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TRANSONIC LAMINAR OPEN
CAVITY INSTABILITY**

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ABSTRACT

A transonic air stream flowing over a rectangular cavity or enclosure is unsteady. At certain flow conditions, large scale oscillations develop in the shear layer which, for an open cavity, spans across the whole enclosure. The convected shear layer instabilities interact with the cavity geometry, they generate resonance, and induce large amplitude wall pressure fluctuations, aerodynamic pressure drag and noise radiation.

A numerical method has been developed to analyse the physics of the cavity flow instability. A specific interest of this study is to investigate the nature of the feed-back loop in the transonic laminar regime, where a flow-resonant feed-back may be complementing the flow-acoustic resonance documented in past work. A Mach 1.5 laminar cavity flow is modelled in which the selected numerical method gives low dispersion and dissipation. Discrete solutions of the short-time averaged laminar Navier-Stokes equations, the flow governing equations, are obtained by a finite volume integral approach. A monotone upwind flux interpolation method by Mensink is used to obtain second order accurate solutions in space. This is based on the Roe approximate Riemann solver. A fourth order Dispersion Relation Preserving scheme time-advances the flow history; this is a multi-step Runge-Kutta type integration method.

The available results indicate that the method is able to reproduce the unsteady self-sustained character of the flow. A dominant cavity mode characterises the instability. The dominant mode frequency is determined by (i) the cavity streamwise length, (ii) the shear layer convection speed, (iii) the feed-back pressure and momentum wave phase speeds in the enclosure, and (iv) the shear layer receptivity phase delay at the upstream cavity edge. A comparison of phase matched ‘instantaneous’ density fields from uniform (baseline) and uniform refined computational grid models shows a similar shear layer motion and

wave pattern outside the enclosure. Inside the enclosure, similar waveforms of wall pressure history are predicted by both models, indicating stationarity. A slightly higher amplitude in the refined grid case is due to the sharper feed-back pressure wave being captured in the enclosure.

The major physics of supersonic cavity flow instability is resolved on the uniform baseline computational grid. Different levels of resolution of the upstream pressure wave, propagating inside in the cavity, lead to overall similar shear layer mode shapes. This is evidence of a flow resonant feed-back being present in the modelled flow, in which the unsteady vorticity and momentum fields in the cavity combine with the upstream propagating pressure wave, complementing the flow-acoustic resonance.

NOMENCLATURE

$\mathbf{F}$	Flux vector
I	Pressure wave intensity, W/m^2
$\mathbf{I}$	Identity matrix
Pr	Prandtl number, 0.726
R	Specific gas constant, $287J/(kgK)$
$\mathbf{W}$	Conservative variable vector

Subscripts

i	Inviscid
v	Viscous
∞	Free stream

INTRODUCTION

A current interest of the aeroacoustic community is to address the resonance that characterizes the unsteady compressible flow over a rectangular cavity or enclosure. At certain flow regimes, this simple geometry is known to generate a complex unsteady motion in the surrounding medium (Fig. 1). This is characterized by a flapping shear layer that may involve vortex shedding, an unsteady flow recirculation in the enclosure, and multiple crossing pressure waves propagating away from the enclosure.

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The pioneering work by Rossiter¹ focused on subsonic cavity aeroacoustics, representative of contemporary landing gear wells and bomb bay flows. Recent studies, including Zhang,² confirm that an open cavity in a transonic aircraft can generate an intense pressure fluctuation, increase drag, pitching moment, and noise. There is a sustained interest in improving enclosure aeroacoustics during weapon delivery, e.g. Suhs³ and Jechura,⁴ stimulating academic research and dedicated flight tests.

Time accurate numerical models have been developed by Zhang et al.,⁵ Sinha et al.,⁶ Takakura et al.⁷ to reproduce and study essential physics of turbulent cavity flow. Early laminar models were obtained by Tam et al.⁸ More recently, Rona & Dieudonné⁹ showed some interesting features of the laminar flow motion, such as significant leading edge shock fluctuations and a variable shear layer separation point. These features stimulated the present investigation which uses a similar laminar model. The absence of an eddy viscosity term and a second order accurate integration method give a moderate numerical dispersion and dissipation in these predictions.

