, $\psi(t)$ is the pulse shaping filter and $h_k(t)$ is the channel response of the k th user and need not be identical for different users. The continuous-time received signal $r(t)$ is sampled to form a length- L_f received signal vector at the n th observation, where L_f is the length of a receiver with tap-weight vector $\mathbf{f}$,

$$\mathbf{r}_n = \sum_{k=1}^K \mathbf{r}_n^{(k)} + \mathbf{w}_n = \sum_{k=1}^K \mathbf{G}_k \mathbf{b}_k[n] + \mathbf{w}_n, \quad (2)$$

where $\mathbf{G}_k$ is the combined code-channel response matrix of the k th user and $\mathbf{b}_k[n] = [b_k[n + L_b - 1], \dots, b_k[n]]^T$ with $L_b = \lceil \frac{L_f + L_h - 1}{N} \rceil$ and $\mathbf{w}_n = [w[nN + L_f - 1], \dots, w[nN]]^T$. Note that

$$\mathbf{G}_k = \mathbf{C}_k \mathbf{H}_k \quad (3)$$

$$= \begin{bmatrix} \tilde{\mathbf{C}}_k & & & \\ & \tilde{\mathbf{C}}_k & & \\ & & \ddots & \\ & & & \tilde{\mathbf{C}}_k \end{bmatrix}^T \begin{bmatrix} \mathbf{h}_k & & & \\ & \mathbf{h}_k & & \\ & & \ddots & \\ & & & \mathbf{h}_k \end{bmatrix},$$

where $\tilde{\mathbf{C}}_k$ represents the block of delayed copies of the code sequence of the k th user with dimension $L_h \times N + L_h - 1$

$$\tilde{\mathbf{C}}_k = \begin{bmatrix} 0 & \cdots & c_k[N-1] & \cdots & c_k[0] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ c_k[N-1] & \cdots & c_k[0] & \cdots & 0 \end{bmatrix}$$

and $\mathbf{C}_k$ denotes the code matrix $\tilde{\mathbf{C}}_k$ with the first $L_h - 1$ columns truncated. The channel response vector for the

k th user has length L_h , i.e., $\mathbf{h}_k = [h_0^{(k)}, \dots, h_{L_h-1}^{(k)}]^T$. For a linear receiver $\mathbf{f}$, the estimate of the transmitted data $\hat{b}[n]$ can be achieved as

$$\hat{b}[n] = z_n = \mathbf{f}^T \mathbf{r}_n = \mathbf{f}^T (\mathbf{G}\mathbf{b}[n] + \mathbf{w}_n), \quad (4)$$

where $\mathbf{G} = [\mathbf{G}_1, \mathbf{G}_2, \dots, \mathbf{G}_K]$ and $\mathbf{b}[n] = [\mathbf{b}_1^T[n], \mathbf{b}_2^T[n], \dots, \mathbf{b}_K^T[n]]^T$. For brevity, we shall consider the first user as the desired user and drop the subscript k in all variables involving the first user.

3. ADAPTIVE STEP-SIZE CMA ALGORITHM

We re-introduce the AS-CMA algorithm as presented in [1, 2]. Consider the CM criterion for real signals,

$$J_{\text{CM}} = \frac{1}{4} E\{(z_n^2 - R_2)^2\}, \quad (5)$$

where $z_n = \mathbf{f}_n^T \mathbf{r}_n$ is the output of the receiver at time n . The dispersion constant R_2 is equal to unity for binary phase shift keying (BPSK) signals. By minimising J_{CM} with respect to $\mathbf{f}_n^\mu$ at a particular step-size μ and taking the instantaneous gradient, the fixed step-size (fixed- μ) CMA receiver weight update equation is given by

