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A NOVEL ORTHOGONAL SET ADAPTIVE LINE ENHANCER TUNED WITH FOURTH-ORDER CUMULANTS

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ABSTRACT

This paper introduces a novel adaptive line enhancer (ALE) structure. The adaptive filter has an autoregressive-moving average (ARMA) structure which is based on classical Laguerre orthogonal functions. The frequencies and radii of the poles within the orthogonal set admit tuning. Tuning of these terms in this contribution involves the use of higher order statistics. Indeed the properties of fourth order cumulants are exploited to aid harmonic retrieval of the sinusoids embedded in the input signal. Furthermore adaptation of the feedforward filter parameters is straightforward since they are linearly related to the filter output. Advantages of stability, short filter length and robustness in the presence of noise are expected. Simulation results are included to show typical performance.

1. INTRODUCTION

The detection of sinusoids in noise is a recurrent problem in signal processing. When no *a priori* knowledge of the possibly time-varying sinusoidal parameters is available an adaptive solution is preferred. The first adaptive filtering approach was proposed by Widrow et al. [1] in a key-note paper in 1975. The operation of this technique, based on an Adaptive Line Enhancer (ALE), is effected by virtue of the predictability of the sinusoidal component(s) compared to the background noise. It has found many application areas which include sonar, biomedical and speech signal processing.

ALEs can be categorised by their structure, namely Finite Impulse Response (FIR) or Infinite Impulse Response (IIR). Initial development of the ALE centered on FIR filters [2,3] because adaptation is straightforward. Unfortunately, such structures are frequently sub-optimal for modelling the underlying process. To achieve a given performance at low signal-to-noise ratio (SNR) increasingly long filter

lengths are required. An IIR filter structure is an optimal solution [4-6], since significantly fewer parameters are needed to achieve a desired performance. However, problems of instability and non-linearity arise due to the feedback in IIR filters. Additionally, the non-linear dependence of the error on the filter parameters can cause local minima in the error performance surface, which influences the system operation.

In this paper a novel ALE structure is proposed which has an ARMA form capable of producing a narrowband frequency response at low SNR combined with linearly related adaptive coefficients. Moreover, this new structure, the Orthogonal Block ALE (OBALE) employs fourth-order cumulants in its adaptation which is expected to improve performance when the input noise is zero mean Gaussian.

2. PROPOSED ORTHOGONAL BLOCK ALE ARCHITECTURE

2.1. Adaptive Filter

2.1.1. Structure

The transfer function, $H(z)$, of an FIR filter can be viewed as the linear combination of a set of functions, the delay elements $\{z^{-i}\}$, orthogonal around the unit circle. In the OBALE the function set z^{-i} is replaced by a set, $G_i(z)$, which obeys the same rules of orthogonality [7]. The orthogonality of the set is important to ensure the optimum error in the least squares sense is achieved [8]. $H(z)$ becomes the weighted sum of the individual components of the orthogonal set

$$H(z) = \sum_{i=0}^{N-1} a_i G_i(z) \quad (1)$$

The linearity between the coefficients $\{a_i\}$ and the function set $G_i(z)$ is retained irrespective of the individual nature of the components of $G_i(z)$. This allows each $G_i(z)$ "block" to be set up as an IIR filter without loss of the overall transversal structure Figure 1. Moreover, it implies that a simple adaptation algorithm such as Least Mean Squares (LMS) or Fast

Least Squares (FLS) can be used to adapt the coefficients $\{a_i\}$ [9].

The chosen set for $G(z)$ can be any suitable orthogonal set preferably with complete properties. In this paper classical Laguerre functions have been chosen. Laguerre functions provide an advantage over other classical series such as Legendre functions in that both real and complex realisations of the filter are possible. These functions have previously been employed to good effect in transient analysis, network synthesis and most recently in adaptive echo cancellation [10,11].