The aim of the current investigation is to gain a further understanding of the cavity flow physics, as the nature of resonance in the flow underpins the development of instability suppression methods.

The review of cavity flows by Rockwell & Naudascher¹⁰ identifies two types of resonance: flow-acoustic and flow-resonant. In flow-acoustic type cavities, feed-back is provided by an upstream travelling sound wave. At flow-resonant regimes the unsteady recirculation in the enclosure, featuring vorticity and momentum fluctuations, perturb the shear layer at the upstream edge, generating a self-sustained instability. The compressible flow regime under investigation was shown by Rona & Dieudonné⁹ to display both large amplitude pressure waves and unsteady flow recirculation in the model flow. It is of interest to further clarify the main feed-back driving mechanism.

FLOW CONDITIONS

A transonic flow develops over a rectangular enclosure of length to depth ratio 3, shown in Fig. 1. All dimensions are normalized by the 15mm cavity depth D . The model flow computational domain extends $15D$ in the streamwise direction and $5D$ in the normal direction. The flow is cold and the free stream stagnation temperature T_s and pressure p_s are 288.5K and 197.51kN/m². At the inlet boundary b1, the free stream speed U_∞ , Mach number M_∞ , density ρ_∞ , static temperature T_∞ and static pressure p_∞ are respectively 425.2m/s, 1.5, 0.937kg/m³, 200K, 53.801kN/m². Close to the wall, a laminar boundary layer approaches the cavity from the inlet b1. At b1

the the boundary layer thickness δ_{99} is $1/3D$, the displacement thickness δ_1 is $0.093D$, and the momentum thickness δ_2 is $0.037D$.

All cavity flow results are normalized by the above free stream values.

NUMERICAL MODEL

A laminar model for the cavity flow is obtained by solving the short-time averaged Navier-Stokes equations for laminar flow:

$$\frac{\partial \mathbf{W}}{\partial t} + \nabla \cdot (\mathbf{F}_i + \mathbf{F}_v) = 0, \quad (1)$$

$$\mathbf{W} = (\rho, \rho \mathbf{u}, \rho e_s)^T, \quad (2)$$

$$\mathbf{F}_i = (\rho \mathbf{u}, \rho \mathbf{u} \mathbf{u} + p \mathbf{I} / \rho, \rho \mathbf{u} h_s)^T, \quad (3)$$

$$\mathbf{F}_v = (0, -\boldsymbol{\tau}, -\boldsymbol{\tau} \cdot \mathbf{u} - \mathbf{q})^T, \quad (4)$$

where ρ , $\mathbf{u}$, p , $\mathbf{q}$, h_s , and e_s are the fluid density, flow velocity, pressure, heat flux vector, stagnation enthalpy and specific stagnation energy. $\mathbf{I}$ is the identity matrix and $()^T$ the transpose operator. The stagnation enthalpy and specific stagnation energy are defined in the following auxiliary relationships:

$$h_s = e_s + p / \rho, \quad (5)$$

$$e_s = C_v T + \mathbf{u} \cdot \mathbf{u} / 2, \quad (6)$$

where C_v is the specific heat at constant volume. The viscous stress tensor $\boldsymbol{\tau}$ is modelled according to Stokes hypothesis¹¹ and the heat flux vector $\mathbf{q}$ models thermal conduction in the flow. The molecular viscosity μ_l is derived from Sutherland's law (e.g. Schlichting¹²). Respectively,

$$\boldsymbol{\tau} = \mu_l [\nabla \mathbf{u} + \mathbf{u} \nabla - (2/3) \mathbf{I} \nabla \cdot \mathbf{u}], \quad (7)$$

$$\mathbf{q} = (\mu_l / Pr) C_p \nabla T, \quad (8)$$

$$\mu_l = 1.458 \times 10^{-6} \frac{T^{1.5}}{T + 110.4}, \quad (9)$$

where C_p is the specific heat at constant pressure. The equation of state, $p = \rho RT$, completes the governing equations.