The weight update equation of the AS-CMA receiver for real signals is given by [1]

$$\mathbf{f}_{n+1} = \mathbf{f}_n - \mu_n(z_n^2 - R_2)z_n \mathbf{r}_n, \quad (6)$$

where μ_n is the *time-varying* step-size. The dispersion constant R_2 is equal to unity for binary phase shift keying (BPSK) signals. The update of the step-size is given by [1]

$$\mu_{n+1} = \left[\mu_n - \alpha(z_n^2 - R_2)z_n Y_n^T \mathbf{r}_n \right]_{\mu_-}^{\mu_+}, \quad (7)$$

where $[\cdot]_{\mu_-}^{\mu_+}$ denotes truncation to the limits of the range $[\mu_-, \mu_+]$ and α denotes the adaptation rate of the step-size μ_n with $\alpha > 0$. Y_n represents the derivative $\partial \mathbf{f}_n / \partial \mu|_{\mu=\mu_n}$ as defined in [3] and, from (6), the update equation of Y_n is given by

$$Y_{n+1} = [\mathbf{I} - \mu_n(3z_n^2 - R_2)\mathbf{r}_n \mathbf{r}_n^T] Y_n - (z_n^2 - R_2)z_n \mathbf{r}_n, \quad (8)$$

where $\mathbf{I}$ denotes the identity matrix with size $L_f \times L_f$. Equations (6), (7) and (8) constitute the adaptive step-size CMA algorithm for real signals [1]. It is straightforward to extend these to the complex case, but we retain the real version for consistency with [4, 3]. For the fixed- μ system, i.e., $\mu_n \triangleq \mu$, we define $Y_n^\mu = \partial \mathbf{f}_n^\mu / \partial \mu$ and its update rule is given by

$$Y_{n+1}^\mu = [\mathbf{I} - \mu(3(z_n^\mu)^2 - R_2)\mathbf{r}_n \mathbf{r}_n^T] Y_n^\mu - ((z_n^\mu)^2 - R_2)z_n \mathbf{r}_n, \quad (9)$$

where $z_n^\mu = (\mathbf{f}_n^\mu)^T \mathbf{r}_n$.

4. INTERPRETATION OF Y_N^μ AND Y_N OF AS-CMA

The behaviour of the AS-CMA algorithm is controlled by the random vector parameter Y_n defined as the derivative of the estimates of the tap-weight vector with respect to μ_n . To study the behaviour of the AS-CMA algorithm, it is essential to understand the meaning of Y_n . In the algorithm development, although we do not assume that the actual system is stationary, we employ the minimisation of the expectation of a *stationary* error with respect to μ_n .

Thus, we study Y_n via Y_n^μ which is introduced as a stationary sequence at $\mu_n \triangleq \mu$ for the fixed- μ process. Following [5], a finite difference approximation is used to study the meaning of Y_n^μ .

Due to the multimodality of the CM cost function, the local minima can be associated with a variety of mean square-error (MSE) values. However, the analysis of [6] has shown a close relationship between the "good" local minima and minimum mean-square error (MMSE) solutions. These local minima are associated with low MSE and are considered as global minima. For a meaningful interpretation of Y_n^μ , we consider the behaviour of Y_n^μ only in the neighbourhood of local minima associated with low MSE.

Following the analysis of [5], Y_n^μ is interpreted by considering its derivative representation. For $\Delta > 0$, $\mu + \Delta \leq \mu_+$, define $\delta \mathbf{f}_n^{\mu, \Delta} = \frac{\mathbf{f}_n^{\mu+\Delta} - \mathbf{f}_n^\mu}{\Delta}$. Therefore,

$$\begin{aligned} \delta \mathbf{f}_{n+1}^{\mu, \Delta} &= \frac{1}{\Delta} [\mathbf{f}_n^{\mu+\Delta} - (\mu + \Delta)\phi(z_n^{\mu+\Delta})\mathbf{r}_n \\ &\quad - \{\mathbf{f}_n^\mu - \mu\phi(z_n^\mu)\mathbf{r}_n\}] \quad (10) \\ &= \delta \mathbf{f}_n^{\mu, \Delta} - \mu \frac{[\phi(z_n^{\mu+\Delta}) - \phi(z_n^\mu)]}{\Delta} \mathbf{r}_n - \phi(z_n^{\mu+\Delta})\mathbf{r}_n, \end{aligned}$$