In the real case the individual set components are calculated using a procedure outlined by Broome [12]. $G_0(z)$ is chosen to be unity. The n th term in the set, $G_n(z)$, $n > 1$ (n odd), is as follows

$$G_n(z) = \frac{(z+1)}{(z - r_n e^{j2\pi\nu_n})(z - r_n e^{-j2\pi\nu_n})} \quad (2)$$

$$\times \prod_{i=1}^{n-1} \frac{(1 - z r_i e^{j2\pi\nu_i})(1 - z r_i e^{-j2\pi\nu_i})}{(z - r_i e^{j2\pi\nu_i})(z - r_i e^{-j2\pi\nu_i})}$$

where r_n is the radius and $\pm\nu_n$ the frequency of the n th complex conjugate pole pair. $G_n(z)$ can be seen to consist of two parts, a bandpass filter centered at frequency ν_n , with a single zero at $z = -1$, and an all-pass filter. The $(n+1)$ th term replaces the zero with one at $+1$, but is otherwise identical.

Similarly we can form a complex 1st-order set of orthogonal function blocks. For the complex case the k th term in the set is as follows

$$G_k(z) = \frac{1}{(z - c_k)} \prod_{i=1}^{k-1} \frac{(1 - A_i(z))}{(z - c_i)} \quad (3)$$

where $G_0(z)$ is unity, $c_k = r_k e^{j2\pi\nu_k}$. The set of coefficients $\{c_k\}$ are therefore complex. When all the radii, $\{r_k\}$, are equal and all the frequencies, $\{\nu_k\}$, are set to zero the discrete Laguerre functions are obtained.

2.1.2 Implementation

The orthogonal block filter can be realised as a "lattice" filter type structure, similar to the expansion of Nie et al. of an orthonormal rational function [13]. In the real case each orthogonal block output, G_{2n+1} , is related to the previous term, G_{2n-1} , as follows

$$G_{2n+1}(t) = G_{2n-1}(t-2) - b_n G_{2n-1}(t-1) \quad (3)$$

$$+ r^2(G_{2n-1}(t) - G_{2n-1}(t-2)) + b_{n+1} G_{2n-1}(t-1)$$

$$n=1,2,3,\dots,p$$

where r is the fixed pole radius, $b_n = 2r \cos(2\pi\nu_n)$ and p the total number of complex pole pairs in the filter. The resulting structure is shown in Figure 2.

Note the same relationship exists between the n even terms, i.e. G_{2n}, G_{2n+2} . Stability of the filter has been ensured by constraining the pole radii to lie on a fixed circle of radius less than but close to unity. A corresponding reduction in the number of adaptation calculations required is achieved.

For the complex case neighbouring terms are related as follows

$$G_k(t) = G_{k-1}(t-1) - c_{k-1}^* G_{k-1}(t) + c_k G_k(t-1) \quad (4)$$

This can be realised in a similar "lattice" type structure to the real case.

2.2. Harmonic Retrieval of Sinusoids

The performance of the OBALE is highly dependent on a good initial choice of the pole frequencies within the orthogonal blocks. To achieve this an initial estimate of the pole positions is formed through the cumulant-based approach to the harmonic retrieval problem of Swami and Mendel [14]. Closely related to Pisarenko's method [15], this method utilises fourth order statistics in place of Pisarenko's second order terms. It does so to take advantage of the cumulant properties of Gaussian noise. Above second order the cumulants of a Gaussian process are zero independent of whether the noise is white or coloured. The OBALE is required to operate at very low levels of SNR, below -15dB, and so this feature is expected to aid pole selection. Fourth order cumulants are necessary as, in the absence of any phase coupling, the third order cumulant, $C_x^3(\tau)$, of a sum of sinusoids is zero.