The cavity inlet boundary flow at b1 is fixed and remains constant throughout the computation. The inlet boundary layer is laminar and the velocity profile was obtained from a numerical solution of the Blasius equation using Chebychev polynomials, as implemented by Zhang & Edwards.¹³ No-slip boundary conditions are imposed at the walls (b2 - b5). As the outflow is mainly supersonic, the conservative variables are extrapolated from the interior (first order), likewise on the normal boundary b8.

The discrete form of the flow governing equations are integrated over the computational domain to obtain a finite volume approximation of the flow field.

Fig. 1 Cavity computational domain.

The computational domain is discretized with a baseline grid and a finer grid, to assess the effects of grid size on the model flow predictions. The baseline model uses a rectangular regular grid 200×160 ($y > 0$) and 40×40 ($y \leq 0$). The 50% finer grid is also of rectangular uniform topology, 300×240 ($y > 0$) and 60×60 ($y \leq 0$). The unit cell aspect ratio is 3 in both models. The baseline and finer grid respective unit cell sizes $(\Delta x, \Delta y)$ are $(0.025D, 0.075D)$ and $(0.017D, 0.05D)$. The gridding of a selected region of size $(4D \times 1.5D)$ is shown in Figs. 2(a, b) for both models.

A multi-domain decomposition is performed to obtain topologically orthogonal grids, defined as adjacent computational domain partitions. The upwind cell centred method implemented by Mensink¹⁴ is adopted to estimate the inviscid fluxes at the cell interfaces. Second order accuracy in space is obtained through the use of the Monotone Upwind Scheme for Conservation Laws (MUSCL) interpolation technique, together with the minmod limiter. For the presented work, the approximate Riemann solver of Roe^{15, 16} is used.

A central difference scheme is used to obtain the gradients of the conservative variables in the viscous fluxes $\mathbf{F}_v$. Flux integration is performed over the finite volume boundary by an explicit multi-step Runge-Kutta method by Hu et al.¹⁷ Standard Runge-Kutta coefficients (1.0, 0.5, 0.33, 0.25) give fourth order formal accuracy. Two additional coefficients ($\beta_5 = 0.39017, \beta_6 = 0.17615$) are used to optimize the dispersion relation preserving characteristics of the time advancement scheme. The two-step low storage implementation detailed in Hu et al.¹⁷ is adopted. A constant time step $\Delta t = 0.01D/U_\infty$ is used to facilitate the prediction post-processing. This corresponds to ~ 0.13 Courant number for the baseline case and ~ 0.26 Courant number for the refined grid case.

At the beginning of the computation, the flow state described in Fig. 2(c) is imposed. The same condition is used for the baseline and grid refined models. Fur-

Fig. 2 Details of the enclosure. (a) baseline grid, (b) refined grid, (c) initial flow conditions.

ther details of the initial flow conditions are given in Rona & Diudonné.¹⁸

RESULTS AND DISCUSSION

After an initial transient, the predicted flow sets in a self-sustained oscillatory mode. The baseline flow unsteadiness is governed by a dominant mode and the computational Schlieren sequence of Fig. 3(a – e) shows four successive phases within one period. Figures 3(a) and 3(e) are taken at the beginning and at the end of one period and are therefore very similar. They are phase referenced to a periodic absolute minimum in the wall pressure history from pressure point 1 in Fig. 1. The respective minima are shown by the arrow in Figs. 7(a, b). All main features in the density

Fig. 3 Computational Schlieren for the baseline model. (a,b,e) mass ejection, (c,d) mass injection. 45° knife edge.

fields of Fig. 3(a) and Fig. 3(e) appear to be phase matched.