where $\phi(z_n) = (z_n^2 - R_2)z_n$ is the prediction error function of the CMA algorithm and $z_n^{\mu+\Delta} = (\mathbf{f}_n^{\mu+\Delta})^T \mathbf{r}_n$. We consider the behaviour of the AS-CMA algorithm within the neighbourhood of an equilibrium point $\tilde{\mathbf{f}}$ in the absence of noise. By using the first-order Taylor expansion at $z_n \approx \tilde{z}_n$ where $\tilde{z}_n = \tilde{\mathbf{f}}^T \mathbf{r}_n$, we arrive at

$$\phi(z_n^{\mu+\Delta}) = \phi(\tilde{z}_n) + \phi'(\tilde{z}_n)\mathbf{r}_n^T(\mathbf{f}_n^{\mu+\Delta} - \tilde{\mathbf{f}}), \quad (11)$$

$$\phi(z_n^\mu) = \phi(\tilde{z}_n) + \phi'(\tilde{z}_n)\mathbf{r}_n^T(\mathbf{f}_n^\mu - \tilde{\mathbf{f}}), \quad (12)$$

where $\phi'(\tilde{z}_n)$ represents the derivative of $\phi(\tilde{z}_n)$ with respect to $\tilde{z}_n$. Substituting (11) and (12) in (10), we arrive at

$$\delta \mathbf{f}_{n+1}^{\mu, \Delta} = \delta \mathbf{f}_n^{\mu, \Delta} - \mu\phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T \delta \mathbf{f}_n^{\mu, \Delta} - \phi(z_n^{\mu+\Delta})\mathbf{r}_n. \quad (13)$$

At $z_n = \tilde{z}_n$, (9) becomes

$$Y_{n+1}^\mu = Y_n^\mu - \mu\phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T Y_n^\mu - \phi(z_n^\mu)\mathbf{r}_n. \quad (14)$$

We define $\mathcal{D}_n^{\mu, \Delta} = \delta \mathbf{f}_n^{\mu, \Delta} - Y_n^\mu$, therefore, at $z_n = \tilde{z}_n$,

$$\begin{aligned} \mathcal{D}_{n+1}^{\mu, \Delta} &= \delta \mathbf{f}_n^{\mu, \Delta} - Y_n^\mu - \mu\phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T (\delta \mathbf{f}_n^{\mu, \Delta} - Y_n^\mu) \\ &\quad - \phi(z_n^{\mu+\Delta})\mathbf{r}_n + \phi(z_n^\mu)\mathbf{r}_n \\ &= \mathcal{D}_n^{\mu, \Delta} - \mu\phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T \mathcal{D}_n^{\mu, \Delta} \\ &\quad - \phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T (\mathbf{f}_n^{\mu+\Delta} - \mathbf{f}_n^\mu) \\ &= \mathcal{D}_n^{\mu, \Delta} - \mu\phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T \mathcal{D}_n^{\mu, \Delta} \\ &\quad - \Delta\phi'(\tilde{z}_n)\mathbf{r}_n \mathbf{r}_n^T \delta \mathbf{f}_n^{\mu, \Delta}, \quad (15) \end{aligned}$$

which is in a similar form to $\mathcal{D}_{n+1}^{\mu, \Delta}$ shown in Section III of [5]. Similar to the moment bounds of Y_n and Y_n^μ discussed in Appendix A of [1], the moment bound for $\delta \mathbf{f}_n^{\mu, \Delta}$ is given by $\limsup_n E\{(\delta \mathbf{f}_n^{\mu, \Delta})^2\} \leq O(1/\mu^2)$. Therefore, as in [5]

$$\limsup_n E\{(\mathcal{D}_n^{\mu, \Delta})^2\} = O(\Delta/\mu)O(1/\mu^2), \quad (16)$$

which implies that for the stationary system and small enough μ ,

$$\lim_{\Delta \rightarrow 0} E\left\{\left(Y_n^\mu - \frac{\mathbf{f}_n^{\mu+\Delta} - \mathbf{f}_n^\mu}{\Delta}\right)^2\right\} = 0. \quad (17)$$

Therefore, for the stationary process and in the neighbourhood of the low MSE local minima, Y_n^μ retains the interpretation as a derivative in the mean square sense.