Briefly the harmonic retrieval approach is as follows. Assume zero mean signals and that the input signal is given as

$$y(t) = x(t) + w(t) \quad (5)$$

where $x(t)$ is the sum of p sinusoids

$$x(t) = \sum_{i=1}^p a_i \cos(\omega_i t + \phi_i) \quad (6)$$

ϕ_i are iid random variables uniformly distributed over $(-\pi, \pi)$ and $w(t)$ is Gaussian noise. Estimate the fourth order cumulants of the input signal, $y(t)$, along a diagonal slice of the trispectrum. Typically all delays are chosen to equal τ . Since $w(t)$ is Gaussian its fourth order cumulant $C_w^4(\tau)$ is zero and thus

$$C_y^4(\tau) = C_x^4(\tau) \quad (7)$$

This leads to a set of linear equations of the form

$$C_y \underline{a} = \underline{0} \quad (8)$$

where C_y is a Toeplitz cumulant matrix and $\underline{a}$ is the eigenvector corresponding to its zero eigenvalue. As with Pisarenko's method [15] the sinusoid frequencies are given by the roots of a polynomial $A(z)$ where $A(z)$ equals

$$A(z) = \sum_{i=0}^{2p} a_i z^{-i} \quad (9)$$

If the additive noise is non-Gaussian, i.e. it has non-zero kurtosis, then the right hand side of equation 6 is replaced by γ_{4w} , the fourth order cumulant of the noise.

2.3 Operation

The OBALE operation consists of two parts as shown in Figure 3. Initially, the cumulant estimator runs alone. An estimate of the number and location of sinusoids within the input signal is made based on the fourth order cumulants of the input signal. The associated frequency estimates are fed to the orthogonal block coefficients $\{b_i\}$, and full operation commences.

As the OBALE runs the output error is passed to the adaptive algorithm. The Normalised Least Mean Squares (NLMS) algorithm [9] varies the weighting of the adaptive coefficients $\{a_i\}$. NLMS is chosen to overcome a change in signal variance due to the orthogonal set. At time t the NLMS algorithm for the OBALE is

$$\underline{a}(t) = \underline{a}(t-1) + \frac{\mu}{\hat{\sigma}_g^2(t)} \underline{g}(t-D) \underline{e}(t) \quad (10)$$

where μ is the adaptive gain constant, $\underline{g}(t)$ the orthogonal block output vector, $\underline{e}(t)$ the ALE output error and $\hat{\sigma}_g^2(t)$ an estimate of the signal power. The signal power is calculated as follows for each block output

$$\hat{\sigma}_g^2(k,i) = \lambda \hat{\sigma}_g^2(k-1,i) + (1-\lambda) |g(k,i)|^2 \quad (11)$$

where λ is a forgetting factor.

The ALE output is feedback into the cumulant estimator which is periodically checked to see if there is any improvement in the pole position estimate. If so, the revised estimate is fed to the coefficients $\{b_i\}$ and operation continues as before.

3. SIMULATION RESULTS

An example of the simulation results achieved with the OBALE is presented in Figure 4 and Figure 5.

The input to the ALE consists of a single sinusoid at normalised frequency 0.1Hz in white Gaussian noise with a SNR of -15dB. The ALE output spectrum is taken after 3000 iterations. A significant increase in the output SNR is observed even though the OBALE has not fully converged. The SNR at the OBALE output improves further as convergence is reached. The levels of improvement in SNR found with the OBALE compare with those of previous workers [4,5]. Its performance at low SNR is particularly promising. No stability problems are encountered with the linear adaptation.

4. CONCLUSION

A novel ALE structure has been presented which links orthogonal functions with fourth-order cumulants. The OBALE allows an IIR structure to be implemented and adapted by a simple algorithm, with no stability problems.

5. ACKNOWLEDGEMENT

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Figures

Figure 1. Orthogonal Block Filter

Figure 2. Real Orthogonal Block Realisation

Figure 3. Orthogonal Block Adaptive Line Enhancer

Figure 4. Power Spectrum of OBALE Input, -15dB SNR

Figure 5. Power Spectrum of OBALE output