Figure 3(a) shows a shear layer spanning across the cavity opening and reattaching on the downstream edge. This is an ‘open’ cavity flow, as defined in Charwat et al.¹⁹ Shear layer flapping at the downstream edge alternates phases of fluid injection and ejection between the cavity and the free stream. In Figs. 3(a,b,e) fluid is ejected past the downstream edge as the shear layer deflects away from the enclosure. Further downstream, the ejection from a previous cycle is being convected over the wall towards the outlet boundary b7. The periodic convection of ejected flow increases the thickness of the reattached boundary layer. Figures 3(c,d) show the fluid injection phase. The shear layer bends towards the cavity interior, impinging against the downstream wall. Fluid is drawn from the free stream and enters the cavity in the neighbourhood of the downstream edge.

Above the enclosure, a complex wave pattern re-

sults from the shear layer unsteadiness. At the upstream edge a shock wave develops at approximately the Mach angle $\theta = \arcsin(1/M_\infty)$, driven by the shear layer normal displacement beneath it. The shear layer motion results in fluctuations in shock strength and position, so that a somewhat steeper angle is captured in Figs. 3(a,b,e). An upstream moving pressure wave propagates through the shear layer and in the supersonic flow above it. It originates on the downstream wall during the fluid injection phase due to mass impingement. In Fig. 3(d) the initial development of this pressure wave is captured. The wave stems from the downstream corner towards a region of relatively uniform flow, its main interference being with a normal wave above the ejected flow over the downstream edge. The angle between the wave front and the downstream direction is less than the Mach angle. The wavefront is therefore propagating upstream.

During mass ejection, Figs. 3(a,b,e), the wave pattern above the downstream edge becomes more complex. At this location, the supersonic flow circumvents the fluid mass being ejected. The b6 wall boundary then turns the circumventing flow on itself, causing a compression shock above the edge. A second arc shaped pressure wave is also visible above the downstream corner. It is a distinct feature from the trailing edge shock as it propagates further in the free stream at a more normal angle to the streamwise direction. Figure 3(a) shows this wave interfering with the most upstream leading edge shock in the (x,y) neighbourhood of $(4D, 3.5D)$. The arc shaped bright line identifies a downstream travelling pressure wave that moves with the shear layer convected instability. The difference in wave strength between the trailing edge shock and the travelling pressure wave is highlighted in Fig. 4(a), where ‘instantaneous’ density contours are shown for the mass ejection phase. The trailing edge shock induces a sharp density change, indicated by the intense contour packing at the cavity trailing edge. The travelling pressure is a weaker compressible feature which is just identifiable in Fig. 4(a): contour lines in the high speed flow above the cavity run mostly parallel to the upstream edge shock. An s-shaped inflection of the contour lines can be noticed just above the cavity ejection at the position of the convecting pressure wave.

A sample ‘instantaneous’ density field from the grid refined model is shown in Fig. 4(b). The result is phase matched to Fig. 4(a), based on the shear layer position and the wave pattern above it. Inside the enclosure, a forward travelling pressure wave is captured close to the upstream wall. In the baseline model such feature is not shown close to the upstream edge, yet the shear layer mode shape and transonic wave pattern in the high speed flow match. This result indicates that, at

Fig. 4 Instantaneous density contours. (a) baseline grid, (b) refined grid. $\rho_{min} = 0.245\rho_{\infty}$, $\Delta\rho = 0.03\rho_{\infty}$.

the selected flow conditions, the fundamental nature of the cavity flow instability is marginally affected by the accuracy in modelling the pressure wave in the enclosure. As such, the instability feed-back loop seems to be influenced by the cavity acoustics, rather than driven by it. At the current test conditions, the model flow is of a flow-resonant cavity in which the dominant instability mode period is mainly dependent on (i) the enclosure streamwise length, (ii) the shear layer convection speed, (iii) the feed-back vorticity and momentum wave phase speeds in the enclosure, and (iv) the shear layer receptivity phase delay at the upstream cavity edge.

The close similarity between the baseline and the refined grid model flows is further examined by the computational Schlieren method. Figures 5(a–e) are a time sequence from the refined grid model. The sequence is phase matched to Figs. 3(a–e). In both computations, the phase is referenced to the wall pressure history at $(2.33D, -1D)$. Figure 3(e) corresponds to Fig. 5(e). The pair shows a wave system outside the enclosure in which the corresponding wave fronts are captured at essentially the same position.