4.1. Subspace Interpretation of Y_n and Y_n^μ

We can study the convergence behaviours of $\mathbf{f}_n$ in the signal subspace via the weight error vector $\boldsymbol{\epsilon}_n = \mathbf{f}_n - \bar{\mathbf{f}}$. It is shown in [2] that mean convergence behaviour of the CMA receiver for DS-CDMA near the global minimum of the CM cost function satisfies

$$E\{\boldsymbol{\epsilon}_{n+1}\} = (\mathbf{I} - \mu_n \sigma_b^2 \mathbf{G} \tilde{\mathbf{B}} \mathbf{G}^T) E\{\boldsymbol{\epsilon}_n\}, \quad (18)$$

where $\sigma_b^2 = E\{b^2[n]\}$ and $\tilde{\mathbf{B}} \triangleq \frac{1}{\sigma_b^2} E\{\mathbf{b}[n] \phi'(\tilde{z}_n) \mathbf{b}^T[n]\}$.

In a similar fashion of studying Y_n via Y_n^μ , we assume the stationary process and study $\boldsymbol{\epsilon}_n$ at $\mu_n \triangleq \mu$. Defining $\boldsymbol{\epsilon}_n^\mu = \mathbf{f}_n^\mu - \bar{\mathbf{f}}$ and $\boldsymbol{\epsilon}_n^{\mu+\Delta} = \mathbf{f}_n^{\mu+\Delta} - \bar{\mathbf{f}}$, (17) can be written as

$$\lim_{\Delta \rightarrow 0} E \left\{ \left(Y_n^\mu - \frac{\boldsymbol{\epsilon}_n^{\mu+\Delta} - \boldsymbol{\epsilon}_n^\mu}{\Delta} \right)^2 \right\} = 0. \quad (19)$$

Therefore, Y_n^μ also retains the interpretation as a derivative of tap weight error vector in the mean square sense.

By using the eigendecomposition of the Hessian matrix $\mathcal{H} \triangleq \mathbf{G} \tilde{\mathbf{B}} \mathbf{G}^T$, we arrive at

$$\mathcal{H} = \tilde{\mathbf{U}} \tilde{\mathbf{\Lambda}} \tilde{\mathbf{U}}^T, \quad (20)$$

where $\tilde{\mathbf{U}}$ is a unitary matrix of normalised eigenvectors and $\tilde{\mathbf{\Lambda}}$ is the diagonal matrix of corresponding eigenvalues. Substituting the Hessian matrix $\mathcal{H}$ defined by (20) in (18) and rearranging, we arrive at

$$E\{\boldsymbol{\epsilon}_n\} = \tilde{\mathbf{U}} (\mathbf{I} - \mu \sigma_b^2 \tilde{\mathbf{\Lambda}})^n \tilde{\mathbf{U}}^T \boldsymbol{\epsilon}_0, \quad (21)$$

where the expectation operator on $\boldsymbol{\epsilon}_0$ is removed since $\boldsymbol{\epsilon}_0$ is not considered a random vector. At $\mu_n = \mu$ and $\mu + \Delta$,

$$E\{\boldsymbol{\epsilon}_n^\mu\} = \tilde{\mathbf{U}} (\mathbf{I} - \mu \sigma_b^2 \tilde{\mathbf{\Lambda}})^n \tilde{\mathbf{U}}^T \boldsymbol{\epsilon}_0, \quad (22)$$