Above the leading edge, Figs. 3(e) and 5(e), an unsteady oblique shock stems from $x \sim -3.4D$. This is due to the shear layer motion at the leading edge transmitting its unsteadiness upstream via the (left) pressure wave characteristic in the boundary layer. In this region, the flow is supersonic above the boundary layer. Only the portion of the pressure wave below the sonic line, close to the surface, may propagate upstream. The intensity I of this two-dimensional wave reduces with upstream propagation by geometric scaling for a two-dimensional wave, that is $I \propto 1/\sqrt{x^2 + y^2}$. The wave generated one period later stems from $x \sim -0.1D$ above the upstream edge in Figs. 3(e) and 5(e). This wave is closer to the

Fig. 5 Computational Schlieren for the refined grid model. (a,b,e) mass ejection, (c,d) mass injection. 45° knife edge.

shear layer and appears brighter in the computational Schlieren. It identifies a more intense pressure wave and provides some confirmation for wave amplitude reduction with upstream propagation in this part of the flow domain.

The shear layer over the cavity open surface appears split at $x \sim 1.5D$ in Figs. 3(e) and 5(e). This change in density gradient is likely to indicate the presence of a convected vortex in the compressible shear flow. The vortex roll up phase is captured in Figs. 4(a, b). At the upstream edge, the density contours from the boundary layer form a hearpin loop around the separating flow. The centre of this loop identifies a low pressure area in the vortex core. In a cavity flow with an upstream laminar boundary layer, vortex shedding is a less prominent feature than with a turbulent boundary layer flow, as discussed by Rona & Dieudonné.⁹ The shear layer mode is essentially of a ‘flapping’ type. This work gives some evidence of the existence of vortex shedding at the laminar flow regime, as prompted

in previous work.⁹

Predictions of wall pressure history are obtained around the enclosure perimeter at the four locations detailed in Fig. 1 (points 1,2,3 and 4). The pressure records on the cavity floor are shown in Figs. 6(a,b) and Figs. 7(a,b). Figures 6(a,b) refer to the pressure tapping at point 1 and Figs. 7(a,b) to pressure point 2. After an initial transient, the flow converges towards a self-sustained oscillatory state. The stationary statistics based on Figs. 7(a,b) confirm the presence of a dominant mode of frequency $f = 0.092U_\infty/D$ (baseline flow) and the abscissa tf in Figs. 6(a,b) and Figs. 7(a,b) indicates the number of dominant mode periods or ‘cycles’ for each model flow.

The initial transient develops in the baseline model flow from the start of the computation, $t = 0$, to $t \sim 200D/U_\infty$, which is about 20 cycles. During this transient, the cavity natural modes grow in amplitude and saturate. The frequency characteristics of the stationary pressure field are measurable after $tf \sim 20$. Close to the downstream edge, where the flow field is most unsteady, the dominant mode amplitude shows some variation up to $tf \sim 40$. At $x = 2.33D$, the pressure fluctuation amplitude is largest among all monitoring positions (Figs. 6-9). The extended duration of the pressure non-stationarity at $x = 2.33D$ over $0 \leq tf \leq 40$ is a local effect which is associated to the non-linear pressure wave generation at the trailing edge. The pressure history at $x = 2.33D$ also shows a step-like change at $tf \sim 54$. This is due to the monitoring position being adjusted by $3D/40$ in the streamwise direction at its final position of $x = 2.33D$. The flow is not forced throughout the computation and the onset and establishment of flow resonance is self-determined by the model.