$$E\{\boldsymbol{\epsilon}_n^{\mu+\Delta}\} = \tilde{\mathbf{U}} (\mathbf{I} - (\mu + \Delta) \sigma_b^2 \tilde{\mathbf{\Lambda}})^n \tilde{\mathbf{U}}^T \boldsymbol{\epsilon}_0, \quad (23)$$

where $\boldsymbol{\epsilon}_0^{\mu+\Delta} = \boldsymbol{\epsilon}_0^\mu = \boldsymbol{\epsilon}_0$. The signal subspace eigenvectors form the column of matrix $\tilde{\mathbf{U}}_s$ and the noise subspace ones form the matrix $\tilde{\mathbf{U}}_n$, i.e., $\tilde{\mathbf{U}} = [\tilde{\mathbf{U}}_s \ \tilde{\mathbf{U}}_n]$. It is shown that the MMSE solution $\mathbf{f}_{\text{MMSE}}$ lies in the signal subspace [7]. For the binary input, it is shown in [8] that $\bar{\mathbf{f}}$ coincides with $\mathbf{f}_{\text{MMSE}}$ implying that $\tilde{\mathbf{U}}_n^T \bar{\mathbf{f}} = \mathbf{0}$. Therefore, if $\mathbf{f}_0$ also lies in the signal subspace, $\tilde{\mathbf{U}}_n^T \boldsymbol{\epsilon}_0 = \mathbf{0}$. Hence, from (22) and (23), we arrive respectively at

$$E\{\boldsymbol{\epsilon}_n^\mu\} = \tilde{\mathbf{U}}_s \left(\mathbf{I} - \mu \sigma_b^2 (\tilde{\mathbf{\Lambda}}_s - \sigma_w^2 \mathbf{I}) \right)^n \tilde{\mathbf{U}}_s^T \boldsymbol{\epsilon}_0, \quad (24)$$

$$E\{\boldsymbol{\epsilon}_n^{\mu+\Delta}\} = \tilde{\mathbf{U}}_s \left(\mathbf{I} - (\mu + \Delta) \sigma_b^2 (\tilde{\mathbf{\Lambda}}_s - \sigma_w^2 \mathbf{I}) \right)^n \tilde{\mathbf{U}}_s^T \boldsymbol{\epsilon}_0, \quad (25)$$

where $\tilde{\mathbf{\Lambda}}_s$ denotes the diagonal matrix of nonzero eigenvalues of $\mathcal{H}$.

Setting $\mathbf{f}_0^\mu$ in the signal subspace implies faster convergence speed for both the least-mean-square (LMS) algorithm [9] and CMA [2]. From (19), (24) and (25), it is shown that Y_n^μ can be bounded within the signal subspace provided that the initialisation $\mathbf{f}_0^\mu$ is in the signal subspace. This bounding of Y_n^μ implies that the convergence speed and time constant of the algorithm can be assessed by considering *only* the eigenvalues associated with the eigenvectors spanning the signal subspace [2, 9].

5. NUMERICAL RESULTS

We considered a synchronous DS-CDMA system with spreading gain 31 ($N = 31$). Without loss of generality, the first user was the desired user with unity power. The background noise was zero mean AWGN with SNR=20 dB referenced to the desired user. We considered the performance of adaptive step-size receivers in a dynamic environment where different user groups switched between three user groups according to a Markov chain. The dynamic environment was set similarly to that of [3] with minor modification. The power of the first user was set such that its SNR was 20 dB and the multiple-access interference (MAI) user groups were designated as follows. User group 1 consisted of 17 MAI users transmitting at 20 dB and two MAI users transmitting at 30 dB. User group 2 consisted of 26 MAI users transmitting at 20 dB. Finally, user group 3 consisted of eight MAI users transmitting at 20 dB and one MAI user transmitting at 30 dB. The MAI structure was constructed according to a Markov chain $x[n]$ with transition probability matrix

$$\mathbf{P} = \begin{bmatrix} 0.998 & 0.01 & 0.01 \\ 0.01 & 0.998 & 0.01 \\ 0.01 & 0.01 & 0.998 \end{bmatrix}. \quad (26)$$

and state space $\{(1, 0, 0)^T, (0, 1, 0)^T, (0, 0, 1)^T\}$. The Markov chain $x[n]$ designates the MAI user group which affects the desired user detection at the n th symbol. The switching between the three MAI user groups occurred at every 500 symbols. Five-ray multipath fading channels were used for both the desired user and all the MAI users for all user groups. The last four rays were uniformly distributed in delay over $[0, 10T_c)$, where T_c is the chip interval, with standard deviation 0.3. Averaged SINR was used as a performance measure and all plots were averaged over 50 Monte-Carlo runs.

We compare the SINR performances of the AS-CMA receiver, the AS-MOE receiver [3] and the AS-SA receiver [5]. All the receiver lengths were chosen to be twice that of the spreading gain. For all AS algorithms, the adaptation rates α were 10^{-7} and the upper and lower step-size limits μ_+, μ_- were set respectively at 10^{-2} and 0. The averaged SINR plots of all algorithms are compared in Fig. 1. Notice that while both the AS-SA and AS-CMA receivers have an ability to track variations of multipath channels, the AS-MOE receiver fails to operate due to mismatch. The SINR plot of AS-CMA almost coincides with that of AS-SA indicating the comparable performance of the two algorithms.

In Fig. 2, the steady-state SINR of the AS-CMA receiver for different initial settings of step-sizes $\mu_0 \in [10^{-8}, 10^{-3}]$ are compared with that of the AS-SA receiver. Despite a 100 000-fold variation in μ_0 , it is shown that the SINR performances of both receivers are maintained at a constant level. The AS-CMA is also shown to achieve a comparable SINR performance as the AS-SA. This result is attributed to the excellent performance of the CM criterion which has been shown to be near the Wiener receiver [10, 6]. Since the SINR results of AS-MOE are approximately at -12 dB throughout the setting range of μ_0 , AS-MOE is therefore discarded in the plot.

Fig. 3 shows the trajectories of μ_n of the AS-CMA receiver and the AS-SA at three different settings of the initial step-size $\mu_0 = \{10^{-8}, 10^{-5}, 10^{-3}\}$. The trajectories were plotted for 80 000 symbols. Insensitivity to the settings of the initial step-size for each algorithm can be observed. The adaptation behaviour of μ_n

Figure 1: The SINR performance of AS-SA [5], AS-MOE [3] and AS-CMA receivers in a Markovian dynamic environment. The AS-MOE receiver fails to operate due to mismatch. AS-SA and AS-CMA are comparable in terms of convergence speed and steady-state performance.

Figure 2: SINR comparison of AS-SA and AS-CMA at different settings of initial stepsizes μ_0 . It is shown that both AS-SA and AS-CMA are not affected by various initial settings despite 100 000-fold of variation of μ_0 . Notice an almost identical performance of both algorithms.

for AS-CMA is similar to that of AS-SA. The convergence speed of both algorithms are approximately the same.

6. CONCLUSION

Convergence behaviours of the adaptive step-size CMA algorithm for a DS-CDMA receiver are studied. Analysis reveals similarity in convergence behaviours of the Y_n process of AS-CMA and AS-SA. Simulation shows the effectiveness of the AS-CMA algorithm in both interference cancellation and channel equalisation for a nonstationary multipath fading channel. A close relation in the step-size trajectories between the blind AS-CMA algorithm and the non-blind AS-SA algorithm is demonstrated. Robustness to variation of the initial settings of the step-size of AS-CMA is also shown.

Figure 3: Trajectories of μ_n for AS-SA and AS-CMA at different settings $\mu_0 = \{10^{-8}, 10^{-5}, 10^{-3}\}$. Insensitivity to variation in the initial step-size settings for each algorithm can be observed. Similarity in adaptation behaviours of μ_n of both algorithms is shown.

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