The predictions from the refined grid model, Fig. 6(b) and Fig. 7(b), disclose a similar trend. The shorter time history reflects limitations in the computational resources. The initial transient appears to extend to $tf \sim 20$ and the pressure history indicates a flow that is slower to converge to stationary self-sustained unsteadiness. This increased randomness in the predicted pressure is also present when selective grid refinement (i.e. grid clustering) is used, as in Rona and Dieudonné.¹⁸ The grid refinement test cases consistently show a reduced level of stationarity, leading to cycle to cycle variations in phase of the dominant mode.¹⁸ The cause of such phase drift and non-stationarity is the dispersion relation preserving characteristics of the second order (space) accurate scheme. In Roe’s second order method, the frequency resolution of the propagating wave characteristics in the computational domain is grid size dependent.^{20,21} As the computational grid is refined, the numerical scheme supports the non-dissipative propagation of

Fig. 6 Wall pressure history on the upstream cavity floor. (a) baseline grid, (b) refined grid.

Fig. 7 Wall pressure history on the downstream cavity floor. (a) baseline grid, (b) refined grid.

Fig. 8 Wall pressure history on the cavity downstream edge. (a) baseline grid, (b) refined grid.

Fig. 9 Wall pressure history near the outflow boundary. (a) baseline grid, (b) refined grid.

higher wavenumber waves. The non-linearity of the Navier-Stokes equations allows interaction between these resolved higher wavenumbers and the dominant cavity modes which lie in a lower wavenumber band. Specifically, the interaction causes a two-way energy transfer across the wavenumber spectrum: Energy is transferred from the dominant modes to higher wavenumbers by dispersion and diffusion, and energy is back-scattered from these high wavenumbers to the lower end of the spectrum. As the refined computational grid better resolves the high wavenumbers, the back-scatter influences the phase coherence of the dominant cavity mode.

The pressure fluctuation peak-to-peak amplitude in Fig. 6(b) and Fig. 7(b) is similar to the baseline case prediction, while the dominant frequency is lower. The result indicates that the kinetic energy in the refined grid unsteady flow is approximately of the same order of magnitude of the baseline model. Therefore the dominant feedback mechanism of the flow instability appears to be captured on the baseline grid. The different levels of grid refinement lead to different acoustic fields inside the enclosure. The refined mesh model is able to capture details of upstream propagating pressure waves that modulates the shear layer instability. At the test conditions, flow-resonance is confirmed to be the driving process that generates flow instability at the observed amplitude. The presence of pressure waves in the enclosure drives the unsteadiness at a lower frequency. This flow-acoustic interaction is a secondary, non-negligible effect.

The predicted pressure history over the downstream edge, shown in Figs. 8(a, b) and Figs. 9(a, b), displays consistent trends for stationarity, cavity mode dominance, and oscillation amplitude with respect to the cavity floor positions. The shape of the stationary pressure trace at $x = 7.66D$, Fig. 9(a) is very similar to the one at the upstream location $x = 5.66D$, Figs. 8(a), and is attenuated in amplitude. Low pressure peaks separate constant pressure plateaus, where $p > p_\infty$. The low pressure peaks signal the passage of the ejected fluid from the cavity. Clockwise vorticity, normal to the (x, y) plane, is generated by shear layer interaction with the free stream during ejection. The low pressure in the convected vortex cores of the ejected flow is recorded as pressure minima in Fig. 8(a) and Fig. 9(a).

CONCLUDING REMARKS

On the basis of the evidence available, conclusions are drawn. These await to be elaborated further on the basis of the work in progress that includes Fourier analysis.

The model flow reproduced salient time-dependent physics of a shallow open cavity tested at Mach 1.5.

Details of the shear layer motion, the moving shocks, and pressure wave system propagating to the far-field were presented.

At the selected flow conditions, a self-sustained instability is predicted. The nature of such unsteadiness appears to be mainly flow-resonant. Different levels of resolution of the acoustic field in the enclosure alter the fundamental mode frequency but have only secondary effects on the predicted instability amplitude.

The model pressure at the cavity walls indicates that a stationary regime is reached within few oscillations from the start of the simulation. Self-similar pressure oscillations show that the flow instability is driven by dominant modes. There is scope to investigate feed-back control techniques for cavity instability suppression, optimized in the narrow frequency band of the leading modes.

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